Modeling Spin-Precessing Neutron Star Binaries: A Generalization of Black Hole Binary Frameworks in `Precession`
2026 LIGO SURF Proposal

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(Dated: May 15, 2026)

I. INTRODUCTION

A current challenge the LIGO/Virgo/KAGRA (LVK) collaboration faces is determining the nature of sources detected with gravitational-waves (GWs). When LVK detects GWs it requires predictions of black hole (BH) and neutron star (NS) binary waveforms to extract a GW signal against background noise – a method called matched filtering [1]. In order to accurately distinguish signal from background via matched filtering, we require highly accurate theoretical waveform templates, which in turn allow us to infer accurate descriptions of the source and its properties. By improving waveform templates, LVK can make the most of GW detections. In order to do this, it is imperative to model the evolution of these binaries from their initial orbits down to the inspiral phase where LIGO can actually detect the emitted GWs.

More interestingly, we are unsure of how compact binary coalescence (CBC) systems actually form. Through accurate evolutions of CBCs we can match formation models with observations, allowing us to more accurately extract information from LVK detections, and constrain formation channels for CBCs.

While our understanding of binary black holes (BBHs) and their orbital evolution under different initial conditions has improved significantly, we remain in need of better models of NS binaries. This is primarily due to the highly complicated, and still not understood, equation of state of NSs. In this project, we aim to build on the work presented in Ref. [2] by creating accurate and computationally cost-effective models of NS binaries.

II. BACKGROUND

A. Gravitational Waves and Compact Binary Coalescence

Gravitational waves, as predicted by Albert Einstein's theory of General Relativity, are ripples in spacetime produced by massive objects accelerating through the fabric of spacetime [3]. The faster and more massive the object, the more pronounced the ripples – analogous to a motorboat creating larger waves than a duck moving across a pond. GWs, though, are quite hard to detect, despite distorting space they have no noticeable effect on Earth. Thus, to detect GWs we focus our attention to the most massive and relativistic sources – compact binaries and compact binary coalescence. Compact binaries are binary systems consisting of BHs and/or NSs. Initially, the compact objects orbit each other at high speeds, emitting weak GWs that reduce the total angular momentum and orbital energy of the system. Due to this loss of angular momentum and energy, their orbital separation decreases as their orbital velocities increase, eventually initiating the process of compact binary coalescence (CBC). In Figure 1 the evolution of CBC is shown compared to the predicted strain on a detector. For this project we will focus our attention on the inspiral phase of the CBC as that is where the key post-Newtonian (PN) approximations we will use to model the inspiral phase can safely be applied.

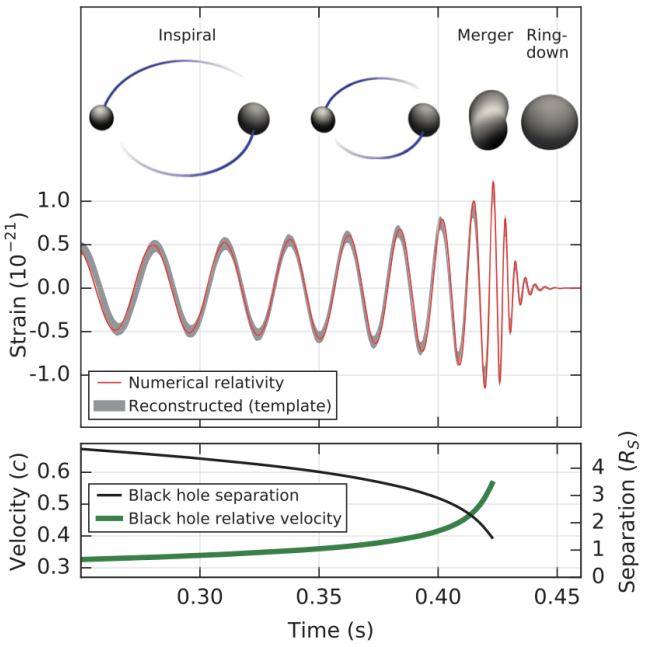


FIG. 1. The predicted strain detected by LIGO from GW150914 – the first detected GW – over inspiral to merger to ring-down as a function of time. Also displayed is the velocity (v/c) compared to the orbital separation [4].

B. Spin-Induced Precession

Compact objects, such as BHs and NSs, can spin, and when in binary systems, the presence of spin can affect the orbit and spin of the other object due to spin-orbit

and spin-spin coupling. These effects, predicted by General Relativity, affect the emitted gravitational signal as first studied by Apostolatos *et al.*. In the absence of precession, the predicted waveform is quite uniform, slowly gaining amplitude overtime due to the faster and faster inspiral as the objects close in on each other. The reason for this uniformity is due to the dependence of the waveform's amplitude on the geometry of the orbit relative to the detector – as described by Equation 7a in Ref. [5]. However, in a precessing binary the wobbling of the orbital plane creates a modulating amplitude over time due to the orbital angular momentum vector's time dependence – Equation 19a in Ref. [5]. This modulation of the waveform over time makes tracking the effect of precession incredibly important for matched filtering. A comparison of non-precessing and precessing waveforms is shown in Figure 2.

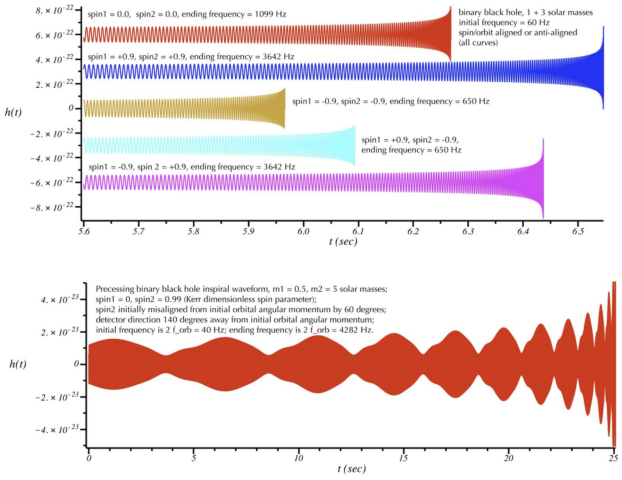


FIG. 2. *Top:* Waveforms of different non-precessing binaries for a range of inital conditions. Note all spins are equal in magnitude and aligned or anti-aligned. The waveform is a plot of predicted detector strain, $h(t)$, over time. *Bottom:* Waveform for a precessing binary showing strain vs. time. Notice how "envelopes" grow and shrink over time but the overall amplitude of the waveform grows as the binary inspirals to merger, this is the direct effect of precession and its modulation of the waveform. Also note the misalignment of the second spin vector with the orbital angular momentum vector by 60^o, hence the induced precession [6].

Precession occurs due to the geometry of the spin and momentum vectors in the binary. Let the spin vectors of compact objects 1 and 2 be $\mathbf{S_1}$ and $\mathbf{S_2}$, respectively, and the system's total spin, $\mathbf{S}$, be defined as $\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$. When $\mathbf{S}$ is not aligned with the vector normal to the orbital plane – the orbital angular momentum vector $\mathbf{L}$ – the system will precess. This precession occurs about the systems total angular momentum vector, $\mathbf{J}$, defined as $\mathbf{J} = \mathbf{S} + \mathbf{L}$. A useful graphic showing these vectors can be found in Figure 3. In summary, due to spin-orbit vector misalignment, precession can occur which has a profound impact on the waveform, making it an important property to understand.

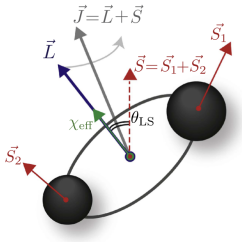


FIG. 3. From Ref. [7] this graphic gives a clear look at the different angular momentum vectors in a spinning binary system as described in Sec. II B. along with $\chi_{\rm eff}$ and θ_{LS} – the effective spin and the spin-orbit misalignment angle, respectively.

C. Modeling Binary Black Hole Orbits

In order to evolve any CBC, we need to account for the effects of General Relativity. While solving Einstein's equations with numerical relativity is the correct and most accurate way to do this, in practice it is incredibly computationally expensive and time-consuming to do so. Instead, we turn to PN approximations to Einstein's equations to model orbital motion, which are accurate to large enough orbital separations, and small enough velocities ($v/c \ll 1$) that for our purposes we can avoid the use of costly super-computing. As worked out in Ref. [5], we can use PN spin precession equations to simplify calculations and model the evolution of these systems efficiently. Luckily for us, these approximations are "unreasonably effective" as Will puts it.

Post-Newtonian approximations alone, though, do not get us far enough. While a faster problem to solve than through numerical relativity, the PN approximation leaves us with a multi-dimensional problem to solve that can still be made far less complex – to do so, a hierarchal timescale analysis can be used. Gerosa *et al.* lay this out clearly in the first paper outlining the `Precession` code used to evolve BBHs, which we will build on for NS binaries. The timescale hierarchy is $t_{\rm orb} \ll t_{\rm pre} \ll t_{\rm rad}$: the objects' orbital periods are much shorter than the precessional period of the orbital plane, which is much shorter than the duration it takes for the orbital separation to decay due to gravitational radiation by a significant amount.

Gerosa *et al.* improved the methods used in Ref. [9] for evolving spinning BBHs by implementing orbit and precession-averaging to the BBH dynamics. This leaves us with a high-dimensional problem describing the orientations and magnitudes of the spin and angular momenta vectors. This problem relies on seven variables: the mass ratio q, the spin magnitudes χ_1, and χ_2,

compacted orbital separation and asymptotic angular momentum terms u and κ, the effective spin $\chi_{\rm eff}$, and the weighted spin difference $\delta\chi$ [2]. On the precessional timescale, five of these are constant: q, χ_1, χ_2, , u, and κ. Due to some special properties of BH spin precession equations – as discussed in Ref. [10] – we also find $\chi_{\rm eff}$ to be a constant of motion on the precessional timescale [2], leaving us with one changing variable on the relevant timescale – $\delta\chi$. In identifying these constants of motion, the previously 7-dimensional problem has been reduced to a 1-dimensional one. Now, with this much simpler analytical treatment of the precessional dynamics, we can average the precessional behavior over a precession cycle, $t_{\rm prec}$. With this average, Equations 61 and 62 in Ref. [2] – which store all the needed information about the binary, such as spin directions – allow us to integrate from some initial orbital separation to a final separation. This integration is of a relatively simple ordinary differential equation – a differential equation dependent on one variable, making it easy to work with and compute. By integrating Equation 61 of Ref. [2], the spin-orbit configuration encoded in Equation 14 of Ref. [2] is determined as a function of orbital separation, thus giving us a full evolution of the momentum vector orientations from arbitrarily large separations to separations close-to-merger, where the PN approximation breaks down.

As described, all of this work with BBHs relies on a key constant of motion in the spin precession equations at 2PN, $\chi_{\rm eff}$. When working with NSs, $\chi_{\rm eff}$ is not constant on the precessional timescale [11], thus the methods used in Ref. [2] to evolve BBH spins are invalid for NSs. While in the BH case we can solve a singular equation analytically, in the NS case we are tasked with solving a system of coupled equations numerically, many times, to precession-average (and thus evolve) their orbits. In our project, we will utilize new spin precession equations generalized to non-BH objects to evolve NS binaries, with the final goal of extending `Precession` to work with NS binaries.

III. OBJECTIVES

Our overall objective is to add to `Precession` a more general model for the evolution of angular momentum vectors in CBCs, namely in NS binaries. In doing so we seek to understand how spin directions evolve and see how they differ from the BH case. To do this, we must first extend the BH precession-averaging approach used in Ref. [2] to NSs. This will involve a few technical challenges described in Section IV. With a way to precession-average NS binaries, we will implement our findings in a functioning framework and validate our developments against real events. With this, we aim to improve future template matching and contribute to future understanding of CBC formation systems through accurate spin evolution models.

IV. APPROACH

In order to recreate the double-averaging approach that Gerosa *et al.* uses, and ultimately expand upon `Precession`, we will take the following steps:

- *Literature review and theoretical setup*: We will study the existing literature on spin precession in compact binaries, focusing on identifying the key differences between BH and NS systems. In the NS case, additional physical effects lead to more complex, highly non-linear and non-polynomial equations, in contrast to the more analytically tractable BH case. This step includes learning how to model the BH case with the current `Precession` code.
- *Implementation of equations and root-finding*: Starting from a preliminary extension of the `Precession` code, we will develop a robust and efficient method to compute precession boundaries, which require solving non-trivial root-finding problems. This will be the central technical challenge of this project. We will use and adapt numerical methods from scipy.optimize and other python-based methods, explore performance improvements using `JAX`, and leverage insights from the BH case to improve initialization and convergence.
- *Code development*: We will integrate these techniques in a new version of `Precession` extending it to deal with generic CBC spin-precession.
- *Testing and validation*: The code will be tested across a range of physical parameters, including masses, spin magnitudes, orbital separations, and NS equations of state. We will validate the implementation by comparing appropriate limits with known BH results.
- *Application to real events*: Finally, we will apply the developed framework to GW observations of NS binaries, with the goal of reconstructing their spin evolution and connecting measurable quantities, such as spin directions measured by LIGO, to their value at the formation of the binary.

V. WORK PLAN

Dates	Phase
June 16 - 19	Introduction
June 22 - July 3	Background, 1st Interim Report
July 6 - July 17	Neutron Stars
July 20 - July 31	Fix Code, 2nd Interim Report
August 3 - 14	Fix Code, Use Code
August 17 - August 21	Final presentation and report

TABLE I. Tentative outline for the project for the summer. The phases are described in more detail in the Work Plan section of this proposal.

1 **Introduction**: Get acclimated to Caltech and the SURF program.

2 **Background**: Finish up familiarizing with `Precession`: review the tutorial, try some inspiral simulations, and attempt to back or forward propagate some posteriors.

3 **Neutron Stars**: Read about NSs and spin-induced quadrupole moments, understand the difference between the BH and NS cases, especially how equations change.

4 **Fix Code**: Start examining the code and identify the problematic points, such as finding the roots, performing quick integrations, and setting boundary conditions.

5 **Use Code**: Perform back and forward propagations of true events (i.e., using true posteriors we already have).

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