



Pulsar Timing Arrays

Bruce Allen

**Max Planck Institute for Gravitational Physics
Hannover, Germany**

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

Physics: 8.012

Fall Term, 1976

Lecture Outline

- Sept. 14 General Intro to Physics, Coordinate Systems, Vectors.
- 16 Kinematics Time-Length.
- 21 Accelerating Motion - Linear-Circular.
- 23 Nature of Forces, Torques, Force Diagrams.
- 28 Statics.
- 30 Elasticity, Scaling.
- Oct. 5 Inertia, Newton's Law.
- 7 EXAM
- 14 Applications of Newton's Law.
- 19 2-Dimensional Motion, Damped Motion.
- 21 Simple Harmonic Motion - Seismometers.
- *26 Collisions and Conservation Laws.
- 28 Conservation Laws.
- Nov. * 2 Conservation Laws and Vibrational Motion.
- 4 Conservative Forces - The Potential, Newtonian Grav.
- * 9 EXAM
- 11 The Eötvös Experiment, Gravitational Red Shift, Principle of Equivalence.
- 16 Central Forces, Angular Momentum Conservation.
- 18 Non-Inertial Frames; Centrifugal - Coriolis Forces.
- 23 Lense-Thirring Effect, Planetary Orbits.
- 30 Planetary and Lunar Ranging Experiments.
- Dec. 2 Perturbation Theory.
- 7 Extended Bodies and Moment of Inertia.
- 9 The Gyro
- 14 EXAM
- 16 Hamilton's Principle.
- * Evening lecture. Special Relativity.

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Information Sheet

Lecturer: Professor R. Weiss Office 20F-006 Phone 3-3527

Recitation Instructors:

Prof. D. Barton	24-511	3-7596
Prof. G. Brandenburg	24-511	3-7597
Prof. C. Canizares	37-501	3-7500
Prof. G. Clark	37-611	3-5842
Prof. J. Friedman	24-512	3-7585
Prof. H. Kendall	24-514	3-7584
Prof. L. Rosenson	24-520	3-7595

Teaching Assistants: Peter Mongeau 6-409A 3-7034
 Dana Roberts 24-612 3-7787

Course Secretary: Susie Fennelly 4-352 3-4855

Schedule:

Lecture	TR	10-11:30	26-100
Recitation Section	01	MW	9 ... Barton	2-132
	02	MW	10 ... Canizares	24-502
	03	MW	10 ... Brandenburg	2-136
	04	MW	11 ... Kendall	8-205
	05	MW	11 ... Canizares	24-502
	06	MW	12 ... Kendall	2-132
	07	MW	1 ... Friedman	2-132
	08	MW	2 ... Friedman	2-143
	09	TR	9 ... Mongeau	2-131
	10	TR	2 ... Rosenson	26-210
	11	TR	1 ... Clark	24-407
	12	TR	12 ... Clark	2-139

Text Book: NEWTONIAN MECHANICS, A.P. French (Norton, 1971).

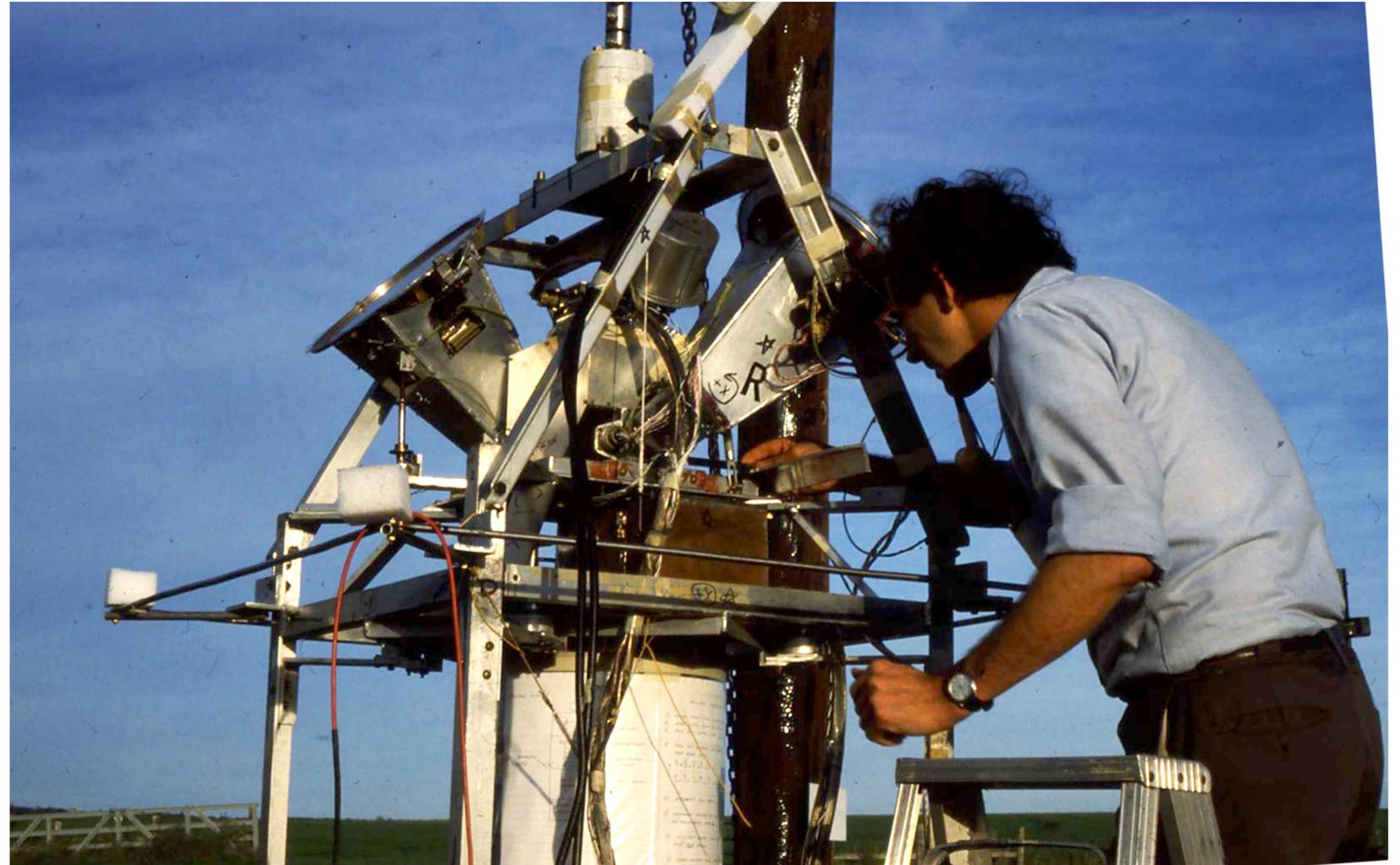
Tutoring: Dana Roberts and others. MW 1-3 p.m. Corridor Lab (4-315)

Homework: Handed out Thursday in Lectures, to be completed and given to Recitation Instructors the following Wed. or Thurs. One problem in each set to be graded by Recitation Instructor on scale 3 → 0. Some problems require work in the Corridor Lab.

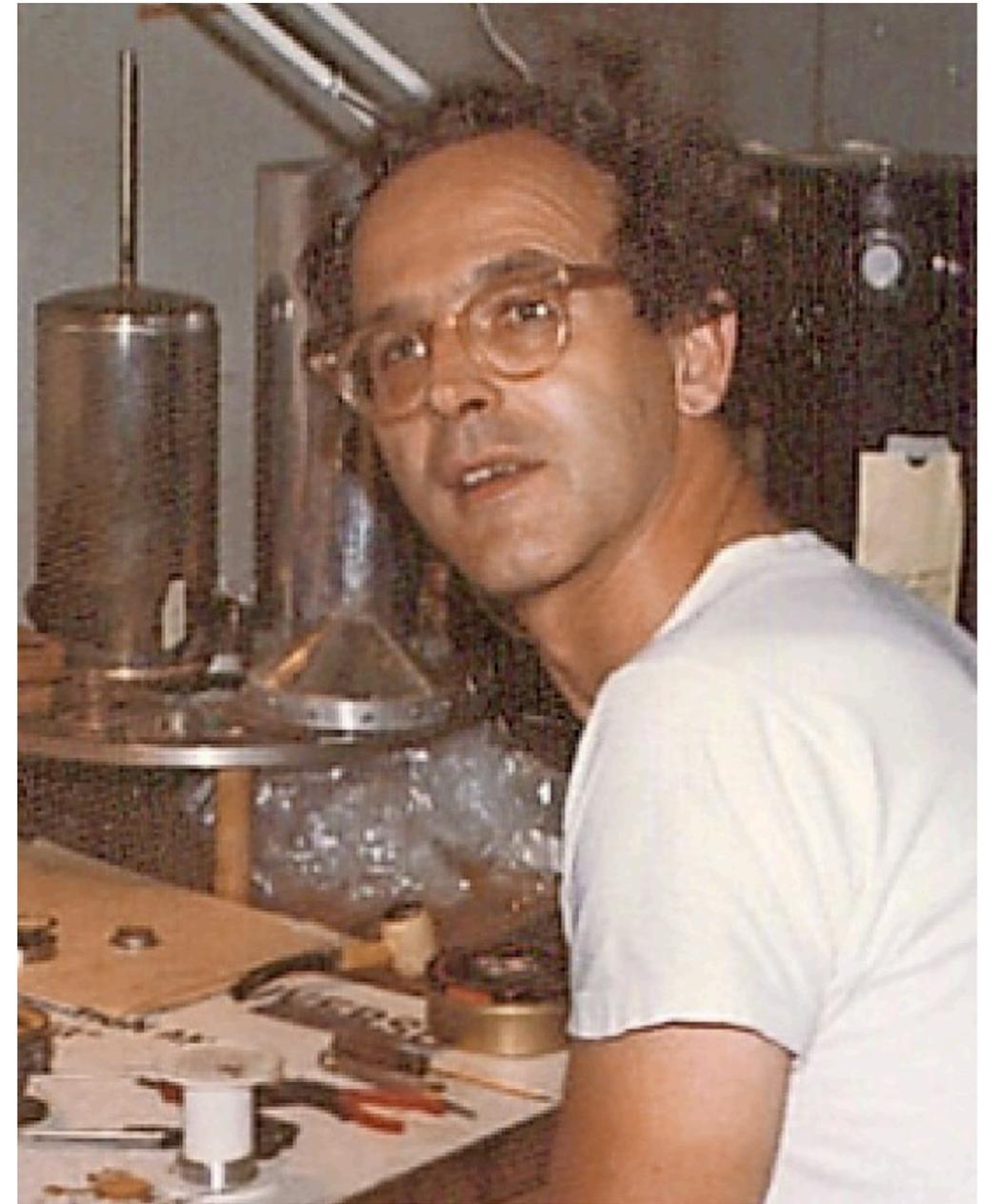
Grading: PASS or FAIL. No Hidden Grades. 75% of evaluation determined by 3 1½ hour exams 25% of evaluation determined by homework.

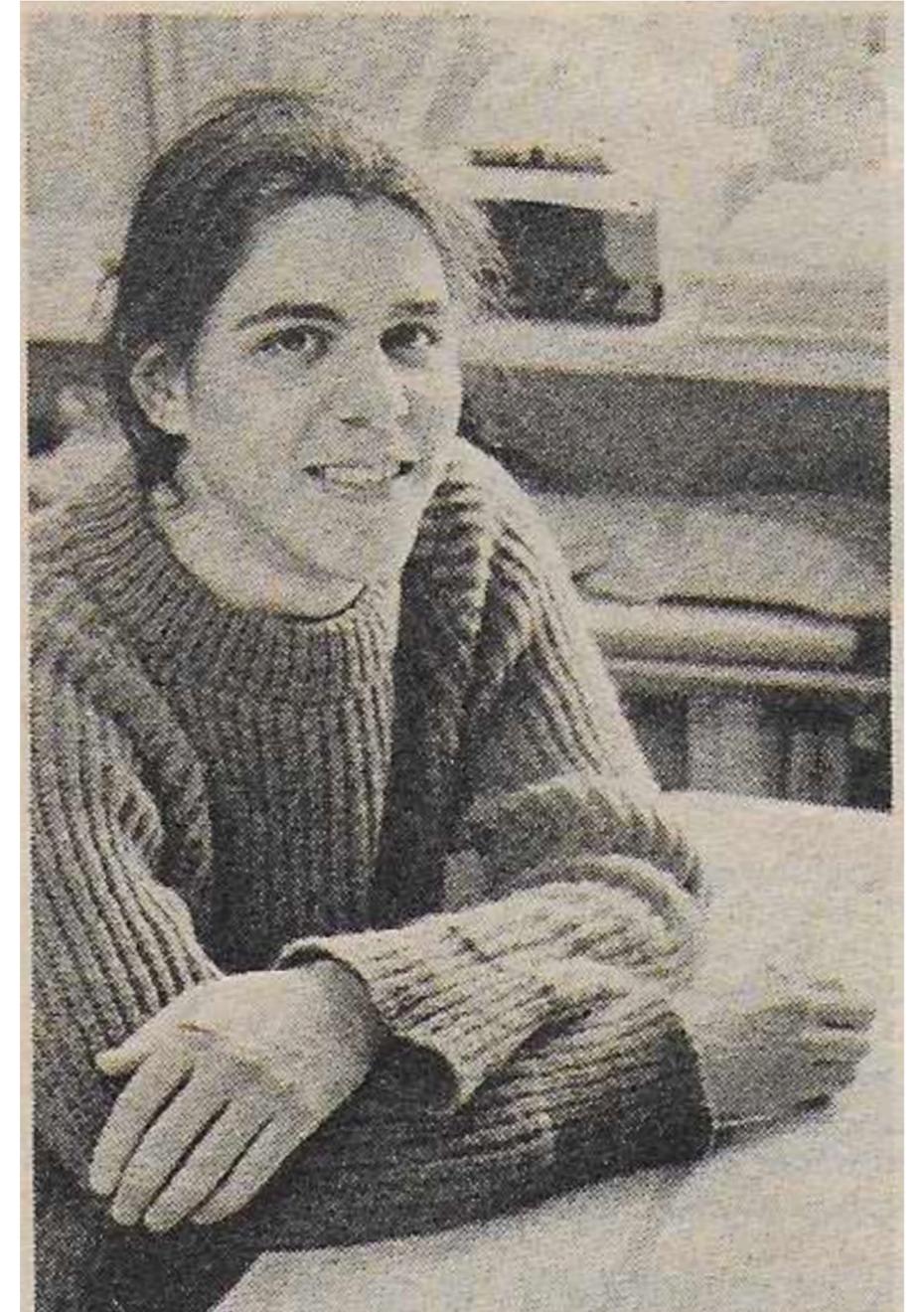


Credit: Ben Weiss



Credit: Mark Halpern





PROGRAM
EIGHTH TEXAS SYMPOSIUM
ON
RELATIVISTIC ASTROPHYSICS

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The National Science Foundation
The New York Academy of Sciences
The University of Texas

TO BE HELD
December 13 through 17, 1976
AT
Copley Plaza Hotel
Copley Square
Boston, Massachusetts

TUESDAY, DECEMBER 14

QUANTUM THEORY IN STRONG GRAVITATIONAL FIELDS

9 a.m. - Noon

CHAIRMAN: B. De Witt, University of Texas

Quantum Field Theory in Curved Space-Time:
an Overview (50 min.)

C. Isham
Imperial College

Fundamental Breakdown of Physics in Gravitational
Collapse (40 min.)

S. W. Hawking
Cambridge University

COFFEE BREAK

Thermodynamics of Black Holes (20 min.)

D. Sciama
University of Oxford

Stress Tensor Calculations (20 min.)

P. C. W. Davies
Kings College, London

Particle Detectors and Black Holes (20 min.)

W. Unruh
University of British Columbia

X-RAY BURSTERS AND GLOBULAR CLUSTERS

2 p.m. - 5 p.m.

CHAIRMAN: J. Bahcall, Institute for Advanced Studies

Observations of X-Ray Bursters (30 min.)

H. Gursky
Center for Astrophysics
Harvard/Smithsonian

Observations of X-Ray Bursters (30 min.)

W. Lewin
Massachusetts Institute of Technology

Physics of Accretion vs Astronomy of X-Ray
Sources (30 min.)

J. Ostriker
Princeton University

COFFEE BREAK

Optical Observations of Globular Clusters (30 min.)

W. Liller
Center for Astrophysics
Harvard/Smithsonian

Neutron Star Models of X-Ray Bursters (15 min.)

D. Lamb
University of Illinois

WEDNESDAY, DECEMBER 15

X-RAY ASTRONOMY

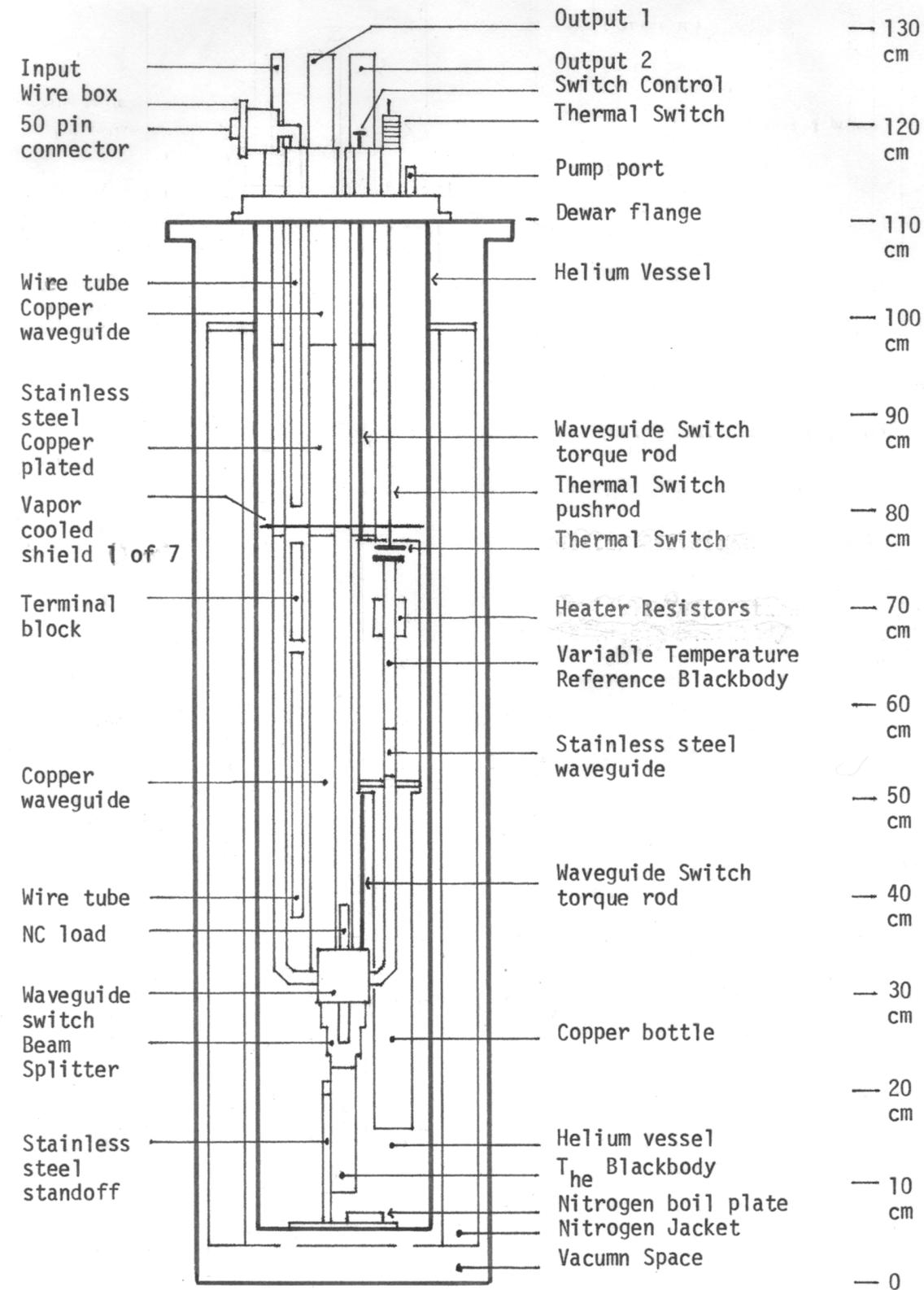
9 a.m. - Noon

CHAIRMAN: G. Garmire, California Institute of Technology

Observations of Extragalactic X-Ray Sources (30 min.)

H. Schnopper
Center for Astrophysics
Harvard/Smithsonian

FIGURE 7. The Liquid Helium Cooled Front End

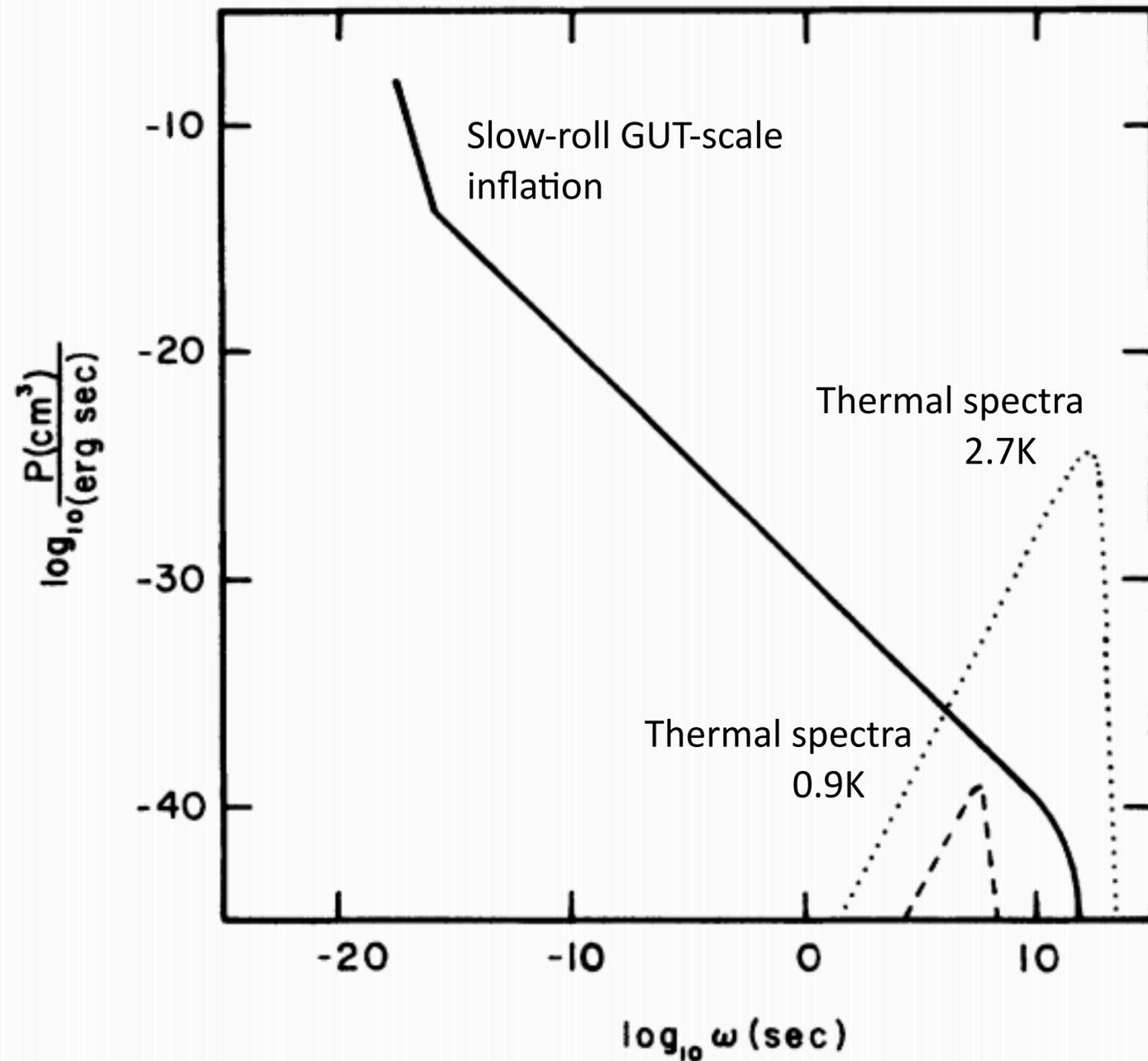


UC Santa Barbara 1984





Stochastic Background Inflation



From B. Allen, "Stochastic gravity-wave background in inflationary-universe models", *PRD* 37, 2078 (1988)

BA, *Vacuum states in de Sitter space*, *PRD* 32 (1985) 3136

BA, Ted Jacobson, *Vector two-point functions in maximally symmetric spaces*, *Comm. Math. Phys.* 103 (1986) 669

BA, *The graviton propagator in de Sitter space*, *PRD* 34 (1986) 3670

BA, *The graviton propagator in homogeneous and isotropic space-times*, *Nucl. Phys. B* 287 (1987) 743-756

BA, Antoine Folacci, *The massless minimally coupled scalar field in de Sitter space*, *PRD* 35 (1987) 3771



Pulsar Timing Arrays (PTAs)

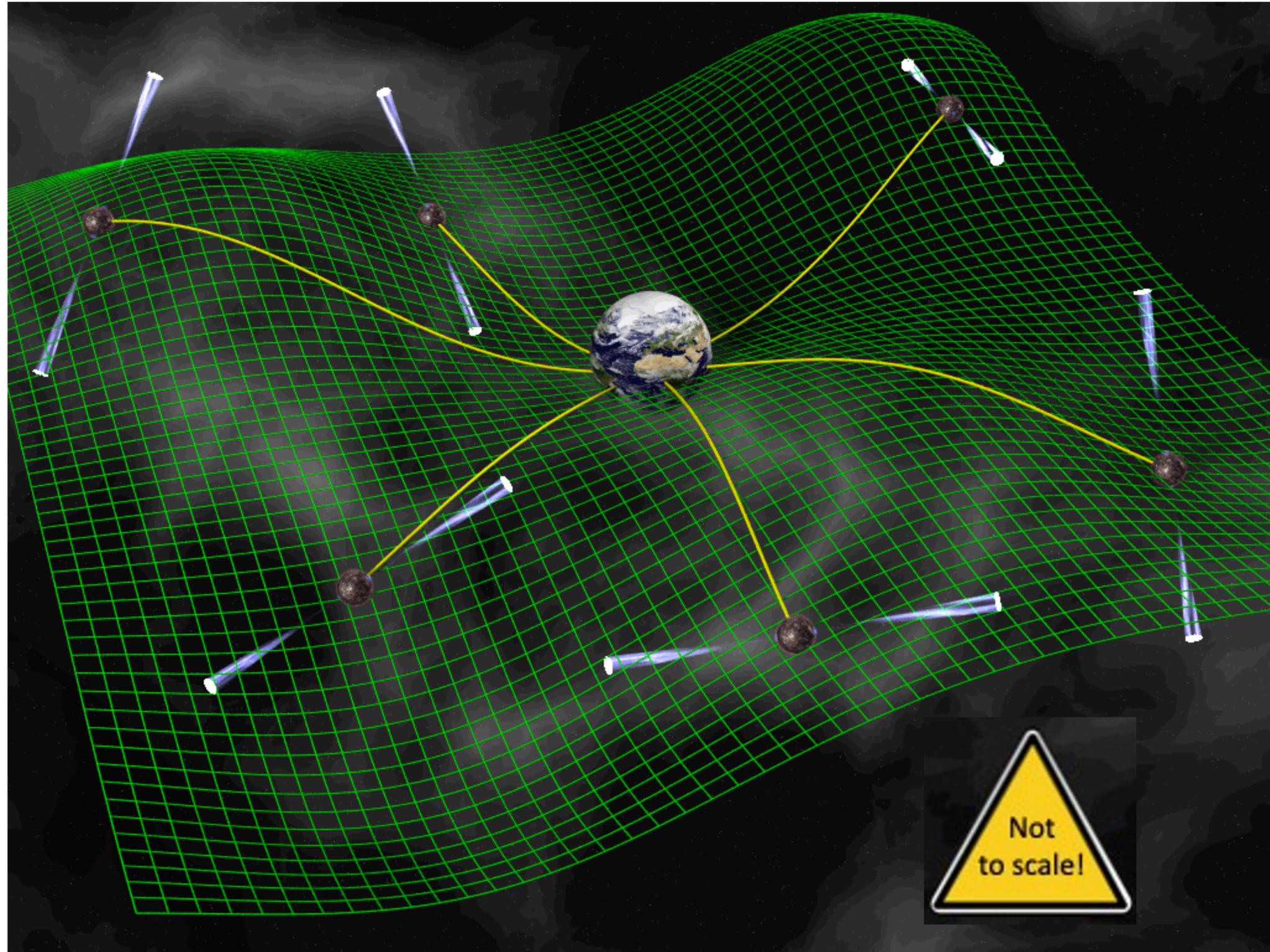


Promise: detect
nano-Hz
gravitational
waves

$$L_{GW} \approx 10 \text{ y}$$

$$L_{pulsar} \approx 1000 \text{ y}$$

$$L_{source} \approx 10^8 \text{ y}$$

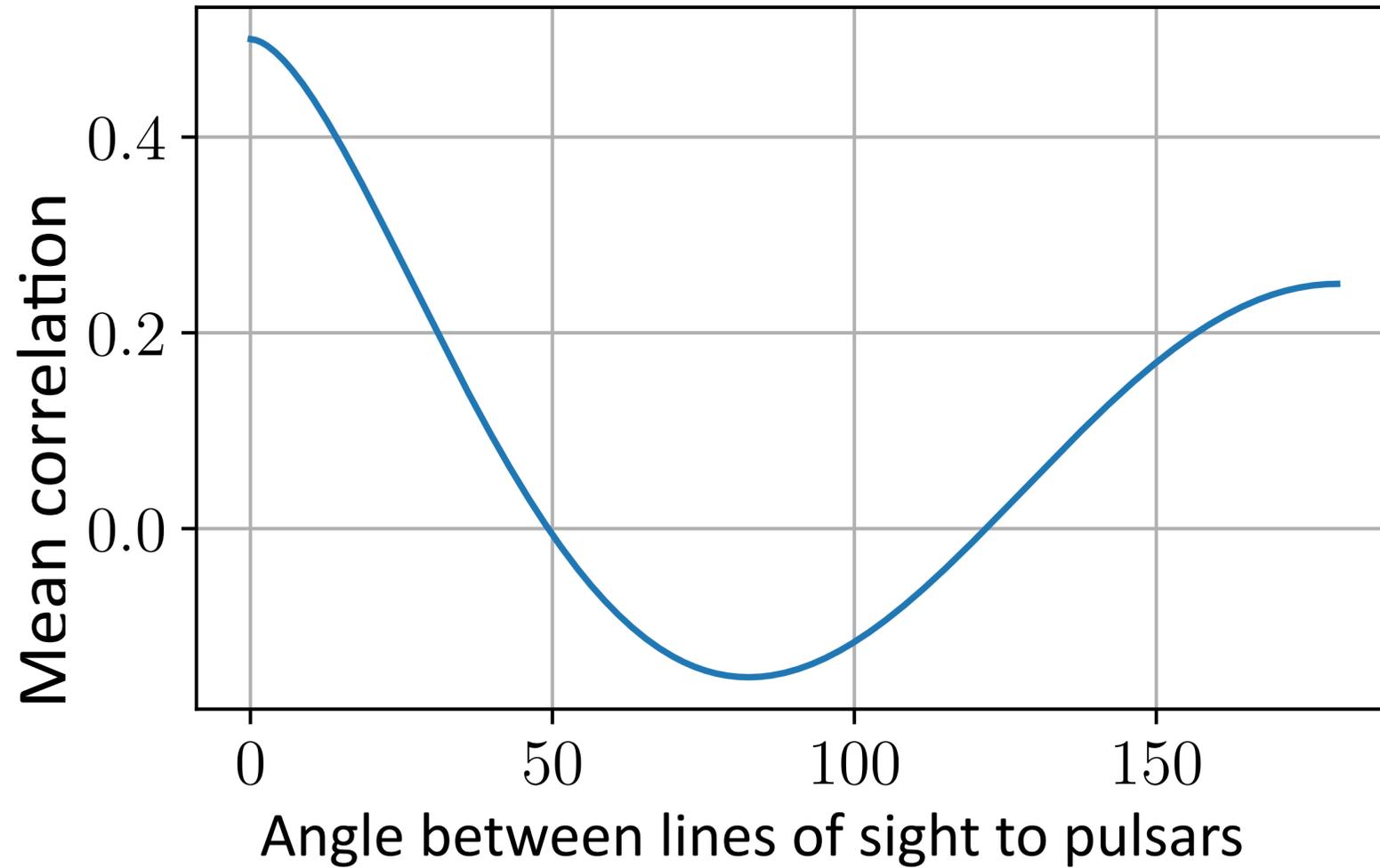




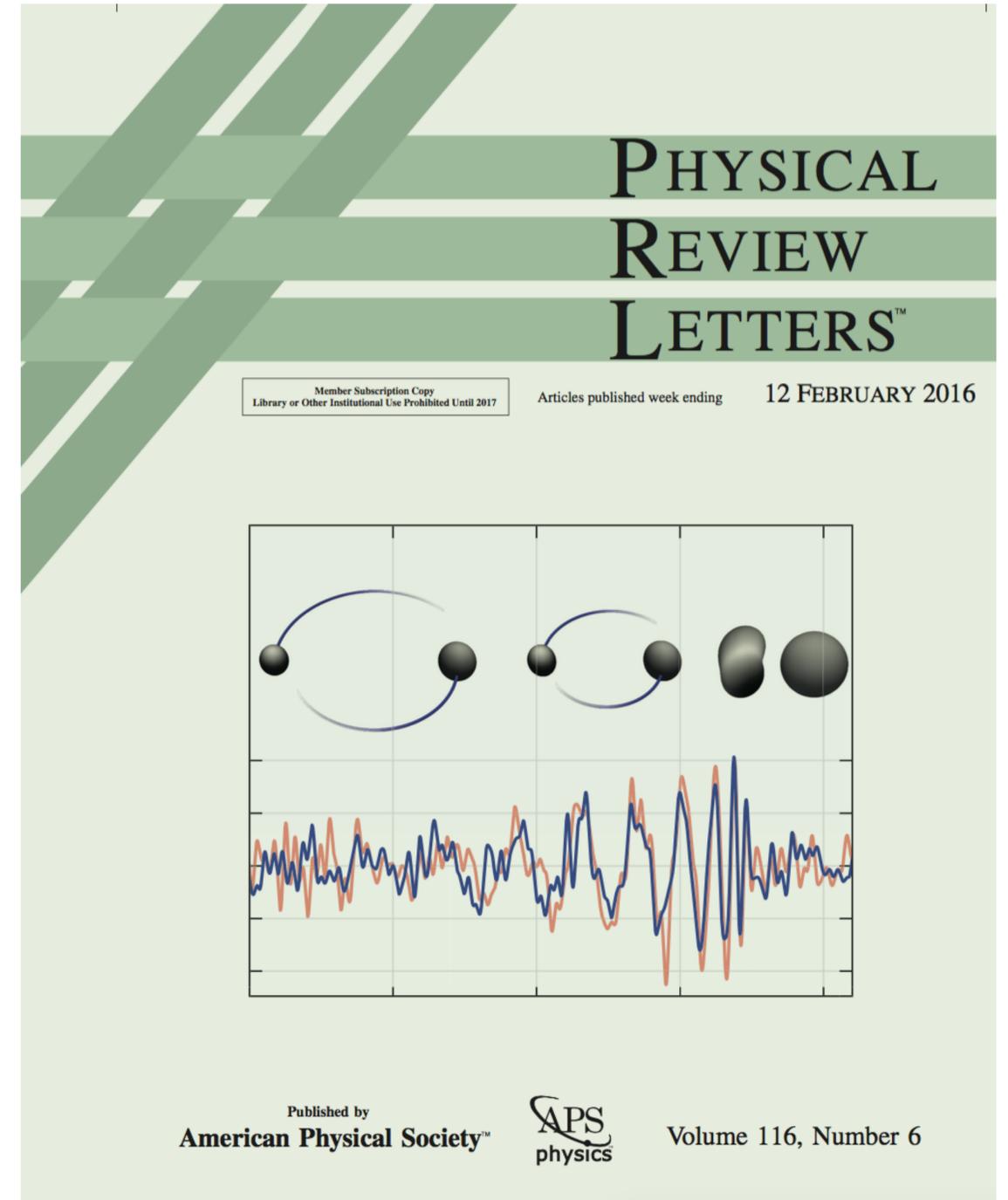
Key: Correlation Pattern in Pulsar Timing Residuals



Hellings and Downs curve



Hellings and Downs, "Upper limits on the isotropic gravitational radiation background from pulsar timing analysis", *ApJ*, 265 (1983)





Current status of six PTAs



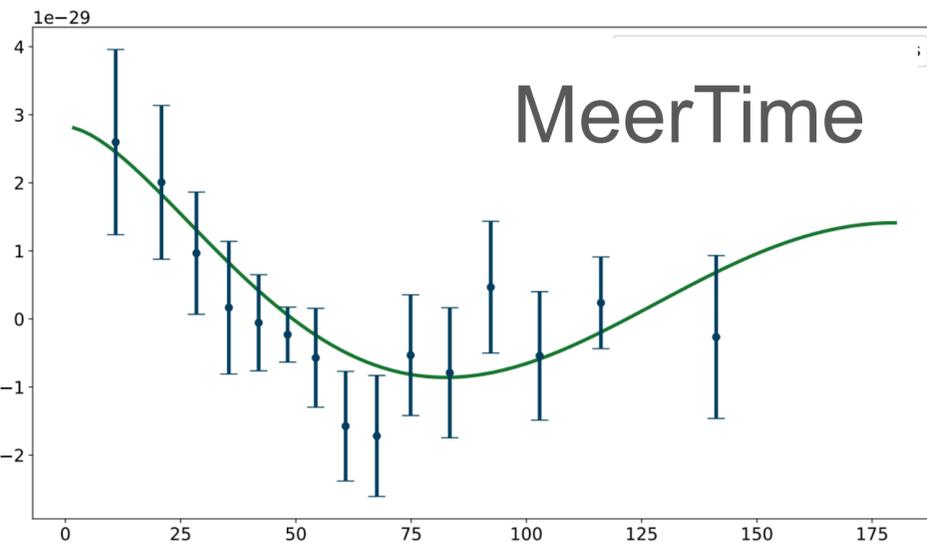
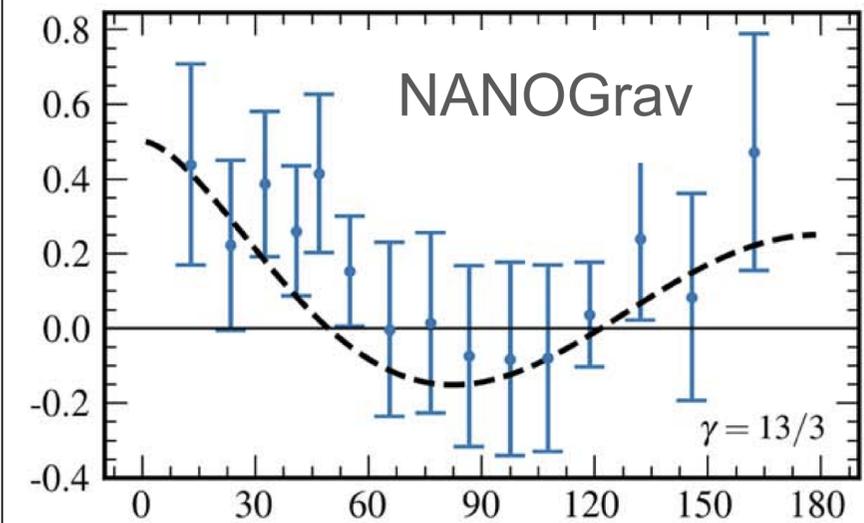
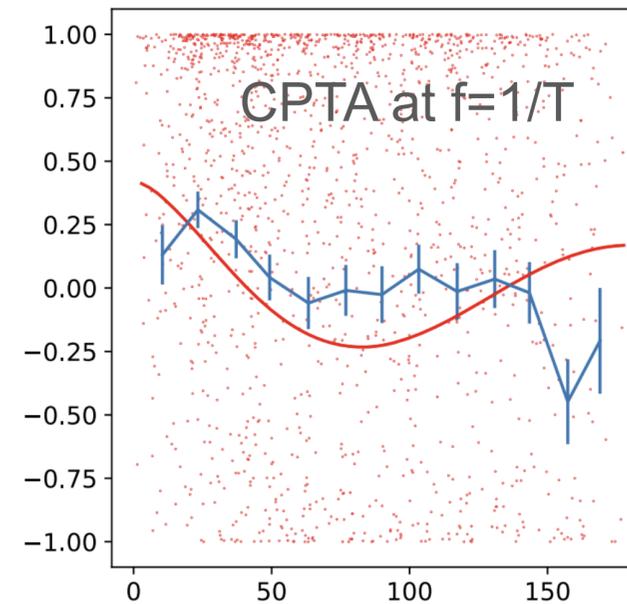
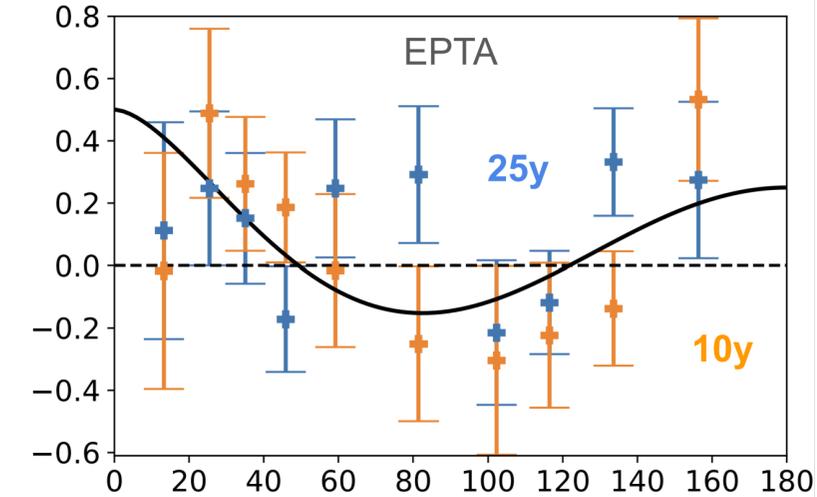
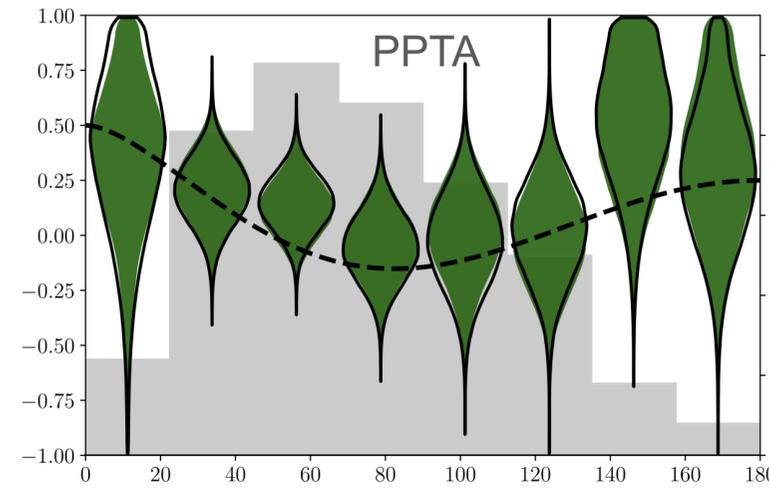
Parkes PTA (PPTA), ApJL 951 L6 (2023), 30 pulsars over 18 years, **no support for or against**

European PTA (EPTA) and Indian PTA, Astron. Astrophys. 678, A50 (2023), 25 pulsars over 25/10 years, **"marginal evidence/evidence"**

Chinese PTA (CPTA), Res. Astron. Astrophys. 23, 075024 (2023), 57 pulsars over 3 years, **"some evidence"**

North American Nano-Hz Observatory for Gravitational Waves (NANOGrav) ApJL 951, L8 (2023), 67 pulsars over 15 years, **"compelling evidence"** [Publication's 3.6σ to 4.2σ later revised to 3.2σ]

MeerTime, MNRAS 536, 1489 (2025), 83 pulsars over 4.5 years, "Under different assumptions about noise processes we can produce **either** what appear to be **compelling Hellings-Downs correlations of high significance 3-3.4 sigma**, with a spectrum close to that which is predicted, **or** surprisingly, under slightly different assumptions, **ones that are insignificant**





First Analysis of five-PTA Data Set

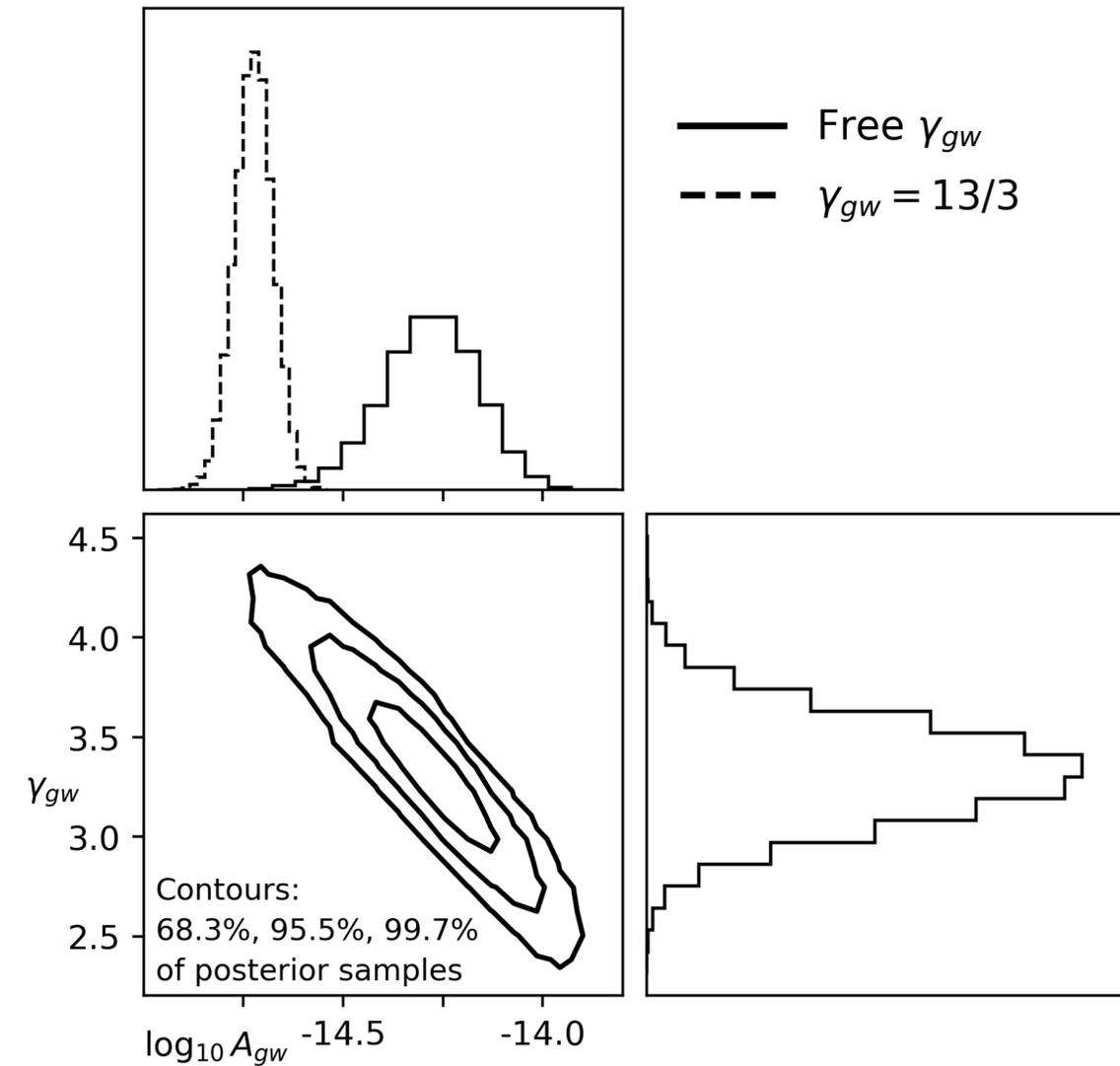


- June 2023 data from 5 of 6 PTAs is public: 121 pulsars in total
- New “direct combination” method from van Haasteren, otherwise vanilla analysis
- Search for a stochastic gravitational wave background which is:
 - isotropic, unpolarized, Gaussian, stationary
 - power-law spectrum:

$$\Omega_{gw}(f) \equiv \frac{f}{\rho_{cr}} \frac{d\rho_{gw}}{df} = \frac{2\pi^2}{3H_0^2} A_{gw}^2 f_r^2 \left(\frac{f}{f_r} \right)^{-\gamma_{gw}+5}$$

$$f_r \equiv 1/\text{year}$$

Yu and Allen, *Stochastic gravitational-wave background search using data from five pulsar timing arrays*, [arXiv2512.08666](https://arxiv.org/abs/2512.08666)



Posteriors on the slope and amplitude





First Analysis of five-PTA Data Set

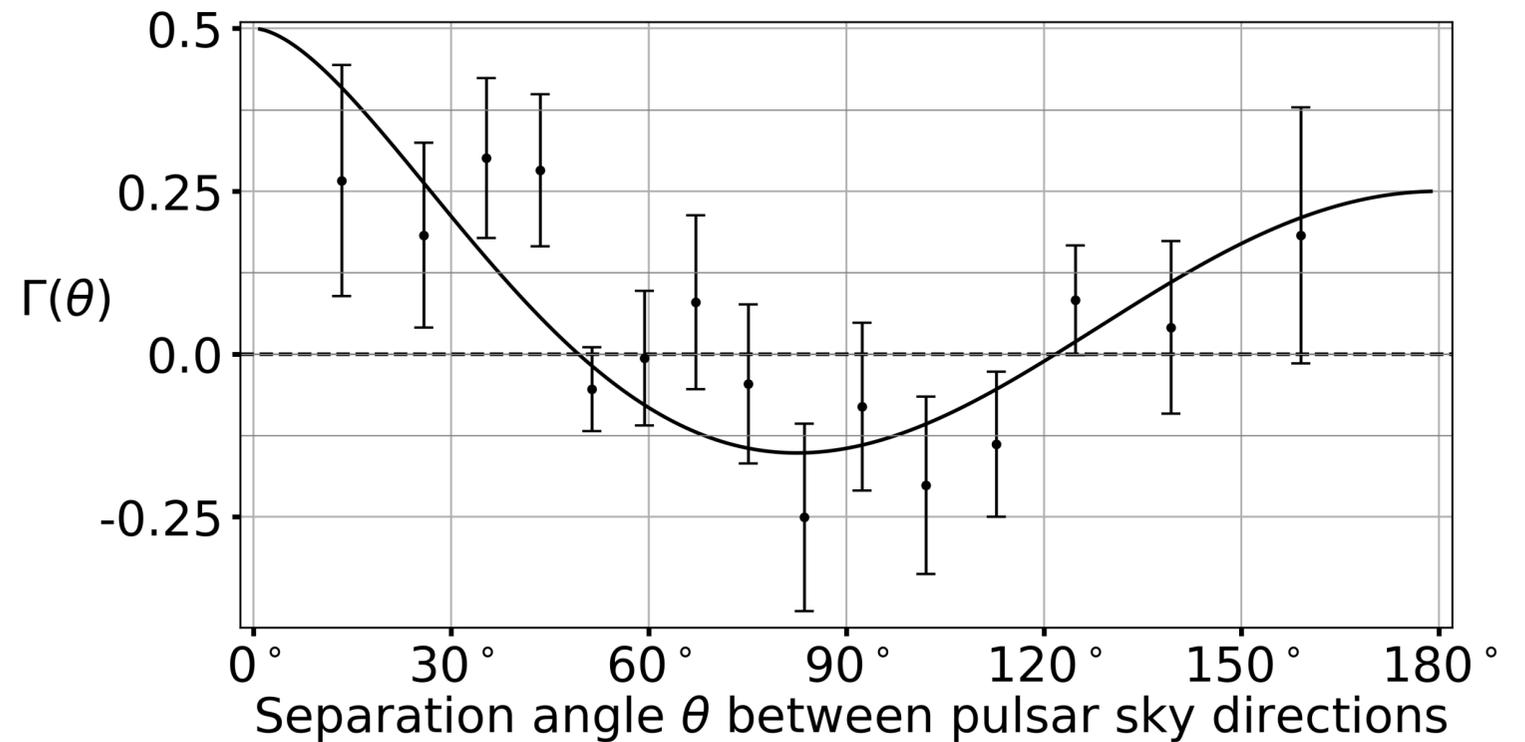


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$$f_r \equiv 1/\text{year}$$

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Reconstruction of the Hellings and Downs curve (reduced $\chi^2 = 0.84$, a good fit)



Yu



van Haasteren



First Analysis of five-PTA Data Set



- June 2023 data from 5 of 6 PTAs is public: 121 pulsars in total
- New “direct combination” method from van Haasteren, otherwise vanilla analysis
- Search for a stochastic gravitational wave background which is:
 - isotropic, unpolarized, Gaussian, stationary

False alarm probability and detection significance

statistic	significance	mean	median	68% range	95% range
OS	σ -units:	4.0	4.5	(4.0, 5.1)	(3.4, 5.6)
	$-\log_{10} p$:	4.4	5.5	(4.4, 6.7)	(3.5, 8.0)

- power-law spectrum:

$$\Omega_{gw}(f) \equiv \frac{f}{\rho_{cr}} \frac{d\rho_{gw}}{df} = \frac{2\pi^2}{3H_0^2} A_{gw}^2 f_r^2 \left(\frac{f}{f_r} \right)^{-\gamma_{gw}+5}$$

$$f_r \equiv 1/\text{year}$$

Bayes factor = $E_{HD}/E_0 = 4400 \approx 4.1\sigma$ significance

Yu and Allen, *Stochastic gravitational-wave background search using data from five pulsar timing arrays*, [arXiv2512.08666](https://arxiv.org/abs/2512.08666)

