

LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
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Cavity-Enhanced Sum Frequency Generation For 2 μm Interferometer Readout		
Final Report		
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1 Introduction

The Laser Interferometer Gravitational-Wave Observatory (LIGO) is one of the most ambitious experiments in modern physics. In 1916, Einstein predicted the existence of gravitational waves as a consequence of General Relativity. More than a hundred years later, LIGO achieved the first direct detection of gravitational waves using laser interferometry precise enough to measure changes in length smaller than one thousandth of the size of a proton.

The performance of gravitational-wave detectors is affected by several fundamental noise sources. Among these, thermal noise is a major limitation especially in the mid-frequency region. The proposed LIGO Voyager upgrade aims to reduce these effects using cryogenically cooled silicon test masses. However, silicon strongly absorbs at the currently used 1064 nm wavelength, making a shift to longer wavelengths necessary. The next generation detectors are therefore exploring interferometry near 2050 nm, where silicon is nearly transparent. The challenge then shifts to photodetection, because 2 micron detection still falls short of the extremely low-loss requirements of GW detectors.

Sum frequency generation (SFG) offers a solution. In a nonlinear crystal, two input fields at frequencies ω_1 and ω_2 can interact to produce a third field at $\omega_3 = \omega_1 + \omega_2$, provided the energy conservation and phase-matching conditions are met. By mixing the 2050 nm signal light with a strong 1064 nm pump in a periodically poled lithium niobate (PPLN) crystal, the signal can be upconverted to ~ 700 nm where silicon photodetectors readily achieve quantum efficiencies exceeding 99%.

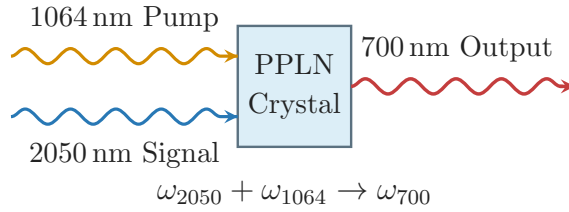


Figure 1: Schematic of upconversion of light via sum-frequency generation.

Sufficiently high optical intensities are required for efficient upconversion inside the crystal. One way to achieve this is through resonant cavity enhancement, in which the 1064 nm pump field can be enhanced inside a bow-tie cavity to build up large circulating power. This project focuses on the development of a cavity-enhanced SFG system for upconversion of weak 2050 nm light to 700 nm using a resonantly enhanced 1064 nm pump field.

Previous efforts within the group have demonstrated single-pass upconversion [1] and initial development of the bow-tie cavity [2]. Building on this, the next stage of this work involves locking and characterizing the bow-tie cavity, integrating the nonlinear crystal, mode matching the signal beam, and detecting the generated 700 nm output.

2 Overview

2.1 Sum Frequency Generation

When an electromagnetic wave propagates through a dielectric medium, the oscillating electric field induces a polarization in the material. For sufficiently weak optical fields, this induced polarization is proportional to the applied field,

$$\mathbf{P}(t) = \varepsilon_0 \chi \mathbf{E}(t), \quad (1)$$

where χ is the linear electric susceptibility. At high optical intensities, however, the polarization is no longer linear in the electric field. Instead, it can be expressed as a power series with coefficients $\chi^{(i)}$ as higher-order susceptibilities,

$$\mathbf{P}(t) = \varepsilon_0 [\chi^{(1)} \mathbf{E} + \chi^{(2)} \mathbf{E}^2 + \chi^{(3)} \mathbf{E}^3 + \dots]. \quad (2)$$

Let us consider an optical field consisting of two monochromatic waves,

$$E(t) = E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t} + \text{c.c.}, \quad (3)$$

where c.c. denotes the complex conjugate. Since the second-order polarization is proportional to E^2 , expanding the square produces four different frequency components,

$$2\omega_1, \quad 2\omega_2, \quad \omega_1 + \omega_2, \quad |\omega_1 - \omega_2|, \quad (4)$$

This is essentially the origin of phenomena such as second harmonic generation, sum frequency generation and difference frequency generation. The process of SFG can be viewed as two input photons combining to produce a single photon whose energy equals the sum of the input photon energies. Besides energy conservation, the conservation of momentum also needs to be satisfied, which leads to the phase matching condition. Thus for SFG, we have

$$\hbar\omega_3 = \hbar\omega_1 + \hbar\omega_2 \quad (5)$$

$$\mathbf{k}_3 = \mathbf{k}_1 + \mathbf{k}_2 \quad (6)$$

A pump field which has much stronger amplitude than the probe field can be treated as constant along the crystal. Under this undepleted pump approximation, the intensity of generated SFG light rises linearly with pump intensity. A weak signal can therefore still convert efficiently if the pump intensity is high enough. This is the motivation for enhancing pump power resonantly in a cavity rather than relying on single-pass power.

2.2 Gaussian Beam Theory and ABCD formalism

Laser beams diffract as they propagate because they have a finite extent in the transverse direction. Under the paraxial wave approximation, the fundamental transverse electromagnetic mode (TEM₀₀) is described by a Gaussian intensity distribution,

$$I(r, z) = \frac{2P}{\pi w^2(z)} \exp\left[-\frac{2r^2}{w^2(z)}\right], \quad (7)$$

where P is the optical power and $w(z)$ is the $1/e^2$ beam radius at propagation distance z .

The beam radius follows a hyperbolic relation,

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}, \quad (8)$$

where w_0 is the beam waist and

$$z_R = \frac{\pi w_0^2}{\lambda} \quad (9)$$

is the Rayleigh range, which represents the propagation distance over which the beam radius increases by a factor of $\sqrt{2}$. Far from the waist, the beam diverges with a half-angle approximately given by

$$\theta \approx \frac{\lambda}{\pi w_0}. \quad (10)$$

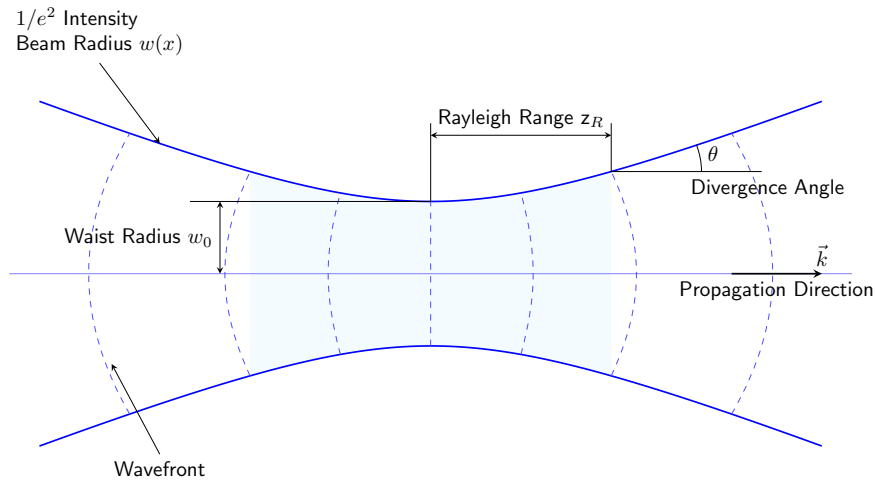


Figure 2: Gaussian beam profile showing the waist radius w_0 , Rayleigh range z_R , wavefront curvature, and divergence angle θ .

A convenient description of Gaussian beam propagation is provided by the complex beam parameter

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \frac{\lambda}{\pi w^2(z)}, \quad (11)$$

where $R(z)$ is the radius of curvature of the wavefront. The advantage of a Gaussian beam's q-parameter is that it can be propagated through arbitrary optical configurations by simple matrix multiplication using the ray transfer (ABCD) matrices,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}, \quad (12)$$

under which the beam parameter transforms according to

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}. \quad (13)$$

The ray-transfer matrices for the cases of our interest are shown in the table below. For a curved mirror, the effective radius of curvature R_e at angle of incidence θ is

$$R_e = R \cos \theta \quad \text{and} \quad R_e = R / \cos \theta, \quad (14)$$

for the tangential and sagittal planes respectively.

Table 1: ABCD matrices for common paraxial optical elements.

Optical Element	ABCD Matrix
Free-space propagation through L	$\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$
Thin lens with focal length f	$\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$
Curved mirror (arbitrary incidence)	$\begin{pmatrix} 1 & 0 \\ -2/R_e & 1 \end{pmatrix}$

This formalism provides a simple and effective method for calculating beam propagation through lenses, mirrors, free space and optical cavities. It also enables us to determine a cavity's stability parameter and the Gaussian eigenmode supported by the cavity.

2.3 Pound-Drever-Hall Locking

Pound–Drever–Hall (PDH) locking is a technique used to stabilize the frequency of a laser to a cavity. The phase of the laser is modulated at radio-frequency Ω using an electro-optic modulator (EOM). The resulting field contains a carrier at ω and two sidebands at $\omega \pm \Omega$. The modulation frequency is chosen such that the sidebands are outside the cavity resonance while the carrier can couple to the cavity mode. This allows the reflected light to carry information about the frequency difference between the laser and the cavity.

The reflected beam from the cavity input is detected using a fast photodetector and demodulated at the same modulation frequency Ω . Mixing the PD signal with the modulation reference and applying a low-pass filter produces the PDH error signal. Near resonance, this signal changes sign as the laser frequency passes through the cavity resonance. The zero crossing therefore provides a frequency reference for the feedback loop. The steep slope around this crossing provides the frequency-discriminating signal required for locking.

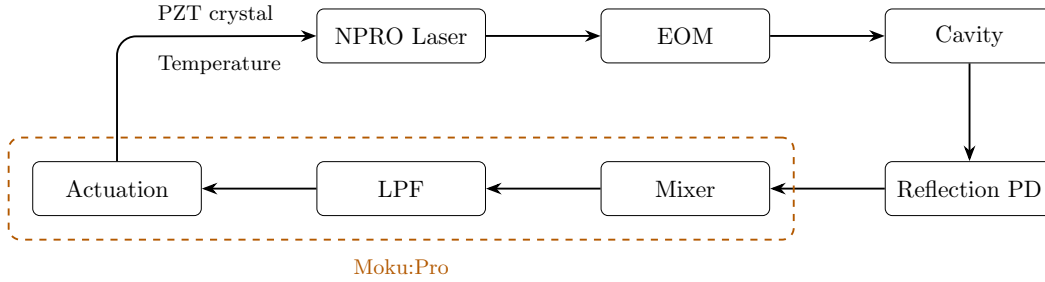


Figure 3: Block diagram of the PDH feedback loop used for cavity locking.

This error signal can be sent to a servo controller, which feeds back to an actuator on the laser or cavity. In our setup, we use two types of actuation on the laser; a fast control via the PZT attached to the NPRO crystal, and a slow control via the laser temperature. The feedback acts to reduce the error signal towards zero and hence suppresses frequency fluctuations. A successful lock is indicated by a stable error signal around the resonance.

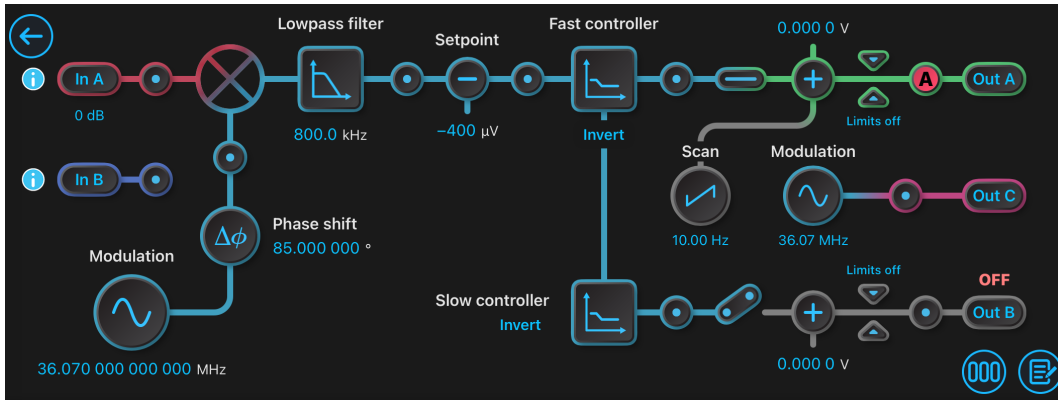


Figure 4: Moku:Pro configuration used for PDH control, showing the modulation source, low-pass filter, and fast and slow feedback paths.

3 Experimental Setup

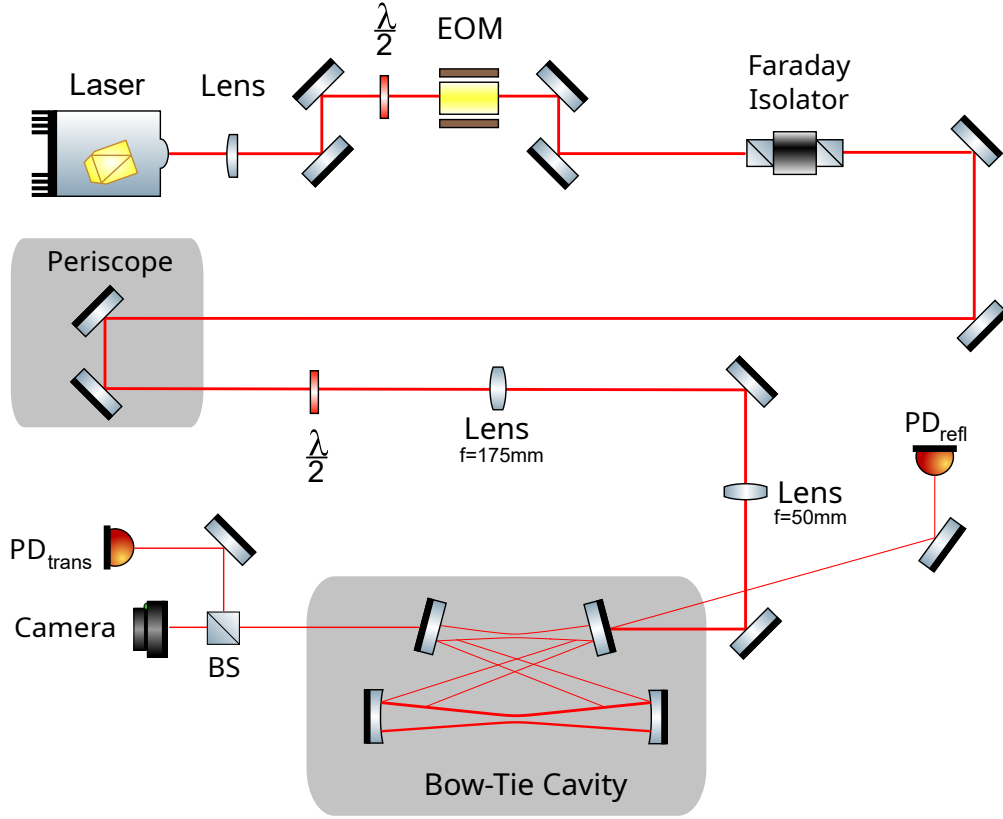


Figure 5: Schematic of the experimental setup for cavity-enhanced SFG. The 1064 nm pump beam is phase-modulated for PDH locking, mode-matched, and coupled into a bow-tie cavity.

The experimental setup used for cavity-enhanced sum-frequency generation experiment is shown above. The 1064 nm Innolight Mephisto NPRO laser provides the pump beam for the experiment. The beam first passes through a lens and an electro-optic modulator (EOM), where phase modulation sidebands at 36.07 MHz are generated for Pound–Drever–Hall (PDH) locking. A Faraday isolator in the path prevents back reflections from returning to the laser. The periscope is used to direct the beam inside a vacuum chamber to be pumped down later for reducing mechanical and acoustic noise. A two-lens telescope then mode-matches the beam to the Gaussian eigenmode of the bow-tie cavity.

Since the generated SFG power scales with the pump intensity, the pump beam is resonantly enhanced inside a bow-tie cavity to obtain a high power buildup and improve conversion efficiency. This bow-tie cavity consists of two flat mirrors and two curved mirrors arranged in an isosceles trapezium geometry. Light reflected from the cavity is directed onto an RF photodetector, where the PDH error signal is generated. The primary objective of this phase of the project is to optimize the mode matching, and acquire a stable PDH lock. Once the cavity is locked, the PPLN crystal can be introduced to achieve cavity-enhanced sum-frequency generation.

4 Progress & Approach

4.1 Bow-Tie Cavity Characterization

The bow-tie cavity is characterized by a few important parameters such as its finesse, free spectral range (FSR), stability, and spatial profile of the Gaussian eigenmode. In this section, these quantities are calculated for our cavity using the geometry and parameters of the four mirrors. The geometry of the cavity is shown in the figure below.

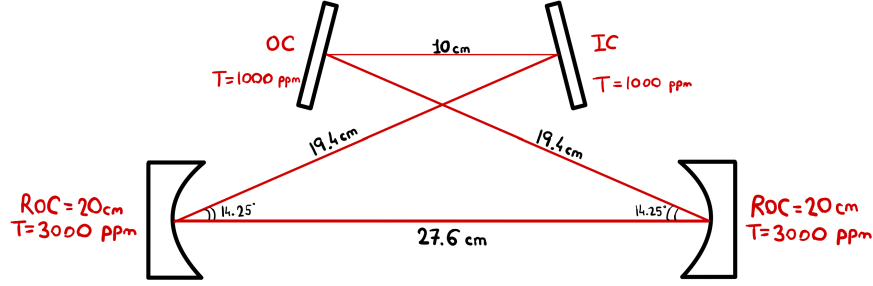


Figure 6: Bow-tie cavity schematic in an isosceles trapezium geometry.

4.2.1 Cavity Finesse

The finesse $\mathcal{F}$ is a measure of how well the cavity stores light. It relates to the average number of round trips a photon makes before leaving the cavity, and therefore determines the sharpness of the resonance peaks. Higher finesse leads to more power enhancement in the cavity, but the linewidth becomes narrower making the resonance harder to maintain.

For a traveling-wave ring cavity, the finesse is determined by the input coupler amplitude reflectivity r_1 and the combined amplitude reflectivity of the remaining components r_{rt} as

$$\mathcal{F} = \frac{\pi \sqrt{r_1 r_{rt}}}{1 - r_1 r_{rt}} \quad (15)$$

For the present bow-tie cavity, using the mirror transmissions (1000, 1000, 3000, and 3000 ppm) and assuming negligible additional round-trip losses, the calculated finesse is

$$\boxed{\mathcal{F} = 783.}$$

In practice, each mirror in the cavity can introduce loss from absorption and scattering. This changes the total round-trip loss and leads to a reduced finesse. Figure 7 shows the calculated variation of finesse with additional round-trip loss.

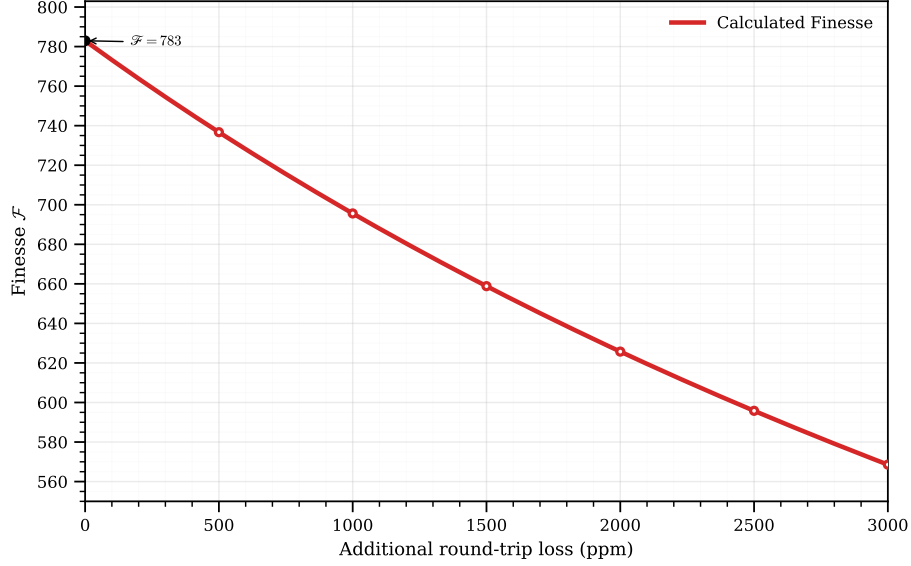


Figure 7: Finesse of the bow-tie cavity versus additional round-trip loss. It falls from an ideal value of $\mathcal{F} = 782.94$ to $\mathcal{F} = 625$ at 2000 ppm loss.

4.2.2 Power Buildup

The power enhancement of a resonant cavity depends on both the fraction of the input light entering the cavity determined by T_1 and the total round-trip loss R . If T_1 is too high, a higher fraction of incident light enters the cavity, but it leaks out just as fast. If T_1 is too low, light stays trapped inside for a long time, but most of the incident light is promptly rejected. To maximize the power held inside, we optimize T_1 with respect to the rest of the round-trip losses. If these two quantities are exactly equal, the cavity is said to be critically coupled. Under this condition the field leaking back through the input coupler destructively interferes with the promptly reflected field giving zero reflected power on resonance.

For our bow-tie cavity, because $T_1 = 1000 \text{ ppm} < 7000 \text{ ppm}$, the cavity is in the under-coupled regime. This does not prevent cavity resonance, but it does result in reduced circulating power and a decrease in amplitude of the PDH error signal used for locking. At this configuration, the cavity has a power enhancement of approximately 63 and reflects roughly 42% of the incident field on resonance.

The plot in figure 8 shows the calculated power enhancement and reflected power as functions of input coupler transmission. The peak in the circulating power coincides with minima in reflected power at critical coupling condition.

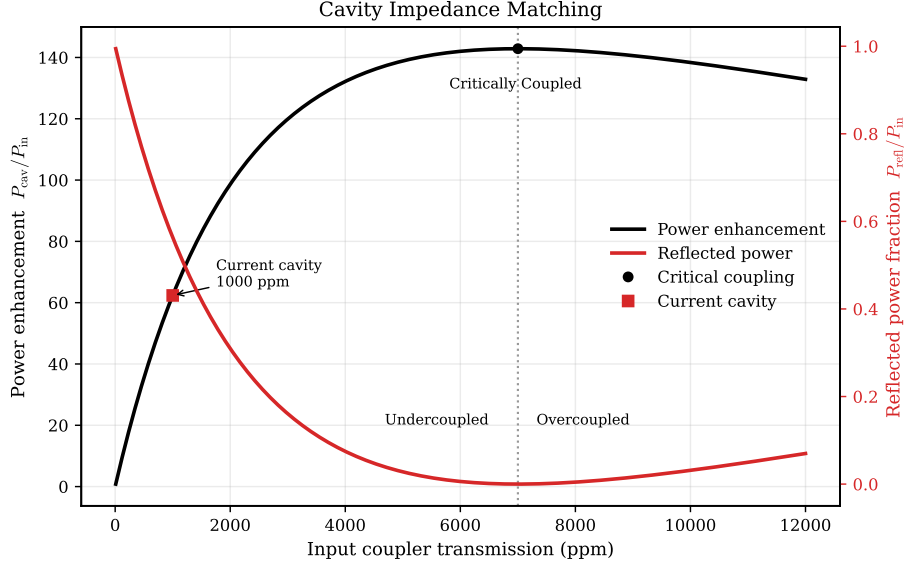


Figure 8: Intra-cavity power enhancement factor and reflected power fraction versus input coupler transmission. Current cavity shown by the red marker is in the undercoupled regime.

4.2.3 Cavity Stability

Cavity stability determines whether a Gaussian beam remains confined within the resonator after repeated round trips. For a two-mirror Fabry–Pérot cavity, the stability criterion is commonly expressed in terms of the mirror g -parameters, $g_i = 1 - L/R_i$, where L is the cavity length and R_i is the radius of curvature of the mirrors. The cavity is stable when $0 < g_1 g_2 < 1$.

For a bow-tie cavity however, this is no longer directly applicable because the beam follows a folded optical path between the curved and flat mirrors, rather than propagating back and forth only between the curved mirrors. We therefore use the more general stability criterion for optical resonators given by the diagonal matrix elements of the round-trip ABCD matrix. For the present cavity geometry, the stability parameter is calculated as,

$$\left| \frac{A + D}{2} \right| = 0.0944 < 1,$$

which satisfies the stability criterion. The cavity is therefore geometrically stable and supports a well-defined Gaussian eigenmode.

4.2.4 Free Spectral Range and Linewidth

The free spectral range (FSR) is the frequency spacing between two successive resonances of the cavity. It is determined simply by the optical round-trip length L of the resonator and is given by $FSR = c/L$, where c is the speed of light. Using our measured round-trip length of 76.4 cm,

$$FSR = 392.4 \text{ MHz}. \quad (16)$$

Using the calculated finesse of $\mathcal{F} = 782.94$, the cavity resonance linewidth (full width at half maximum) is obtained from

$$\delta\nu = \frac{\text{FSR}}{\mathcal{F}} = 0.501 \text{ MHz}, \quad (17)$$

Figure 9 shows the calculated cavity transmission spectrum over several free spectral ranges.

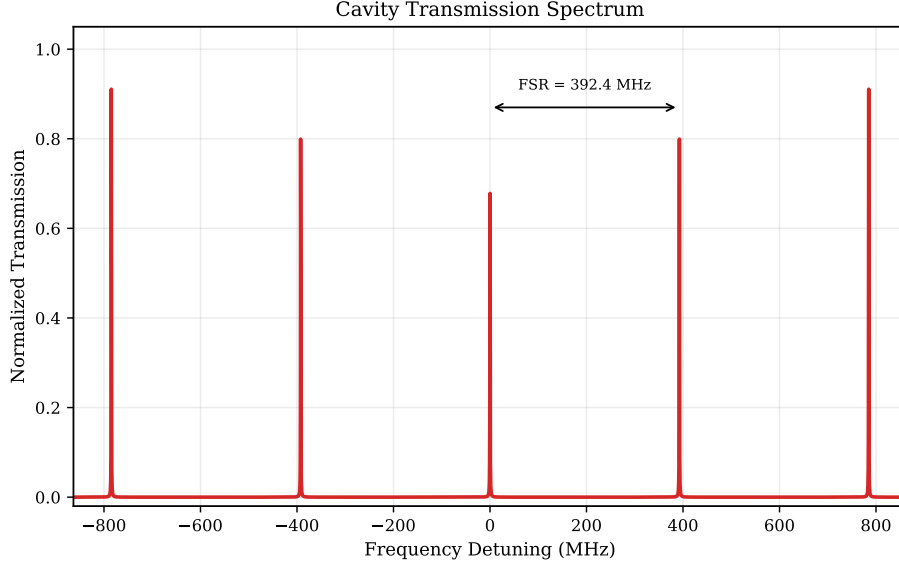


Figure 9: Calculated transmission spectrum of the bow-tie cavity. Adjacent longitudinal resonances are separated by the free spectral range of 392.4 MHz and calculated linewidth for each resonance is approximately 0.5 MHz.

4.2.5 Beam Profile of Spatial Eigenmode

The cavity eigenmode is obtained by requiring that the complex Gaussian beam parameter remains unchanged after one complete round trip through the resonator. If the round-trip propagation is represented by the ABCD matrix

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix},$$

then the eigenmode satisfies the self-consistency condition

$$q = \frac{Aq + B}{Cq + D}.$$

Solving this equation yields the complex beam parameter q . The cavity eigenmode was calculated as:

$$q_{\text{cav}} = -1.978 + i 5.117 \text{ in.} \quad (18)$$

The real and imaginary parts of q give the waist location and Rayleigh range, from which the beam radius along the cavity was calculated using the Gaussian beam propagation.

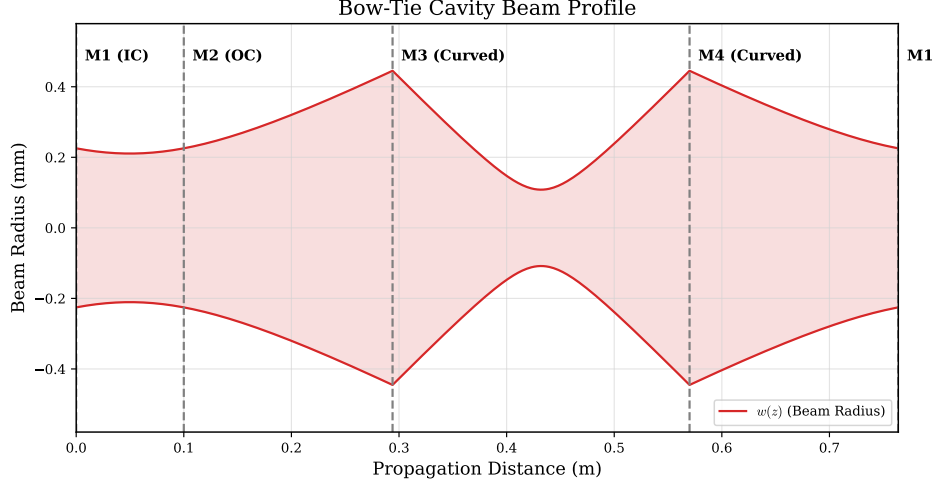


Figure 10: Calculated $1/e^2$ beam radius $w(z)$ along propagation distance inside cavity. The gray vertical lines mark the location of flat mirrors (M_1, M_2) and curved mirrors (M_3, M_4).

The plot above shows the beam radius for the eigenmode along one complete round trip. The calculated beam waist is $105\mu\text{m}$. This eigenmode is used as the target beam for determining the mode-matching efficiency presented in the following section.

The properties of our bow-tie cavity are summarized in the table below.

Table 2: Cavity Parameters Summary

Parameter	Value
<i>Design Parameters</i>	
Round-Trip Length	76.40 cm
Radius of Curvature (Curved Mirrors)	20 cm
Angle of Incidence	14.25°
Input Coupler Transmission	1000 ppm
Round-Trip Loss	7000 ppm
<i>Calculated Parameters</i>	
Finesse	782.94
Free Spectral Range	392.4 MHz
Resonance Linewidth (FWHM)	0.501 MHz
Power Enhancement	~ 63
Stability Parameter	0.0944
Beam Waist	$105.3\mu\text{m}$
Eigenmode Q-parameter at Input Coupler	$0.05 + 0.130i$ m

4.2 Mode-matching Efficiency

Proper spatial mode-matching between the incident laser beam and the eigenmode of the cavity is necessary for efficient coupling into the cavity. In practice, achieving perfect 100% mode matching is quite difficult due to alignment errors. Generally, a mode-matching efficiency above 95% is considered good and values exceeding 99% indicate excellent mode matching. Higher mismatch reduces the amount of power coupled into the fundamental mode and causes reflection and excitation of higher order transverse modes.

Initial profiling of the beam which was done before the telescope showed that the beam waist was too close to the EOM, due to which sufficient data could not be obtained on both sides of the waist. We instead placed a pickoff mirror right after the lens telescope. This gave a direct measurement of the beam that was coupled into the cavity which included the effect of all errors introduced upstream by the telescope and other optics. We recorded 23 beam radius measurements at 1-inch intervals. The standard Gaussian beam propagation model was used to fit the data as a function of propagation distance:

$$w(z) = w_0 \sqrt{1 + \left(\frac{z - z_0}{z_R} \right)^2}, \quad (19)$$

where w_0 is the beam waist, z_0 is the waist location relative to the profiling reference, and z_R is the Rayleigh range.

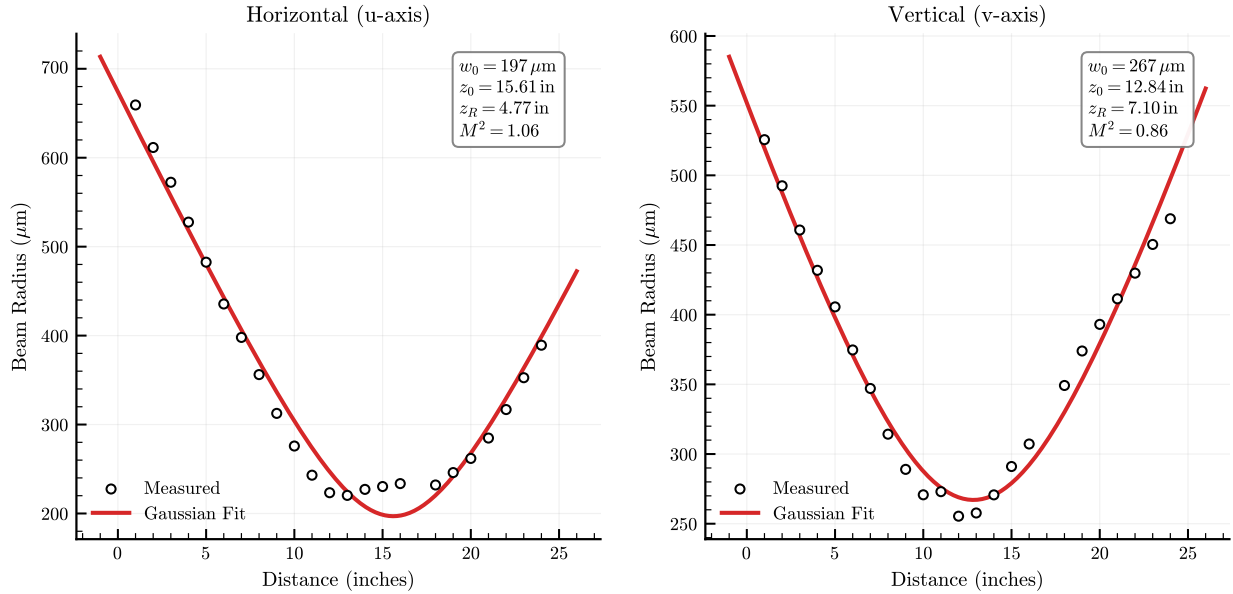


Figure 11: Beam profile along both axes measured after the mode-matching telescope, with Gaussian beam propagation fit used to determine beam parameters.

The values of beam waist at $z_0 = 15.60$ inches and Rayleigh range $z_R = 4.76$ inches from the fit determine the complex beam parameter at the pickoff mirror. To find the beam parameter at cavity input, we propagate it through the remaining distances using translation matrices.

Resulting beam parameter at the cavity input was found to be

$$q_{\text{beam}} = -1.645 + i 4.769 \text{ in.} \quad (20)$$

From Table 2, the calculated cavity eigenmode at the input coupler is known as:

$$q_{\text{cav}} = -1.978 + i 5.117 \text{ in.} \quad (21)$$

The overlap efficiency between measured beam and cavity eigenmode can be calculated as

$$\eta = \frac{4 \text{Im}(q_1) \text{Im}(q_2)}{|q_1^* - q_2|^2}, \quad (22)$$

where q_1 and q_2 are the complex beam parameters of the incident beam and cavity eigenmode, respectively. Substituting our measured and theoretical q -parameters into this relationship, the calculated overlap efficiency comes out as

$$\boxed{\eta = 0.99762} \quad (23)$$

which corresponds to an exceptionally low mode mismatch of only 0.23%. This indicates that the beam delivered to the cavity is exceptionally well matched to the cavity eigenmode. Even if we do the same calculation using the less reliable vertical axis fit, its mismatch comes out as $\sim 8.4\%$. Thus, mode mismatch is unlikely to be the limiting factor behind our weak PDH error signal.

4.3 PDH Signal Investigation

4.3.1 Expected PDH Error Signal

In this section, we model the expected theoretical Pound-Drever-Hall (PDH) error signal for our setup in which the phase modulation frequency of $\Omega = 36.07$ MHz is used.

To find the modulation index (β) for our setup, we used the phase modulation specification from the EOM datasheet, which states a modulation depth of 15 mrad/V at a wavelength of 1000 nm. Adjusting this value for our laser wavelength of 1064 nm gives a phase modulation coefficient of:

$$K_\phi = 15 \text{ mrad/V} \times \left(\frac{1000 \text{ nm}}{1064 \text{ nm}} \right) = 14.1 \text{ mrad/V} \quad (24)$$

During our runs, the Moku:Pro lock-in amplifier drove the EOM driver with an RF amplitude of 190 mV_{pp}. Given the driver's voltage gain of 130, the peak voltage applied across the electro-optic crystal is calculated as:

$$V_{\text{peak}} = \frac{190 \text{ mV}}{2} \times 130 = 12.35 \text{ V} \quad (25)$$

Multiplying the modulation coefficient by this peak driving voltage gives the dimensionless modulation index:

$$\beta = K_\phi \times V_{\text{peak}} \approx 0.174 \quad (26)$$

The responsivity and transimpedance gain for our photodetector are known from the datasheet and used to convert the optical power to a voltage scale. The list of parameters used for generating the expected signal curve is summarized in Table 3.

Table 3: Parameters used for Expected PDH Error Signal Calculation

Parameter	Value
Free Spectral Range (FSR)	392.4 MHz
Finesse ($\mathcal{F}$)	782.94
RF Modulation Frequency (Ω)	36.07 MHz
Phase Modulation Coefficient	14.1 mrad/V
EOM Driver Peak Voltage Output	12.35 V
Calculated Modulation Index (β)	0.174
Incident Laser Power (measured)	70 μ W
Photodetector Responsivity	0.75 A/W
Photodetector Transimpedance Gain	40 kV/A
Mixer Conversion Factor	0.5

The resulting expected error signal is plotted in Figure 12. The plot shows that we should ideally expect a maximum peak-to-peak error signal swing of roughly 180 mV.

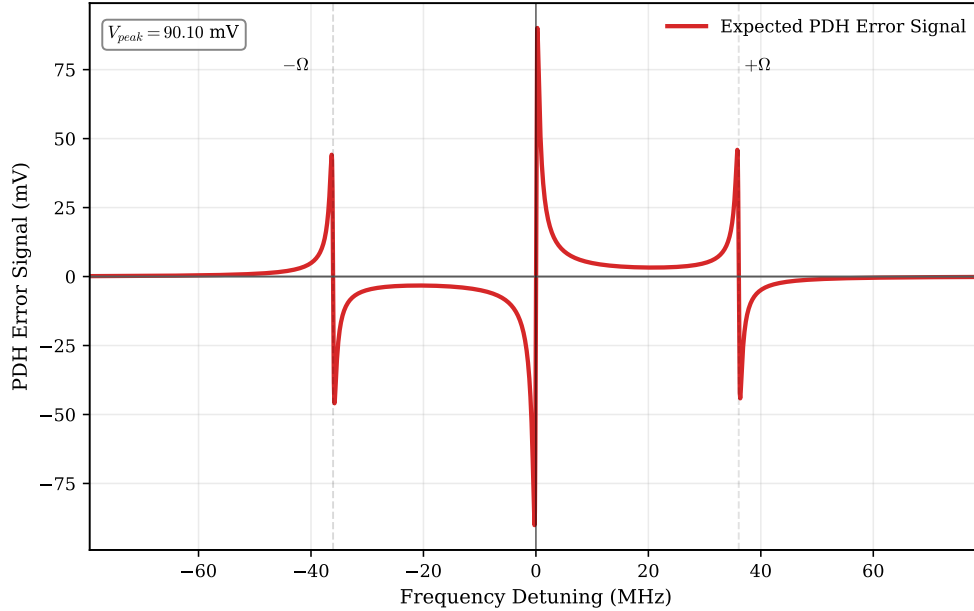


Figure 12: Expected PDH error signal modeled for our cavity configuration and EOM parameters. The plot shows a peak error signal voltage of $V_{peak} = 90.10 \text{ mV}$ with the modulation sidebands at $\pm 36.07 \text{ MHz}$.

4.3.2 EOM Characterization

Since the measured PDH error signal was much weaker than expected despite good mode matching, the RF modulation chain was investigated. We started by measuring the frequency response of the EOM driver using the swept-sine method. The spectrum confirmed that the EOM driver was resonant at the modulation frequency of 36 MHz.

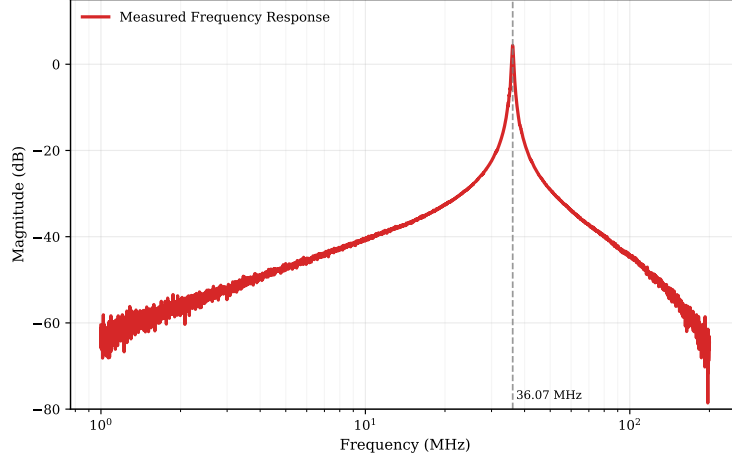
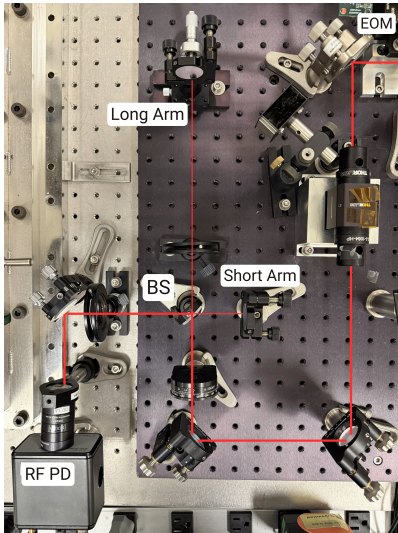
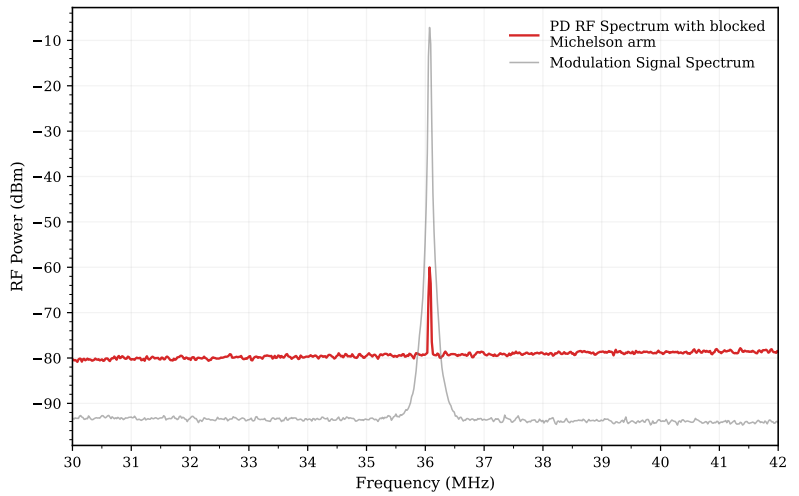


Figure 13: Measured frequency response of the EOM driver using the swept sine method.

Next, to verify that the EOM was producing the required phase modulation, we assembled an asymmetric Michelson interferometer along the beam path. The path length difference between the two arms converts the optical phase modulation into measurable intensity modulation. A picture of the setup is shown below. The photodetector output was analysed using the Moku Spectrum Analyzer.



(a) Interferometer setup



(b) RF spectrum with one arm blocked.

Figure 14: (a) Schematic of the asymmetrical Michelson Interferometer setup, and (b) RF PD spectrum measured with one arm of the interferometer blocked.

When one arm of the Michelson interferometer was blocked, the 36 MHz peak was still found to be present (Fig 14b) which suggested the presence of amplitude modulation via the EOM. This is possible if the incident beam into the EOM is not s-polarized to within ± 1 degree. We replaced the half-wave plate before the EOM with a polarizer and varied the axis of transmission. The plot below shows the spectrum obtained at p- and s- input polarizations of light into the EOM. The desired 36 MHz modulation in the RF spectrum became prominent while the component causing amplitude modulation was greatly reduced.

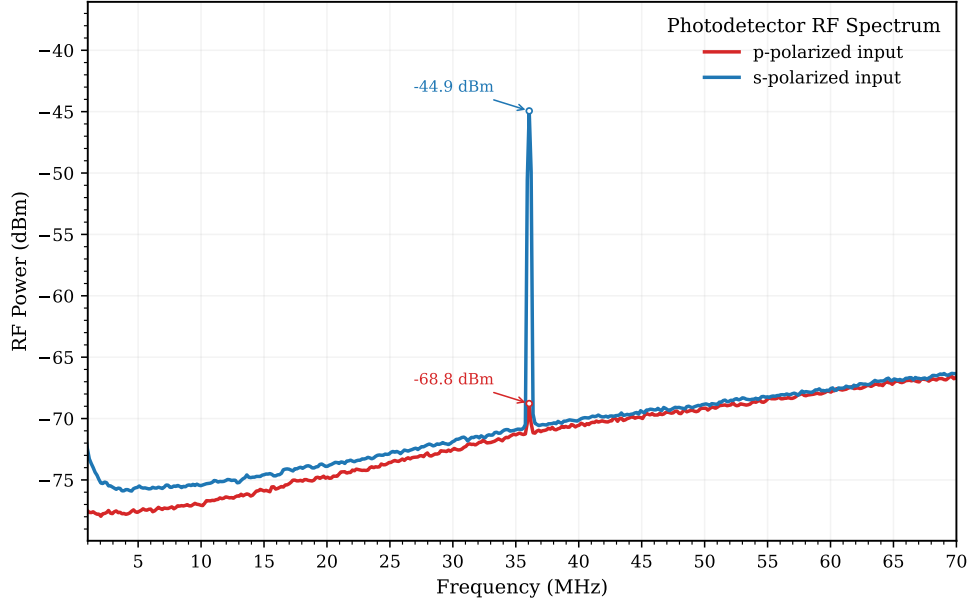


Figure 15: RF PD Spectrum comparison for p- and s- polarized transmission from the polarizer into the EOM.

The presence of the resonant peaks in these measurements showed that the EOM was generating modulation at the modulation frequency. This confirmed that the RF modulation chain was functioning as intended.

4.3.3 Observed PDH Error Signal

After verifying the mode-matching efficiency and the RF modulation chain, the cavity was aligned again and the PDH locking was attempted. The measured error signal is shown in Figure 16.

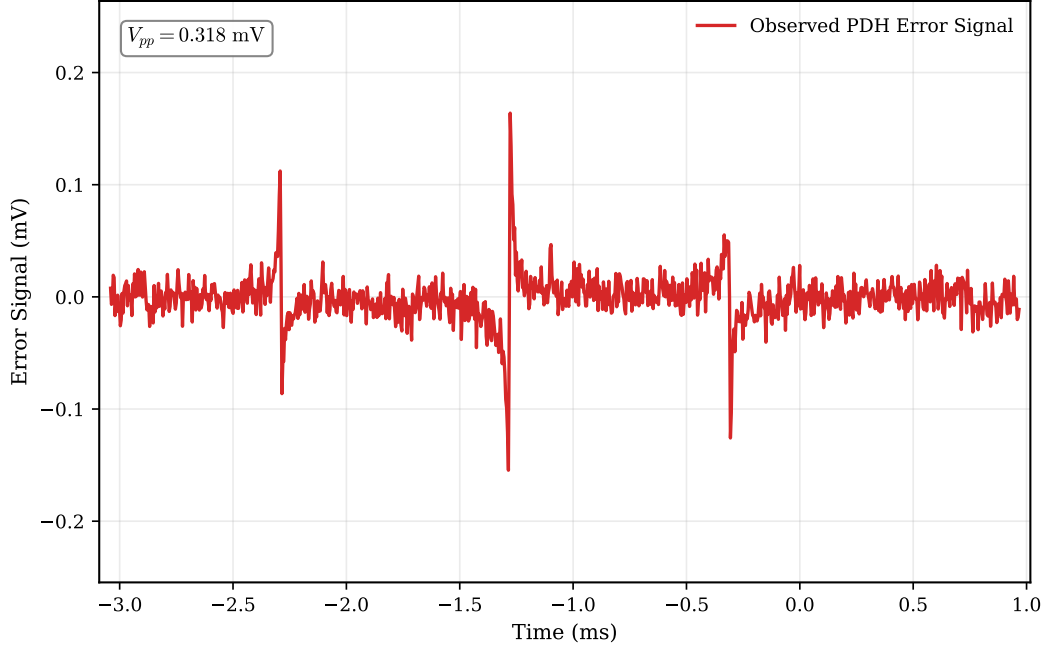


Figure 16: Measured PDH error signal. Peak-to-peak voltage of 0.318 mV is few orders of magnitude smaller than the expected signal of approximately 180 mV.

Although a clear PDH error signal was observed which confirmed that the cavity was resonating and that the modulation and demodulation chains were functioning correctly, the peak-to-peak amplitude was approximately 0.318 mV which is nearly three orders of magnitude smaller than the expected value of approximately 180 mV predicted in Section 4.3.1. Following this, the decision was made to replace the aging InnoLight Mephisto NPRO with the RIO laser and shift the experiment to a new setup.

4.4 NPRO Laser Investigation

The Innolight Mephisto 800NE NPRO used in the experiment is approximately 25 years old. Several observations indicative of the laser's degradation were made throughout the course of the project. This section summarizes three such indications.

4.4.1 Laser Spatial Mode

When the beam profile of the NPRO laser was first measured before the Faraday isolator, it was found to deviate significantly from an ideal Gaussian beam. The horizontal and vertical axes fits gave different waist locations differing by approximately 8 cm indicating astigmatism.

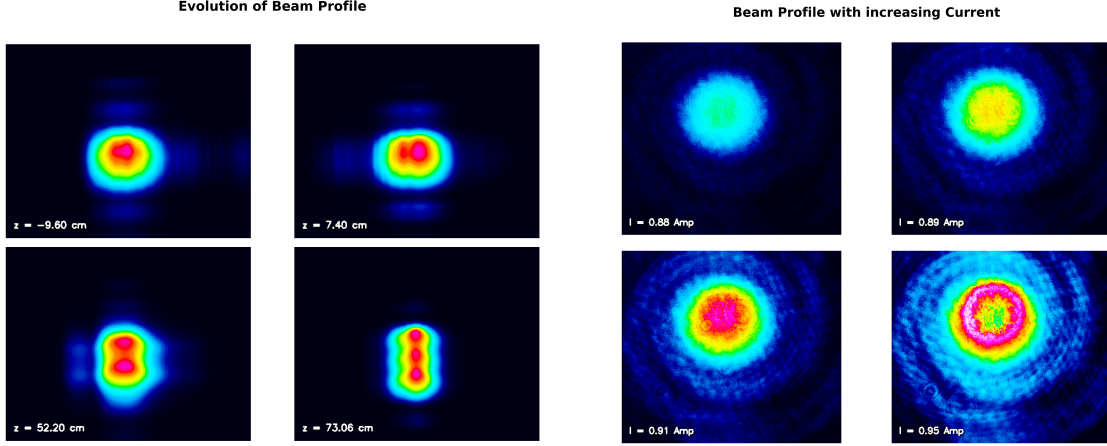


Figure 17: Representative beam profiles of the NPRO laser. Left grid shows non-Gaussian spatial features appearing at various distances along the path. On the right are beam profiles measured at different laser diode currents showing annular structures evolve at high currents.

More importantly, the transverse profile images appeared to contain higher-order spatial features such as lobes and annular discs. These distortions persisted despite adjustment in the alignment of upstream optics, laser operating temperature, and diode current, suggesting that they originated from the laser itself. Nevertheless, profiling of the beam immediately before the cavity showed that a mode-matching efficiency higher than 95% could still be achieved (section 4.2).

4.4.2 Polarization

As touched upon in Section 4.3.2, the RF spectrum measurement suggested presence of residual amplitude modulation (RAM). One possible cause was that the EOM was not being driven with purely *s*-polarized light. Throughout the experiment, the polarization entering the EOM had been adjusted using a half-wave plate (HWP) by rotating it to minimize the transmitted power through a polarizing beam splitter. However, this procedure assumes that the incident beam is linearly polarized. If the laser output itself is elliptically polarized, no rotation of the HWP alone can produce a purely *s*-polarized beam.

To investigate this, the HWP was removed and replaced with a linear polarizer. Power transmitted was measured while rotating the transmission axis of the polarizer through 360°. The maximum and minimum transmitted powers were measured to be approximately 20 mW and 5 mW respectively.

Assuming fully polarized light, the ellipticity can be estimated from the ratio of the electric field amplitudes along the major and minor polarization axes,

$$\frac{E_{\text{minor}}}{E_{\text{major}}} = \sqrt{\frac{P_{\text{min}}}{P_{\text{max}}}} = \sqrt{\frac{5}{20}} = 0.50. \quad (27)$$

This indicated that the laser output was not purely linearly polarized. We retained the linear polarizer in the path to ensure that only *s*-polarization entered the EOM, although this reduced the optical power available. In principle, a quarter-wave plate could be used to convert the elliptical polarization into a purely linear state without this loss.

4.4.3 Relaxation Oscillations

We had observed a strong and sharp noise peak near 200 kHz in the RF spectrum of the photodetector throughout the experiments. The peak was present during PDH locking attempt, Michelson interferometer measurements, and even when the EOM modulation was switched off. Since relaxation oscillations of diode-pumped solid-state lasers are expected in the few hundred kilohertz range, this suggested that the noise peak originated from the NPRO laser itself.

For a four-level laser such as Nd:YAG, the relaxation oscillation frequency is given by

$$f_{\text{RO}} = \frac{1}{2\pi} \sqrt{\frac{\rho(\sigma - 1)}{T_{\text{rt}}T_g} - \left(\frac{\sigma}{2T_g}\right)^2}, \quad (28)$$

where $\sigma = P_{\text{pump}}/P_{\text{th}}$ is the normalized pump parameter, T_{rt} is the cavity round-trip time, ρ is the fractional round-trip loss, and T_g is the upper-state lifetime. Using the cavity damping time,

$$\tau_c = \frac{T_{\text{rt}}}{\rho}, \quad (29)$$

and noting that $T_g \gg \tau_c$ for Nd:YAG lasers, the second term under the square root is negligible, giving

$$f_{\text{RO}} = \frac{1}{2\pi} \sqrt{\frac{\sigma - 1}{\tau_c T_g}}. \quad (30)$$

The output power above threshold satisfies

$$P_{\text{out}} \propto P_{\text{pump}} - P_{\text{th}}, \quad (31)$$

It follows that

$$\sigma - 1 = \frac{P_{\text{pump}}}{P_{\text{th}}} - 1 \propto P_{\text{out}}, \quad (32)$$

and therefore

$$\boxed{f_{\text{RO}} \propto \sqrt{P_{\text{out}}}} \quad (33)$$

To test this prediction, the RF spectrum of the photodetector was recorded while gradually varying the laser diode current. The frequency peak and its amplitude were extracted from the spectrum while the corresponding laser output power was also recorded for each operating current.

Figure 18 plots the results from this experiment. Frequency peak follows a closely linear dependence on $\sqrt{P}$ which supports the conclusion that the ~ 200 kHz noise peak originates from relaxation oscillations of the NPRO laser. The oscillation amplitude increases rapidly near threshold, but remains approximately constant over most of the range while the laser output power continues to increase.

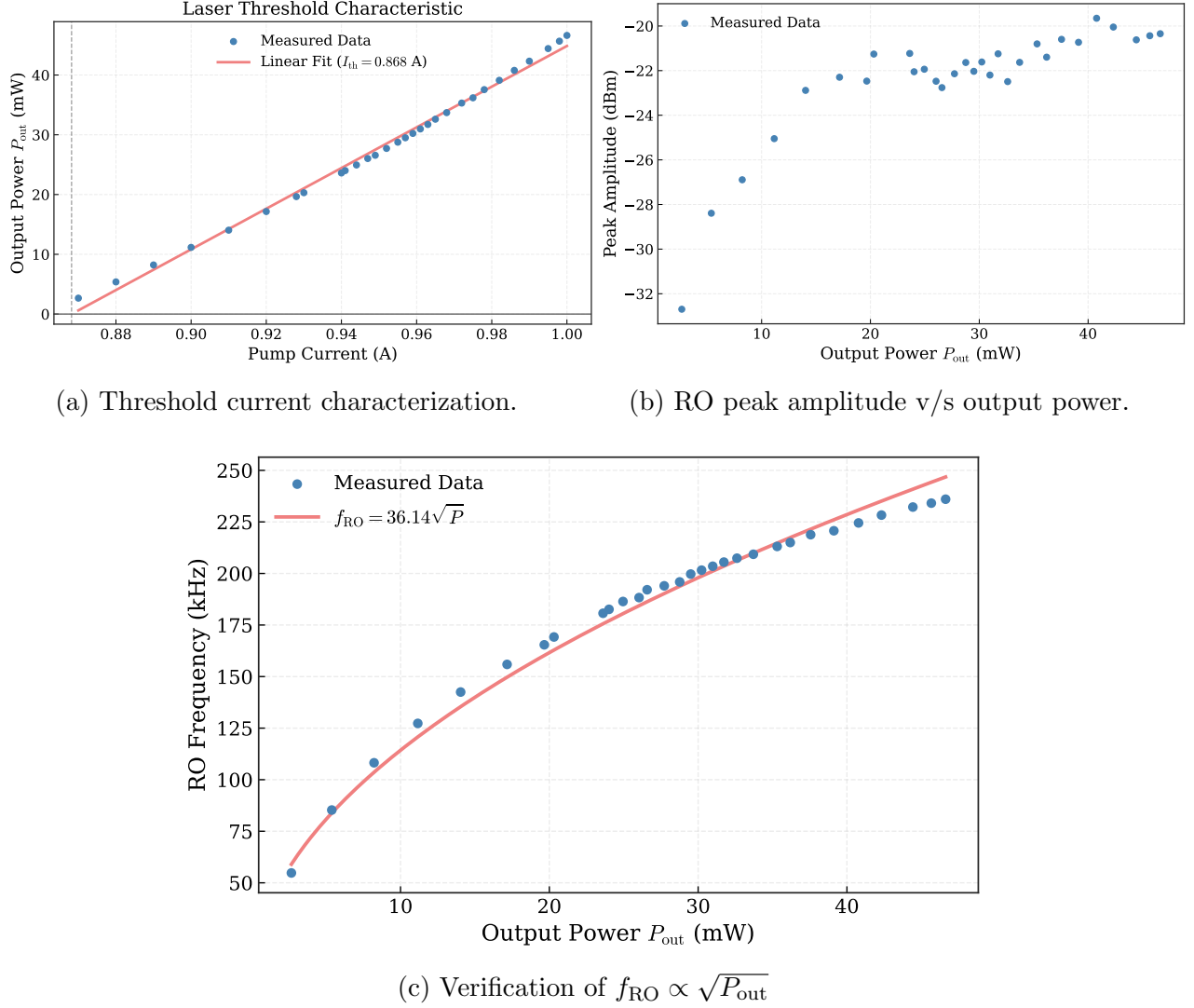


Figure 18: Relaxation oscillation analysis of the NPRO laser. (a) Laser power v/s pump current. Linear fit yields the threshold current $I_{\text{th}} = 0.868$ A. (b) Evolution of the peak relaxation oscillation amplitude measured as a function of output power. (c) RO frequency peak plotted with P_{out} showing excellent agreement with a square-root dependence.

Relaxation oscillation noise can be suppressed by actively stabilizing the laser intensity through feedback to the pump diode current. The Mephisto laser provides a built-in *Noise Eater* function for this purpose. Unfortunately, documentation for this particular 800NE laser was unavailable, due to which we could not safely activate the feature. An alternative approach would be to implement an external intensity stabilization loop using a pick-off photodiode and the current modulation input of the laser driver.

4.5 Rebuilding the Experimental Setup

Despite detailed investigation of the laser, mode-matching efficiency into the cavity, and functioning of the RF Modulation chain, a PDH error signal of expected magnitude could not be obtained. In addition, several characteristics of the aging NPRO laser made it difficult to identify the major limiting factor that was preventing cavity locking. Rather than continue troubleshooting this source, a decision was made to transition the experiment to a new setup with a fibre-coupled RIO diode laser. This shift was also utilized as an opportunity to integrate a functional vacuum chamber for the cavity optics.

4.5.1 Transition to the RIO Laser

The spatial profile of the RIO Laser was characterized by the same procedure as before. Figure 19 shows the measured beam radius along the horizontal and vertical axes together with Gaussian beam fits. The measured beam is nearly circular, with $M^2 \approx 1$ along both axes which indicates a high-quality Gaussian spatial mode suitable for cavity coupling.

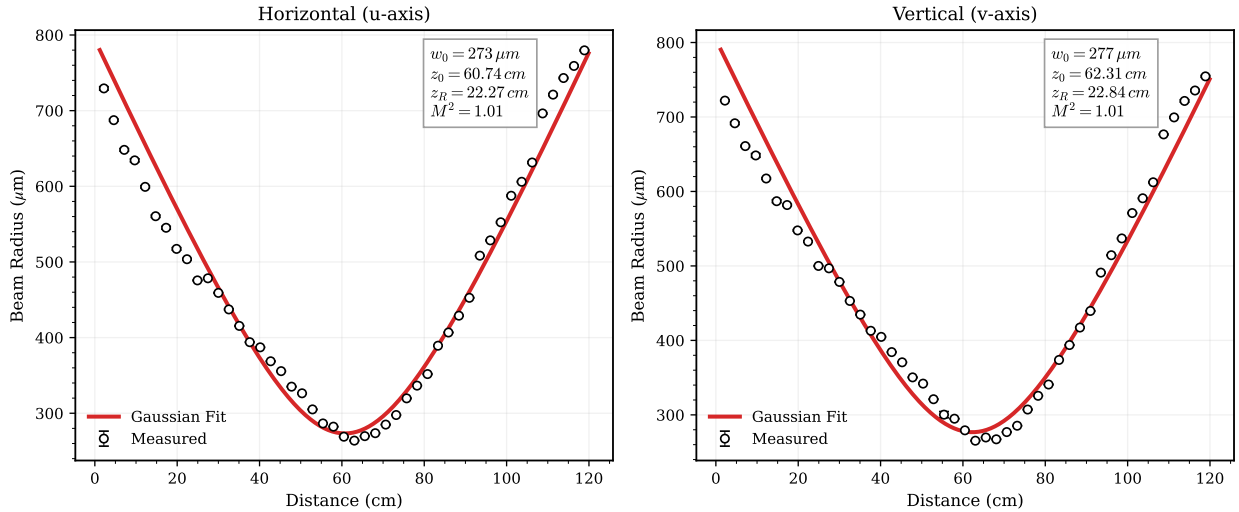


Figure 19: Measured beam profile of the RIO laser after a focusing lens of $f = 500 \text{ mm}$

The next step was to design a new mode-matching lens solution to couple the beam into the Gaussian eigenmode of the bow-tie cavity.

4.5.2 Designing a Mode-Matching Solution

Finding a mode-matching solution is usually an optimization problem over combinations of lenses and their placement in space. We set up the problem as follows. Beam parameters measured from the RIO profile were used as the starting point. The target was the cavity eigenmode calculated earlier, defining the required radius at the cavity's input coupler. There were several practical constraints on the placement of the lenses. For instance, the EOM and its baseplate occupied part of the available optical path, while the region containing the vacuum chamber and periscope also could not be used for placing lenses. These were given as constraint regions to be excluded from the optimization space available.

A relatively large length of optical table was available between the input at EOM and the chamber input. Rather than designing the telescope to fit into the smallest possible space, we chose to make use of this available length and search for solutions with reasonably large distances between the lenses, as this is expected to make the mode matching less sensitive to small errors in lens position. A lens catalog containing all available focal lengths in the lab was provided as input to the search. For each pair-wise combination of lenses, the beam was propagated through the free-space sections and lenses using the ABCD matrix formalism, and the resulting beam parameter at the cavity input was compared with the desired cavity mode. The search was first used to identify solutions with high mode overlap. Among these, the solutions were further ranked by a robustness metric, which measured the loss in overlap for a given error in lens positions. To quantify this sensitivity, the positions of the lenses were perturbed around their locations and the overlap efficiency was re-calculated.

The solution which yielded an overlap of 100% and ranked highest in robustness was selected for the initial implementation. This used lenses of focal lengths $f_1 = 1000$ mm and $f_2 = 400$ mm, with a total propagation distance of approximately 1.40 m. The beam propagation and the positions of the two lenses relative to the EOM and vacuum-chamber region are shown in Fig. 20.

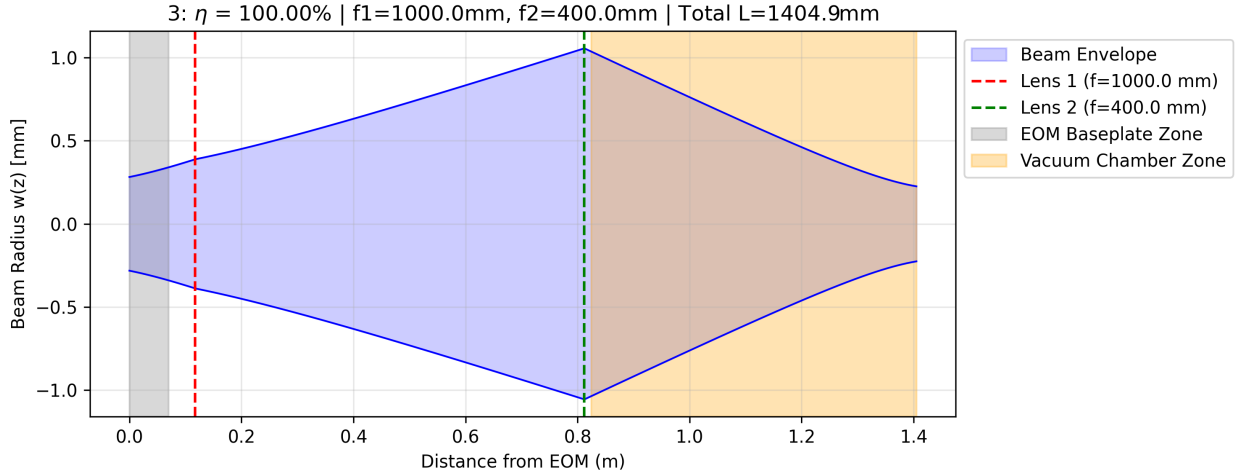


Figure 20: Computed propagation for the optimal mode-matching solution, showing positions of the two lenses and the available optical path including constraints. The solution uses $f_1 = 1000$ mm and $f_2 = 400$ mm, with a calculated mode-matching efficiency of 100%.

Mounting the first lens at its calculated position, the beam was profiled again immediately upstream of it. This provided a new measurement of the actual beam after the first lens, which was then used as the input to a second numerical search. This allowed the position of the second lens to be optimized using the beam actually produced by the first stage, rather than relying on the propagation of the original measured beam alone. This two-step procedure reduced the effect of errors in the initial beam measurement and in the physical placement of the first lens.

4.5.3 Final Setup and Integrated Improvements

The setup was reassembled on a new optical table with the RIO laser as the source. We first mounted the RIO laser fibre head and directed the beam through a half-wave plate into the Faraday isolator. A 500 mm focal length lens was then used to keep the beam comfortably small through the EOM. A second half-wave plate was placed before the EOM to control the input polarization. The transmission through the EOM and Faraday isolator was optimized separately for maximum power.

The vacuum chamber was then mounted on the table to house the cavity. The new mode-matching solution was implemented along the remaining beam path. A periscope was built to direct the beam into the chamber. The cavity optics were carefully aligned inside the chamber, along with the reflection photodiode and transmission camera. Higher-order cavity modes were initially observed on the transmission camera, and alignment was fine-tuned until the TEM_{00} mode was obtained. The complete assembled setup is shown in Fig. 21. The rebuilt setup incorporated several improvements based on the lessons learned from earlier configuration:

- A well-characterized laser source along with a robust mode-matching solution.
- Polarization control before the Faraday isolator, the EOM, and cavity input.
- Two mirrors placed upstream of each component for complete beam steering control.
- Cavity placed inside a vacuum chamber to reduce airflow and acoustic disturbances.

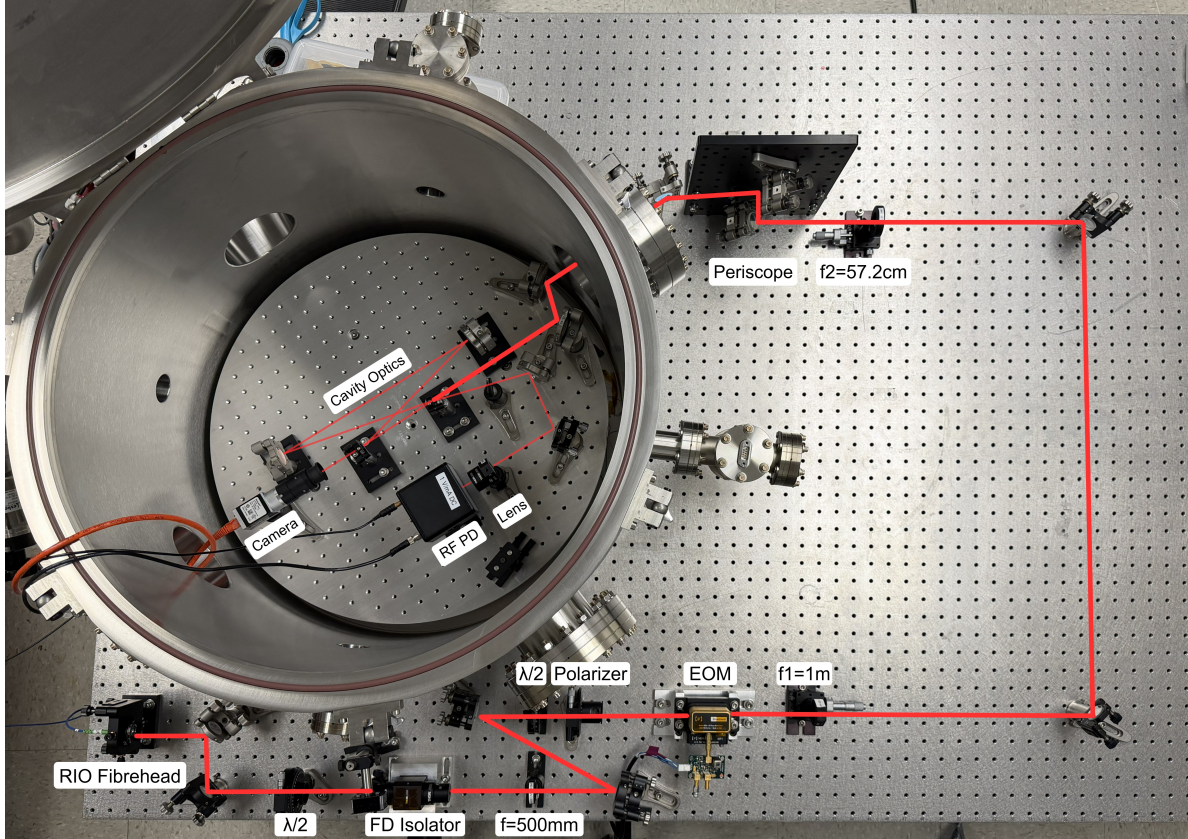


Figure 21: Rebuilt experimental setup showing 1064 nm beam path from the RIO laser to the bow-tie cavity inside the vacuum chamber.

5 An Alternate Cavity Design for SFG

The current SFG setup uses a bow-tie cavity. This geometry is useful because it provides total four ports, which are required to inject the 2050 nm signal and extract the 700 nm output. However, it also introduces an important requirement on the cavity mirrors. The two curved mirrors surrounding the nonlinear crystal would need to be highly reflective for the pump while transmitting both the 2050 nm signal and the generated 700 nm light. Such trichroic mirrors are more expensive to procure and add complexity to the optical design.

An alternative configuration, suggested by Prof. Martin Fejer and discussed with Prof. Rana Adhikari, can realize SFG in a much simpler and elegant manner. The basic idea is to use a two-mirror linear cavity with two Brewster-angle wedges, as shown in Fig. 22.

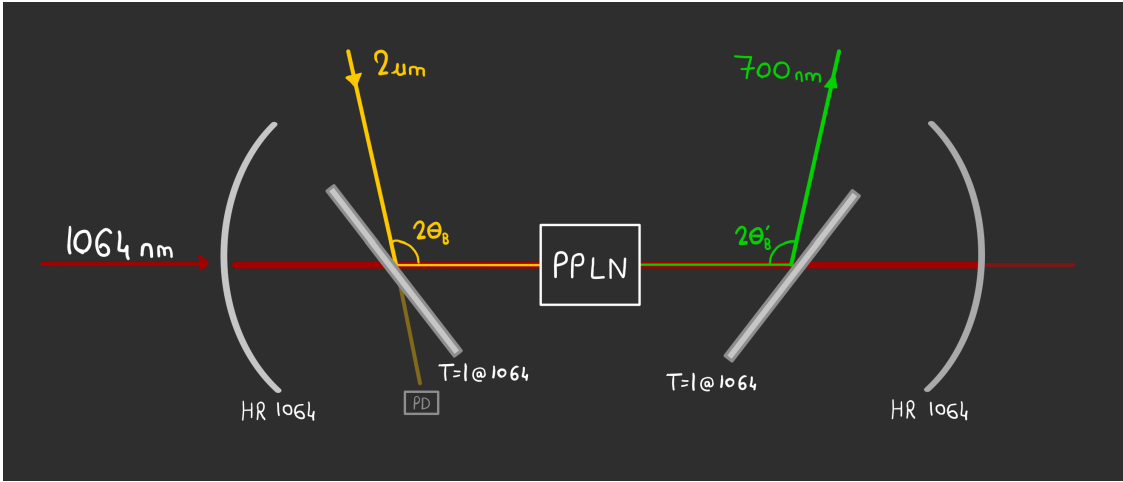


Figure 22: Schematic of a Brewster-wedge configuration for SFG, the 1064 nm pump (red) resonant between curved mirrors; the 2050 nm signal (yellow) and 700 nm output (green) are coupled through the two wedges.

Both curved mirrors are highly reflective at 1064 nm and form the resonant cavity for the pump. The pump field is coupled into the cavity through one of the curved mirrors. It then passes through the first wedge with near-unity transmission. Both wedges only cause a correctable change in the cavity eigenmode. The 2050 nm signal is injected through the first wedge, which is oriented at twice the Brewster angle, causing part of the s-polarized signal to be reflected towards the PPLN crystal. Similarly, the second wedge is oriented at $2\theta'_B$ so that part of the generated 700 nm light is reflected out of the cavity ready to be detected.

However, there is an important trade-off. At the Brewster angle, the s-polarized component has finite reflection. Assuming the SFG interaction requires s-polarized fields, a fraction of the 2 μm and 700 nm fields is lost to transmission at the wedges.

Despite this, the configuration has several useful features for an initial demonstration of SFG. The reflection and transmission of the wedges can be measured precisely, allowing the losses at the input and output to be accounted for when calculating the actual upconversion efficiency. Further, the power of rejected signal in the transmission path can be measured using a photodiode, allowing us to determine the fraction of s-polarized signal coupling towards the crystal, which can then be used for calculating upconversion efficiency.

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