

LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Technical Note	LIGO-T2600158-vX	2026/05/20
Validating LIGO Interferometer Models		
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1 Introduction

Gravitational waves (GWs) are distortions in the fabric of space-time, first predicted by Albert Einstein in 1915 as a result of general relativity. These waves propagate from energetic physical processes (colliding black holes, supernovae, colliding neutron stars) at the speed of light, carrying information about the events that created them. GWs are difficult to measure due to the minimal spatial distortion (strain) that they impose on Earth’s surface, on the order of 10^{-21} . The Laser Interferometer Gravitational-Wave Observatory (LIGO) was the first experiment to observe a GW in 2015 [1].

To reach a strain sensitivity capable of observing gravitational waves, LIGO must overcome many sources of noise including but not limited to: seismic noise, optical vacuum fluctuations, cosmic rays, and thermal noise [2]. This project will focus on simulating how mirror thermal fluctuations affect observed optical signals, in order to reproduce experimental data.

While previous studies have been done on the theoretical modeling of mirror thermal fluctuations [3, 5], there is still a mismatch between simulation predictions and experimental measurements. This is believed to originate from the nonstationary evolution of thermal distortions in the optics, due to incomplete knowledge of their initial thermal state and incomplete modeling of their thermal dynamics which can introduce time-dependent systematic errors into the calibration and optical modes [9].

The predictions of this project’s optical model can be validated against real LIGO data, specifically from stretches of data where the interferometer remains locked and the mirrors are allowed to reach a thermal steady-state. Data that satisfies these criteria is not trivial to obtain, as whenever significant transient noise is introduced into the system, the laser will not be locked. Ultimately, these optical simulations will improve our understanding of thermal effects in LIGO and may enable the development of more robust active thermal compensation techniques, mitigating the impact of thermal noise and allowing for even higher strain sensitivity needed for future gravitational wave detectors.

2 Background

2.1 LIGO Optical Setup

The LIGO detector is a Dual Recycled Michelson Interferometer (DRMI). Following the input laser path, the light travels through several key components [7]:

1. Input Mode Cleaner: Filters out higher-order laser spatial modes (see Sec 2.3).
2. Power Recycling Mirror: Forms a resonant cavity in the space before the beam splitter to amplify the injected laser power.
3. Beam Splitter: Divides and redirects the injected beam into the two Fabry-Perot arm cavities.
4. Signal: After exiting the cavities, the beams recombine and are sent through a signal recycling mirror and an output mode cleaner before measurement by a photodetector.

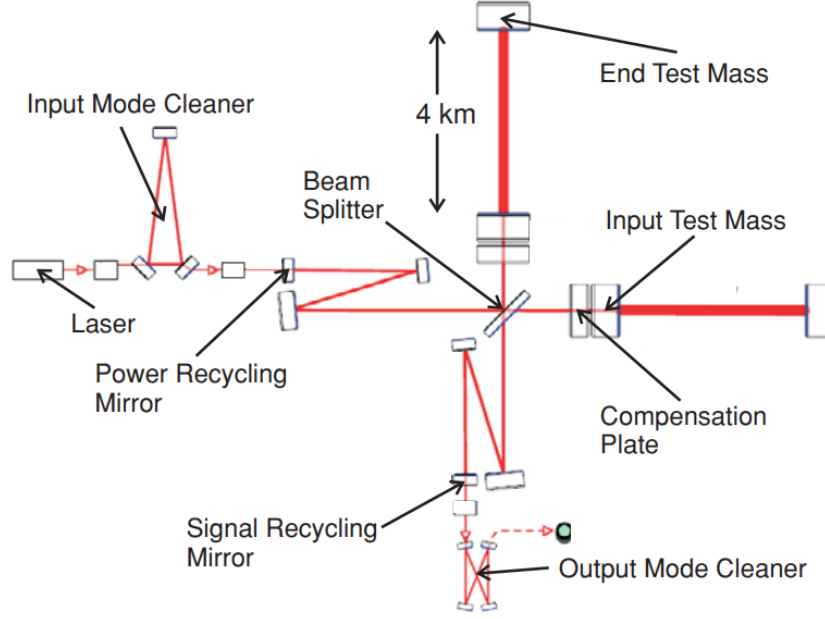


Figure 1: Diagram of LIGO's dual recycled Michelson interferometer optical setup [7].

When the interferometer arms are at the same length, it is tuned near a dark fringe, where the lasers recombine destructively at the anti-symmetric port, minimizing the signal received by the photodetector. When a gravitational wave passes by the interferometer, the two Fabry-Perot cavities may be length contracted by different amounts causing a perturbed phase difference when the lasers recombine leading to an increase in output laser power.

2.2 Laser Modes

The spatial distribution of a laser beam can be decomposed into an orthonormal basis, most commonly either the Hermite-Gauss (HG) modes or Laguerre-Gauss (LG) modes. The derivation for these modes follows from solving the paraxial approximation for an electromagnetic wave traveling in free space (Bond et al. [6]):

$$(\partial_x^2 + \partial_y^2)u - 2ik\partial_z u = 0 \quad (1)$$

One solution to Equation (1) are the Hermite-Gauss modes, which are rectilinearly symmetric. In the equations below $w(z)$ denotes the beam waist and $q(z)$ denotes the complex beam waist parameter [6].

$$\text{HG}_{lm}(x, y, z) = u_l(x, z)u_m(y, z) \quad (2)$$

$$u_J(x, z) = \left(\frac{\sqrt{2/\pi}}{2^J J! w_0} \right)^{1/2} \left(\frac{q_0}{q(z)} \right)^{1/2} \left(\frac{-q^*(z)}{q(z)} \right)^{1/2} H_J \left(\frac{\sqrt{2}x}{w(z)} \right) e^{\frac{-ikx^2}{2q(z)}} \quad (3)$$

Another solution to Equation (1) are the Laguerre-Gauss modes which are cylindrically

symmetric.

$$u_{p,l}(r, \phi, z) = \left(\frac{2p!}{\pi(p+|l|)!w_0^2} \right)^{1/2} \left(\frac{q_0}{q(z)} \right) \left(\frac{-q^*(z)}{q(z)} \right)^{\frac{2p+|l|}{2}} \left(\frac{\sqrt{2}r}{w(z)} \right)^{|l|} L_p^{|l|} \left(\frac{2r^2}{w^2(z)} \right) e^{\frac{-ikr^2}{2q(z)}} e^{-il\phi} \quad (4)$$

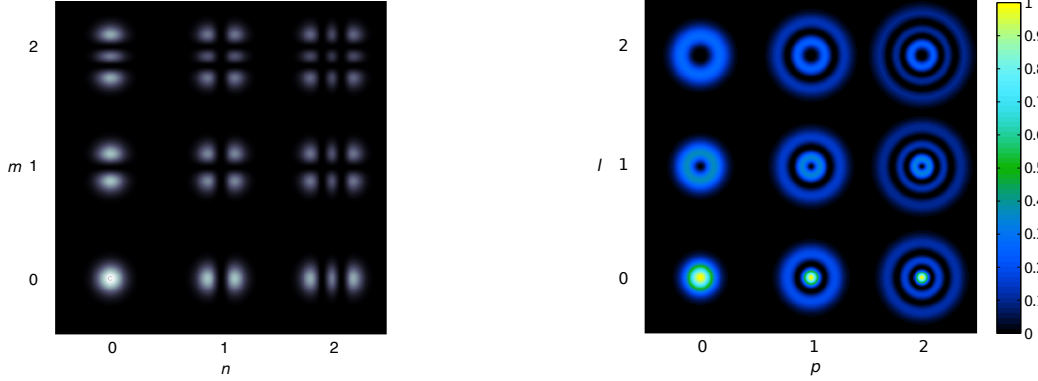


Figure 2: Normalized intensity distributions of Hermite-Gauss modes u_{nm} (left) and Laguerre-Gauss modes u_{pl} (right) [6].

The fundamental mode in both bases is a Gaussian intensity distribution which we can denote as the Transverse electromagnetic fundamental mode: TEM_{00} . This mode is relevant for the interferometer as we want the laser to remain in this fundamental mode throughout its optical path.

2.3 Optical Cavities

Optical cavities are combinations of reflecting optical components that allow for light to form standing waves. When the resonant wavelength of light is reflecting in an optical cavity, the amplitude/intensity will be increased, greatly increasing the laser power. These cavities are utilized through LIGO since the sensitivity of an interferometer-based GW detector is proportional to the laser power utilized [6].

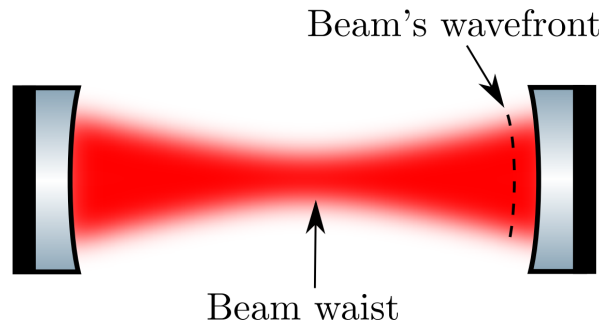


Figure 3: Diagram of optical cavity and corresponding spatial eigenmode [6].

Optical cavities also require the spatial wavefront of the light to match the geometry of the reflecting components to minimize loss in the gain. For example, in Figure 3 we can see that the beam wavefront must match the curvature of the mirror. Every optical cavity has an optical eigenmode, a combination of the basis modes described in Sec 2.2 that resonates in the cavity without loss.

One source of signal loss/noise in LIGO is mode-mismatch in an optical cavity. This is when the light in the cavity differs from the optical eigenmode. For example, thermal fluctuations in a cavity mirror's coating can scatter a resonating eigenmode into higher-order, non-resonating modes.

2.4 Optical Thermal Modeling

Theoretical modeling of the test masses in LIGO show that the thermal fluctuations in the bulk material are negligible in comparison to surface coating thermal fluctuations [3]. The dominant thermal effects arise from two coupled physical mechanisms: thermo-elastic deformation and the thermo-optic effect. In thermo-elastic deformation, laser absorption in the mirror coating causes local heating which expands the substrate, physically deforming the mirror surface. In the thermo-optic effect, the same heating alters the local refractive index of the coating material, changing the effective optical path length for the reflected beam. Both effects produce a spatially varying phase shift across the beam profile, distorting the reflected wavefront away from the ideal spherical geometry required for cavity resonance [8].

The primary thermal distortion in the test mass can be approximated as a quadratic surface deformation, equivalent to a change in the mirror radius of curvature (RoC). This shifts the cavity eigenmode waist size and position, reducing the overlap integral between the circulating field and the ideal cavity mode. Beyond the quadratic approximation, higher-order surface deformations, arising from the non-uniform intensity profile of the laser beam and the finite aperture of the mirror substrate, produce more complex mode-coupling and are more difficult to model analytically. These higher-order effects are where the mismatch between current simulations and experimental data is most significant [8]. It is difficult to quantify how these thermal effects manifest in experimental data because the initial thermal state is not well known. For example, whenever significant transient noise is introduced into the interferometer, the system may fall out of lock further complicating our understanding of the initial thermal state. Improved knowledge of the thermal initial state would allow simulations to make better predictions about the dynamic thermal state of the interferometer during its operation.

2.5 Thermal Compensation System (TCS)

To mitigate the effects of mirror thermal fluctuations, the Thermal Compensation System (TCS) was introduced. This instrument attaches to the mirrors and provides compensatory heating to the mirror surface to counteract thermal fluctuations caused by the laser. As the test masses absorb a fraction of the circulating laser power, they develop radial temperature gradients that cause thermo-elastic surface deformations in the Fabry-Perot arms and

thermo-refractive substrate lenses in the recycling cavities [12]. To maintain optimal interferometer sensitivity, the TCS must constrain residual wavefront distortion to below 5.4 nm RMS, ensuring that round-trip scattering of the fundamental TEM₀₀ mode remains strictly under 0.1% [12]. The system continuously monitors these thermal lenses using independent Hartmann Wavefront Sensors (HWS) [12]. To counteract the distortions, the TCS utilizes a combination of ring heater (RH) actuators to adjust the overall surface RoC, alongside CO₂ laser projectors that apply spatially tunable heat profiles to the compensation plates to clean up higher-spatial-frequency residuals [12]. Incorporating the dynamics of these specific TCS sensors and actuators into our optical simulations will be helpful for accurately reproducing the nonstationary thermal state of the interferometer.

2.6 Markov chain Monte Carlo (MCMC)

MCMC is a particularly useful technique for systems where the observable data exhibits a highly non-linear dependence on the underlying physical parameters. In the context of complex physical models, simulation-based Bayesian optimal experimental design relies heavily on Markov chain simulations to characterize these non-linear system responses [10, 11].

Within our FINESSE model, the relationship between the thermal state of the test masses and the resulting optical signal is inherently non-linear. As discussed in Section 2.3, thermal fluctuations in the mirror coatings alter the cavity geometry, leading to non-trivial mode-mismatch and scattering of the resonating optical eigenmode. The heating process itself is non-linear and knowledge of the thermal state is obscured by incomplete knowledge of the initial thermal state. By leveraging MCMC algorithms, we can hopefully capture this non-linear temperature dependence, isolating the specific thermal and design parameters that have the greatest impact on the signal variation observed in LIGO operational data.

Following conventions of Bayesian inference methods, we can introduce three spaces [10]:

- **The Design Space ($\mathcal{D}$):** The set of all controllable experimental variables. A specific design, denoted as $d \in \mathcal{D}$, represents a unique configuration of the apparatus. This would encompass the design choices for the interferometer we implement in our simulated optical model.
- **The Parameter Space (Θ):** The underlying physical states of nature or model parameters that the experiment seeks to infer. A specific parameter set, denoted as $\theta \in \Theta$, represents quantities the experimenter may not directly control but wishes to learn about (e.g. thermal fluctuations).

Additionally, we define the **Data Space ($\mathcal{Y}$)**, where $y \in \mathcal{Y}$ represents the observable measurements generated by the experiment (e.g. observed interferometer signal). The relationship between these spaces is governed by a forward model $F(d, \theta)$, which simulates the expected physical data y .

The goal of the experiment is to update our knowledge about the physical parameters θ after observing data y collected under a specific design d . This update is governed by Bayes' theorem:

$$p(\theta|y, d) = \frac{p(y|\theta, d)p(\theta)}{p(y|d)} \quad (5)$$

Metropolis-Hastings (MH) and Hamiltonian Monte Carlo (HMC) are specific types of MCMC algorithms designed to draw representative samples from Θ .

3 Objectives

1. Develop a realistic optical model of the interferometer optics
2. Incorporate thermal effects into the model
3. Use thermal steady-state data for validation
4. Identify missing systematics in current models
5. Reproduce observed signals through simulation
6. Determine the most critical parameters

4 Approach/Method

Using previous optical models as reference, we will create an optical model of the Full Interferometer using FINESSE, a Python-based interferometer simulation package [4]. The model must quantitatively reproduce observed signals such as arm cavity power, optical gains, and RF sideband behavior. Within the FINESSE model, we will introduce thermal distortions in the optics. For example, by including thermal fluctuations in the mirror dielectric coatings and observing the corresponding change in signal power. To judge the accuracy of this thermal model, we can compare its output to stretches of LIGO data taken while the system was in a known thermal steady state, i.e., laser remains locked and heats components past a 24 hour transient heating period. This validation period will inform us on what physics our model is missing, which we will subsequently incorporate into the model. With a corrected model we can attempt to produce results in line with experimental data. By leveraging MCMC algorithms, we can vary interferometer design parameters to identify those which have the greatest impact on the measured signal (e.g., beam positioning, thermal distortions, finite mirror apertures, and baffles). This can inform the current LIGO design and future designs on priorities for on-site commissioning measurements and detector improvements.

5 Timeline

1. **Week 1:** Literature review and familiarization with FINESSE package.
2. **Weeks 2-3:** Set up FINESSE model and incorporate thermal effects. Implement MCMC methods into simulation and test which technique (e.g. MH or HMC) is most accurate.
3. **Weeks 4-5:** Obtain experimental LIGO data for model validation and compare to simulation results.

4. **Weeks 6-7:** Analyze simulation and experimental data to improve model accuracy while tracking parameters of interest.
5. **Weeks 8-9:** Collect final results and prepare final report and project presentation.

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