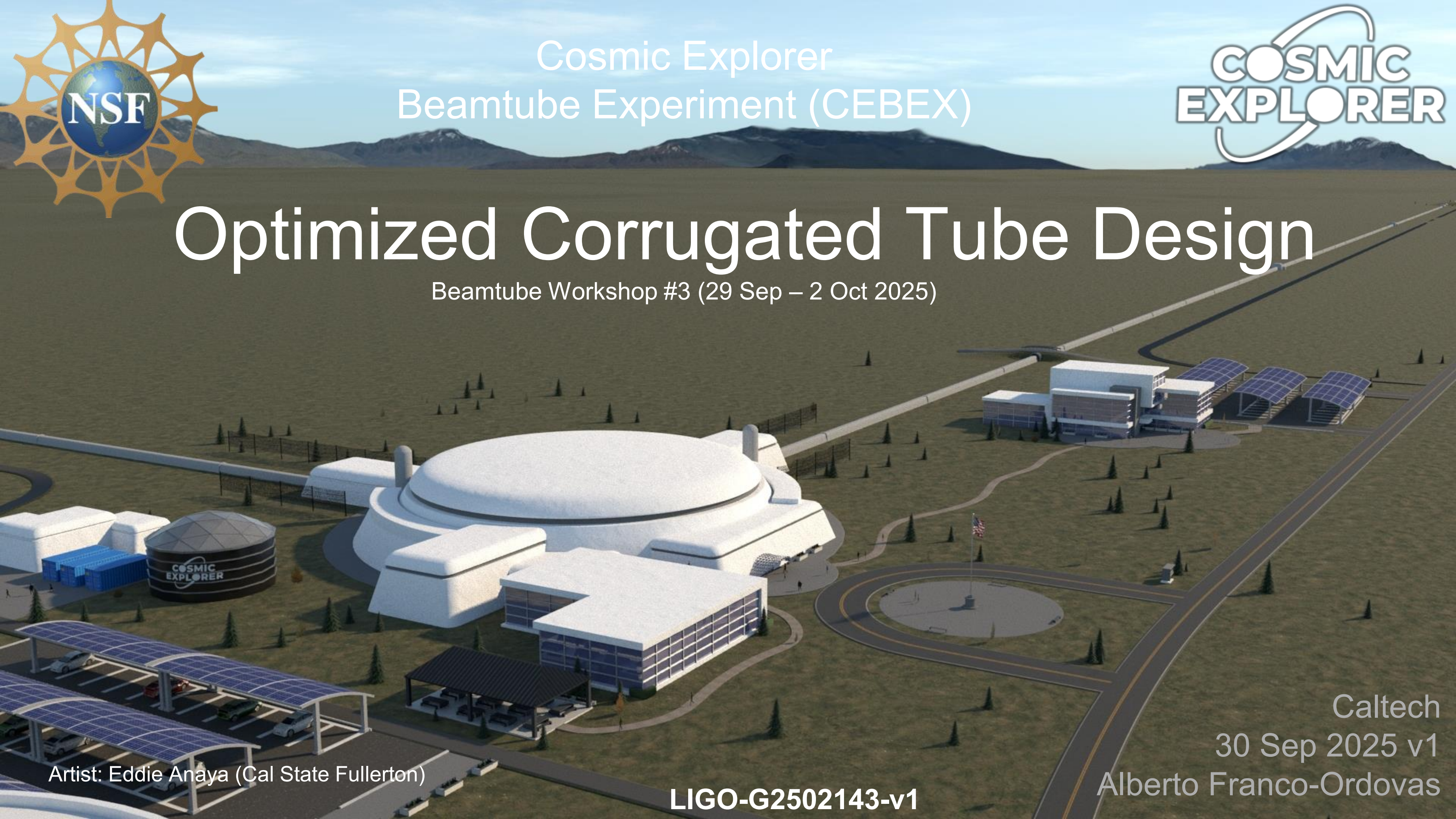
Cosmic Explorer
Beamtube Experiment (CEBEX)

Optimized Corrugated Tube Design

Beamtube Workshop #3 (29 Sep – 2 Oct 2025)



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30 Sep 2025 v1
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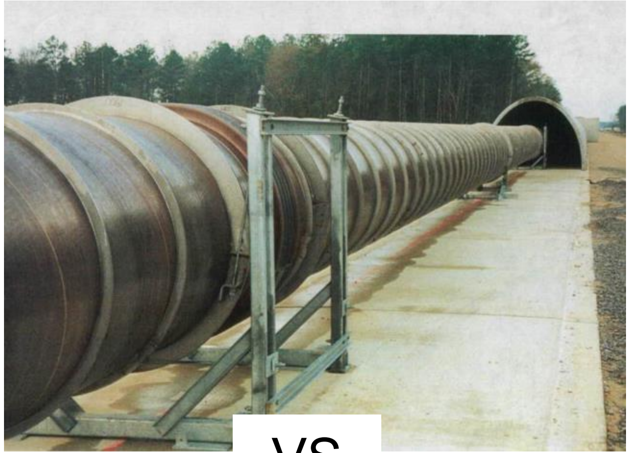
LIGO-G2502143-v1

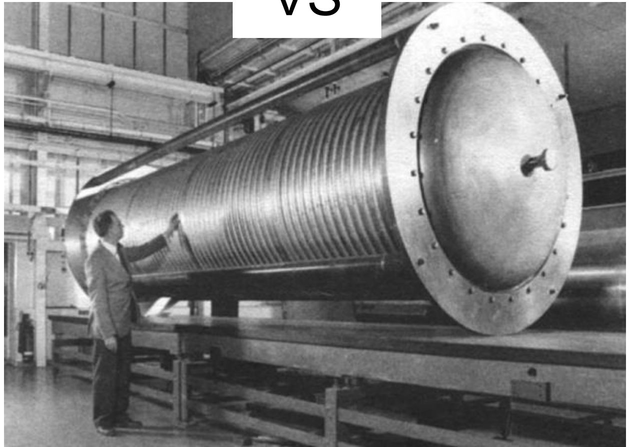
Scope

- ❑ Corrugated design: Why?
- ❑ Convolution geometry: Parameters & Selection
- ❑ FEA Model:
  - ❑ Geometry & Python Script
  - ❑ BC's + Set Up
  - ❑ Results
- ❑ Parametric runs
- ❑ Design Explorer (DX)
  - ❑ Results + Response Surfaces
- ❑ Optimization

Corrugated Tube: WHY!?

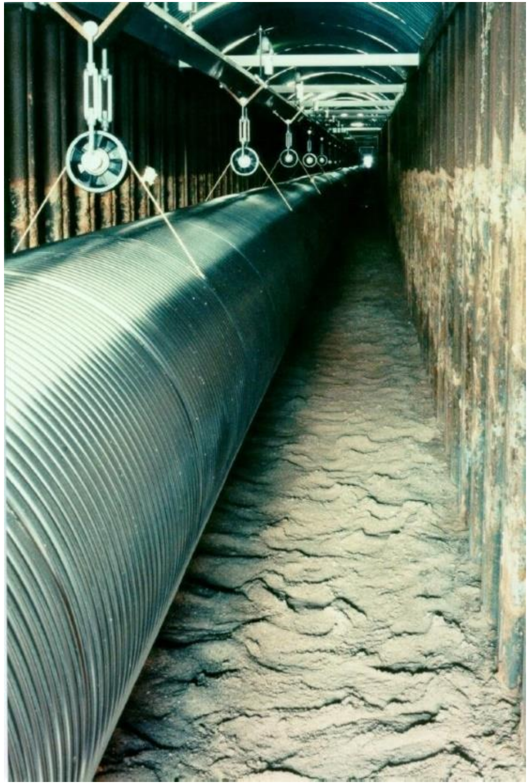


VS

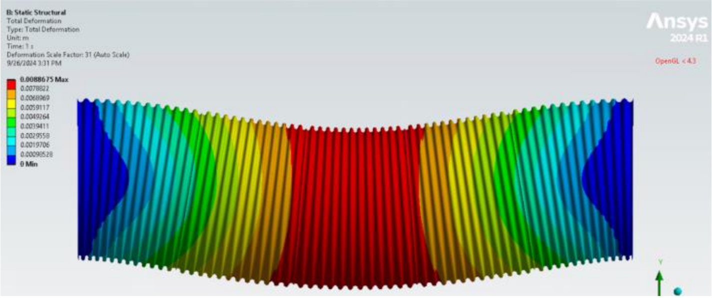


- Original LIGO:
  - 3.2 [mm] wall thickness x 20 [m] x 1.2 [m]
  - Stiffener rings (welded)
  - Expansion joints (40 [m])
- Corrugations could help with:
  - Improving buckling response (NO stiffener rings)
  - Absorbing thermal expansion (NO/Less exp. Joints)
  - Thinner wall
- 90’s vs 2020’s: better computational & manufacturing processes to help exploring

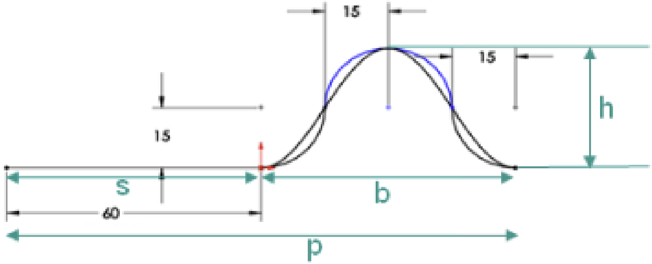
Previous Corrugated Tubes



GEO 600

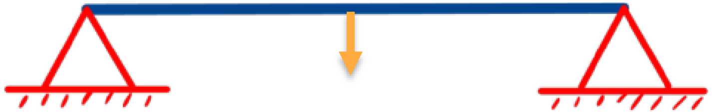


- GEO 600: Hung from the ceiling → For 40kms?!?!
- Supports are also to be taken into account
- Continued corrugation: too slinky if supports are not placed close enough
- Trade-off: singular convolution modules: Flat span + Corrugation

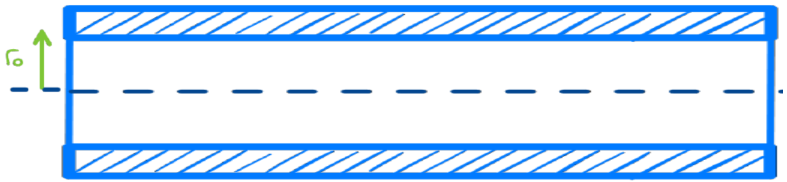


Convolution Geometry: Shape?

- Deflection (Sag): $y = -\frac{FL^3}{48EI}$



- Global Buckling (Euler): $P_{cr} = \frac{\pi^2 EI}{(KL)^2}$



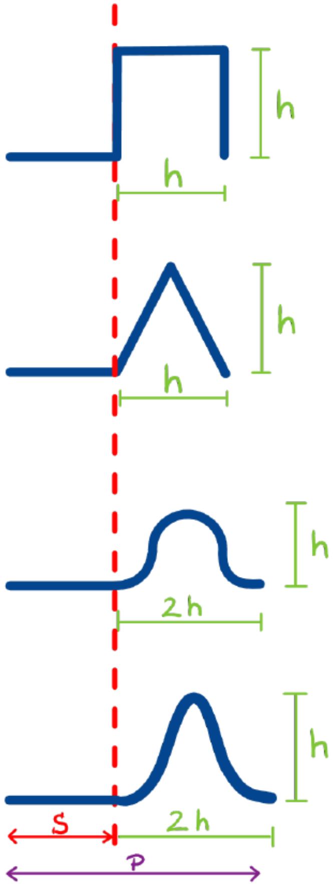
- For Thin Wall Members:

$$I = \pi R_m^3 t$$

  - R_m : Mean radius
  - t : Thickness

$$\bar{r} = \frac{\text{AREA}}{\text{LENGTH}} = \frac{\int_0^P r(p) \cdot dp}{L_{Total}}$$

Convolution Geometry: Sinusoidal



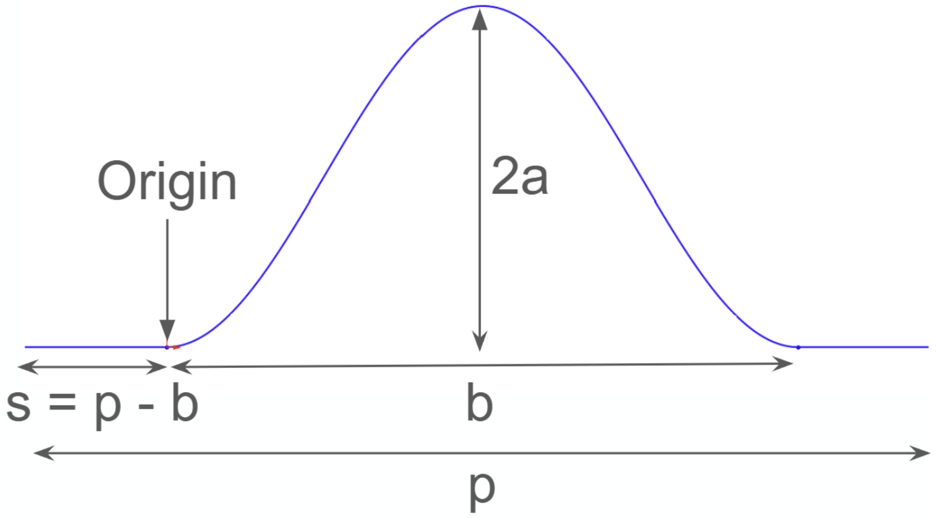
	R	I ($\propto R^3$)
$\bar{r}_{Sq} = \frac{r_0 \cdot P + h^2}{S + 3h} = 445$ [mm]	-22%	-80%
$\bar{r}_{Tri} = \frac{r_0 \cdot P + h^2/2}{S + \sqrt{5} \cdot h} = 507$ [mm]	-7%	-22%
$\bar{r}_{Cir} = \frac{r_0 P + h^2}{S + \pi \cdot h} = 519$ [mm]	-4%	-14%
$\bar{r}_{Sin} = \frac{r_0 \cdot P + h}{87.82^*} = 542$ [mm] #		

- Numerical values for:
  - h = 30 [mm]
  - P = 120 [mm]

- The sinusoidal corrugation provides a better performance

Convolution Geometry

- Sinusoidal Corrugation Parameters:
  - Amplitude: a
  - Period: b
  - Pitch: p
- Beamtube Parameters:
  - Thickness: t
  - Length: L (*)



$a*\{\sin[2*\pi*x/b-\pi/2]+1\}$ from 0 to b

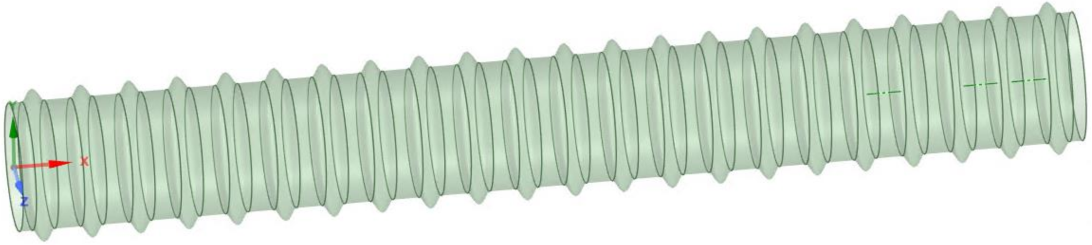
<u>4 Parameters:</u> How to find an "optimal" space? Through Parameter Correlation!

FEA Model → Input/Output Param → Design Points → Response Surface → Optimization

FEA Model: Geometry Definition

- Initial geometry: 2D Sketch

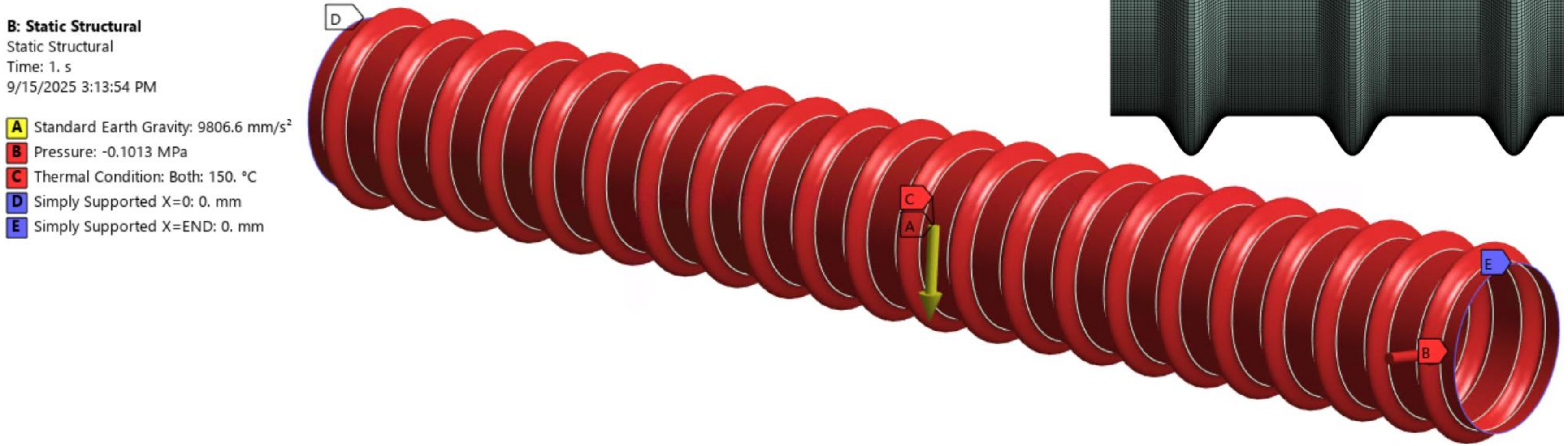
- Final geometry: 3D Shell



- A Python Script is needed to generate the geometry from scratch every run
- Otherwise, the input parameters (a, b, p) cannot be modified once the 2D sketch has been revolved and became a 3D Shell body.

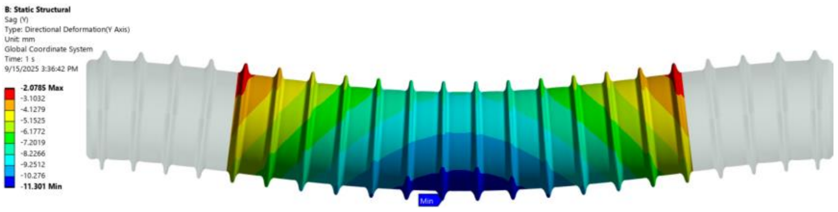
FEA Model: BC's & Configuration

- Boundary conditions implemented:
  - Gravity + Bakeout (@ 150ºC) + Inner Vacuum
  - All through "Named Selections": automatization
- Mesh defined: 10x10 [mm]

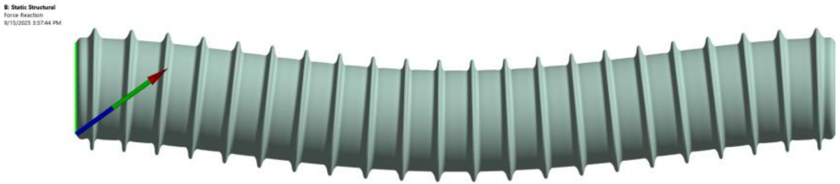


FEA Model: Results to observe

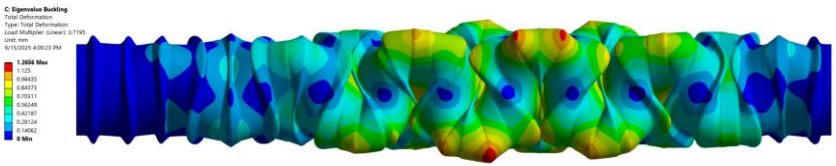
The following results were set as Output Parameters



- ❑ Sag: Deflection at the beamtube center



- ❑ Reaction Forces: X & Y axis at Supports

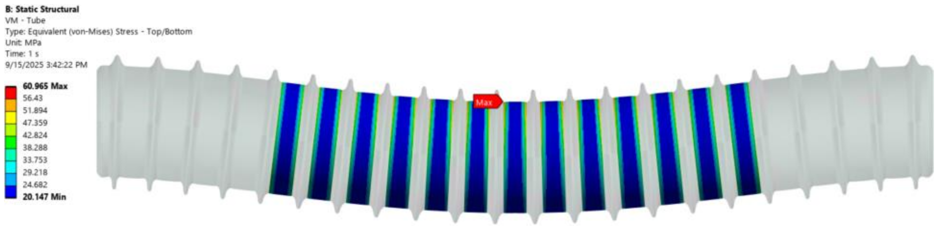


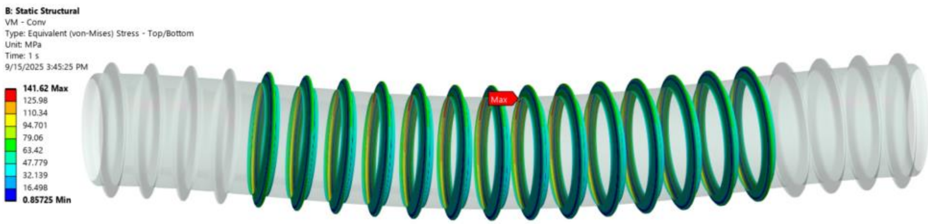
- ❑ Buckling Factor: Global buckling

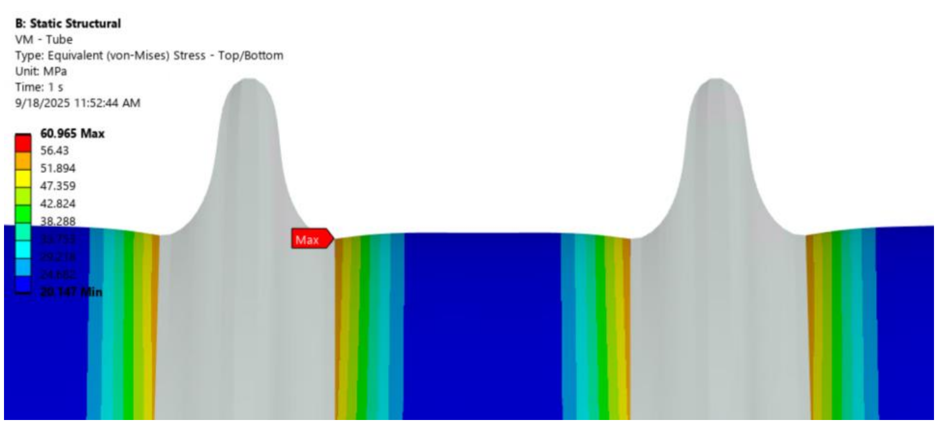
FEA Model: Results to observe

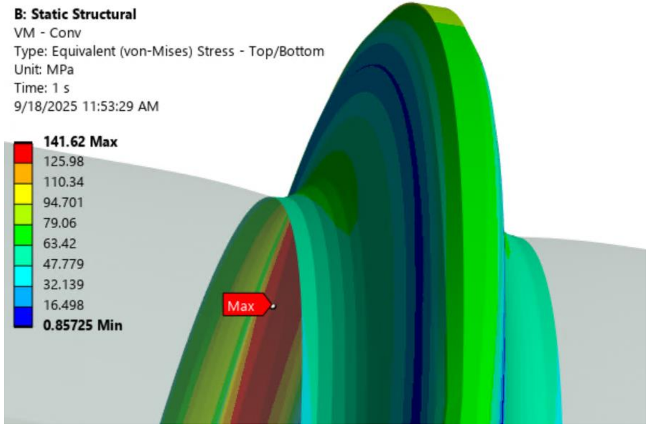
The following results were set as Output Parameters

- ❑ Von-Mises: Max Stress at flats & convolutions, for a better understanding





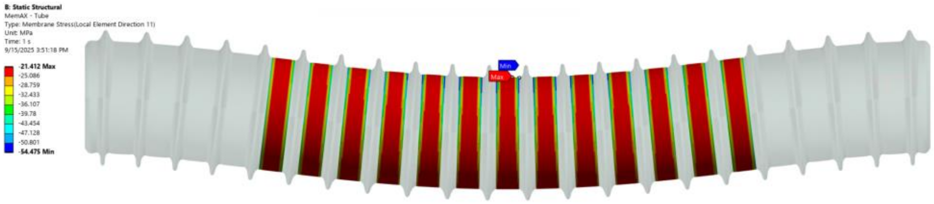


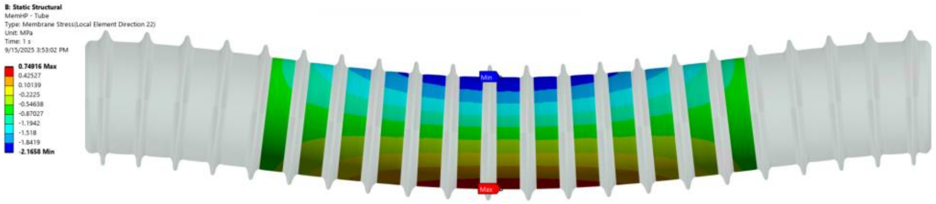


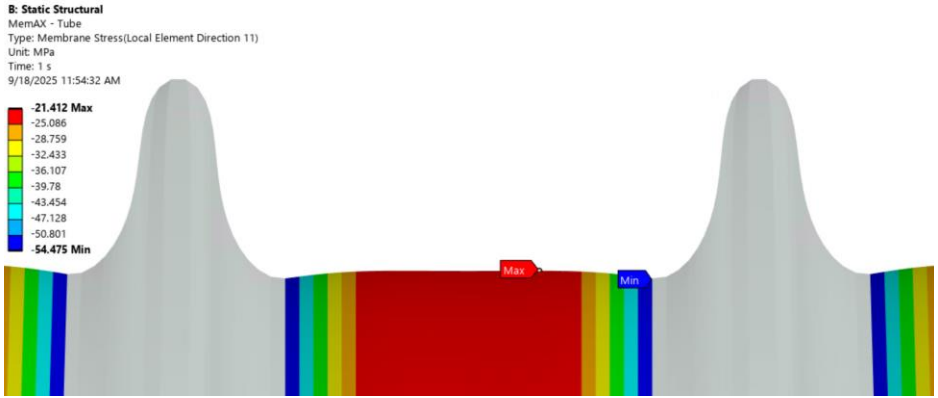
FEA Model: Results to observe

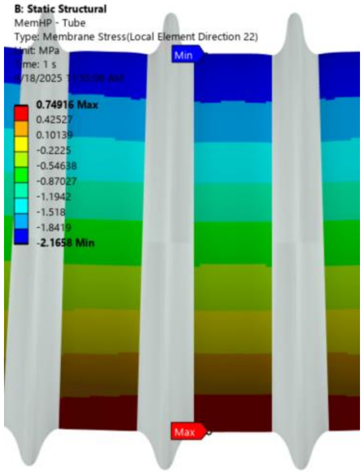
The following results were set as Output Parameters

- Membrane: Max/Min at flats for Axial (left) and Hoop (right)





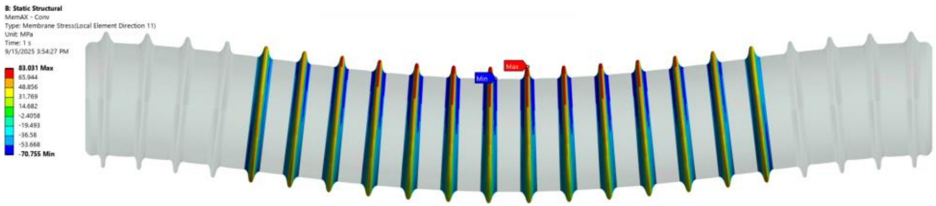


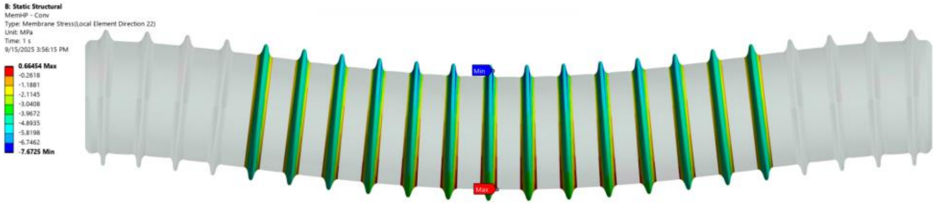


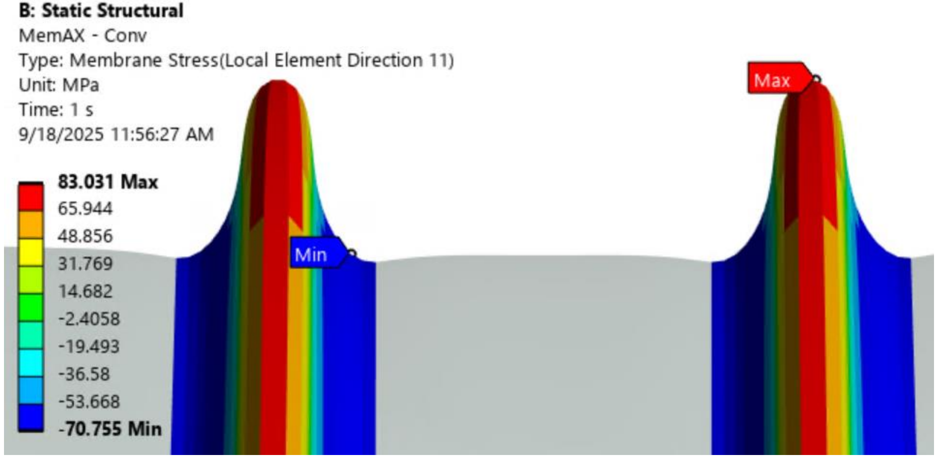
FEA Model: Results to observe

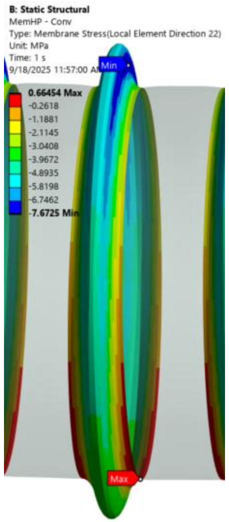
The following results were set as Output Parameters

- Membrane: Max/Min at corrugations for Axial (left) and Hoop (right)









FEA Model: Results to observe

The following results were set as Output Parameters

- To wrap-up: 14 output parameters total

RESULT	PARAMETERS	LOCATION
Sag (Y)	Min	Mid beamtube
Force Reaction	X, Y	Ends
Buckling Factor	Load Multiplier	Global
Von Mises – Tube	Max	Central flat: boundary w conv
Von Mises – Convolution	Max	Central conv: peak, inner face
Tube – Membrane Axial	Min, Max	Central flat
Tube – Membrane Hoop	Min, Max	Central flat
Conv – Membrane Axial	Min, Max	Central conv: peak & valley
Conv – Membrane Hoop	Min, Max	Central conv: valleys

Parametric Runs: Creating a data pointcloud

Outline of All Parameters

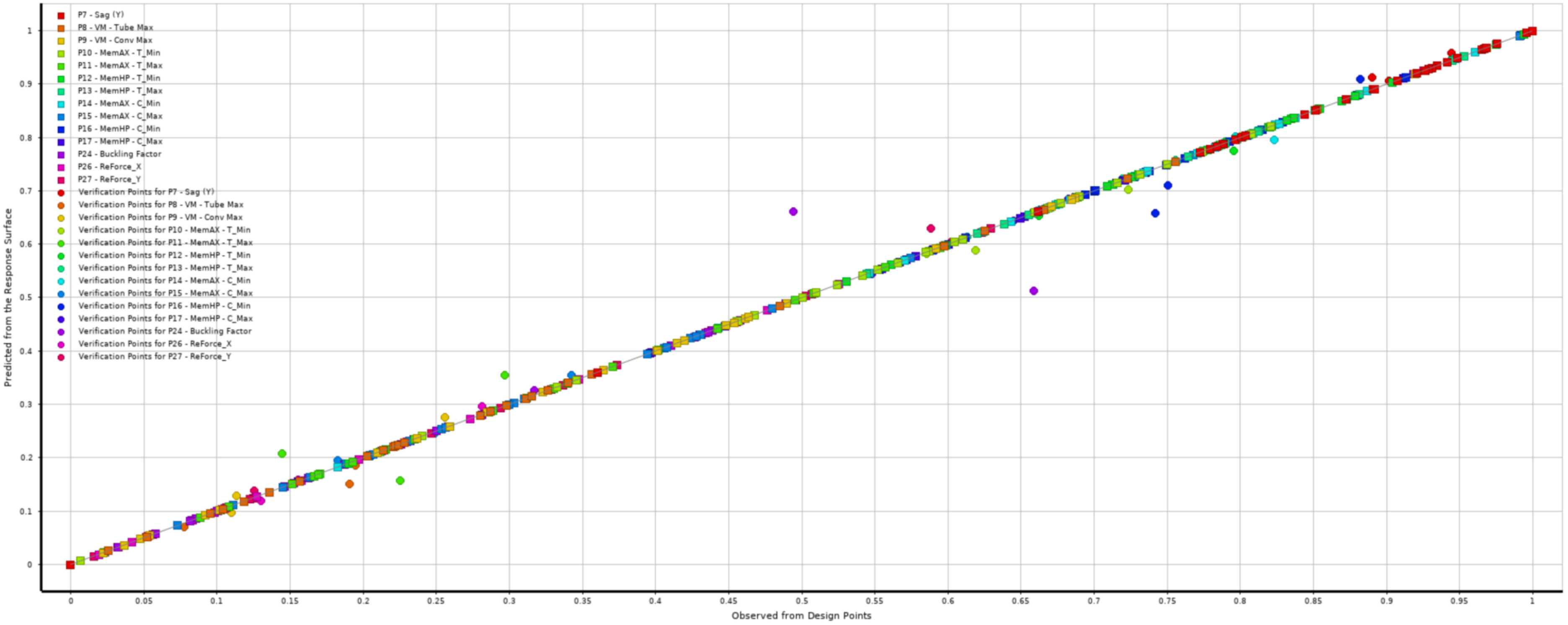
	A	B	C	D
1	ID	Parameter Name	Value	Unit
2	⊟ Input Parameters			
3	⊟ Geometry (A1)			
4	P20	a	58	mm
5	P21	b	215	mm
6	P22	p	470	mm
7	⊟ Static Structural (B1)			
8	P25	t	2.7	mm
*	New input parameter	New name	New expression	
10	⊟ Output Parameters			
11	⊟ Static Structural (B1)			
12	P7	Sag (Y)	-11.301	mm
13	P8	VM - Tube Max	60.965	MPa
14	P9	VM - Conv Max	141.62	MPa
15	P10	MemAX - T_Min	-54.475	MPa
16	P11	MemAX - T_Max	-21.412	MPa
17	P12	MemHP - T_Min	-2.1658	MPa
18	P13	MemHP - T_Max	0.74916	MPa
19	P14	MemAX - C_Min	-70.755	MPa
20	P15	MemAX - C_Max	83.031	MPa
21	P16	MemHP - C_Min	-7.6725	MPa
22	P17	MemHP - C_Max	0.66454	MPa
23	P26	ReForce_X	7781.5	N
24	P27	ReForce_Y	5577.4	N
25	⊟ Eigenvalue Buckling (C1)			
26	P24	Buckling Factor	3.7195	

- Parameters Set: all parameters displayed
- Need to define range of study for input Par
  - a: 40 to 80 [mm], b: 120 to 240 [mm], p: 300 to 700 [mm]
  - t: 2 to 3.5 [mm]
- Need to define Design of Experiment Method
  - Central Composite Design: Face centered
  - This settles the way DX will select the Design Points to calculate so they properly cover the design space.
- Once all the DPs are done: Meta Model
  - Surface type: Kriging, variable.
  - This defines the algorithm used to study the correlation among DPs to create response surfaces (prediction)

Design Explorer (DX) Results

Goodness of Fit

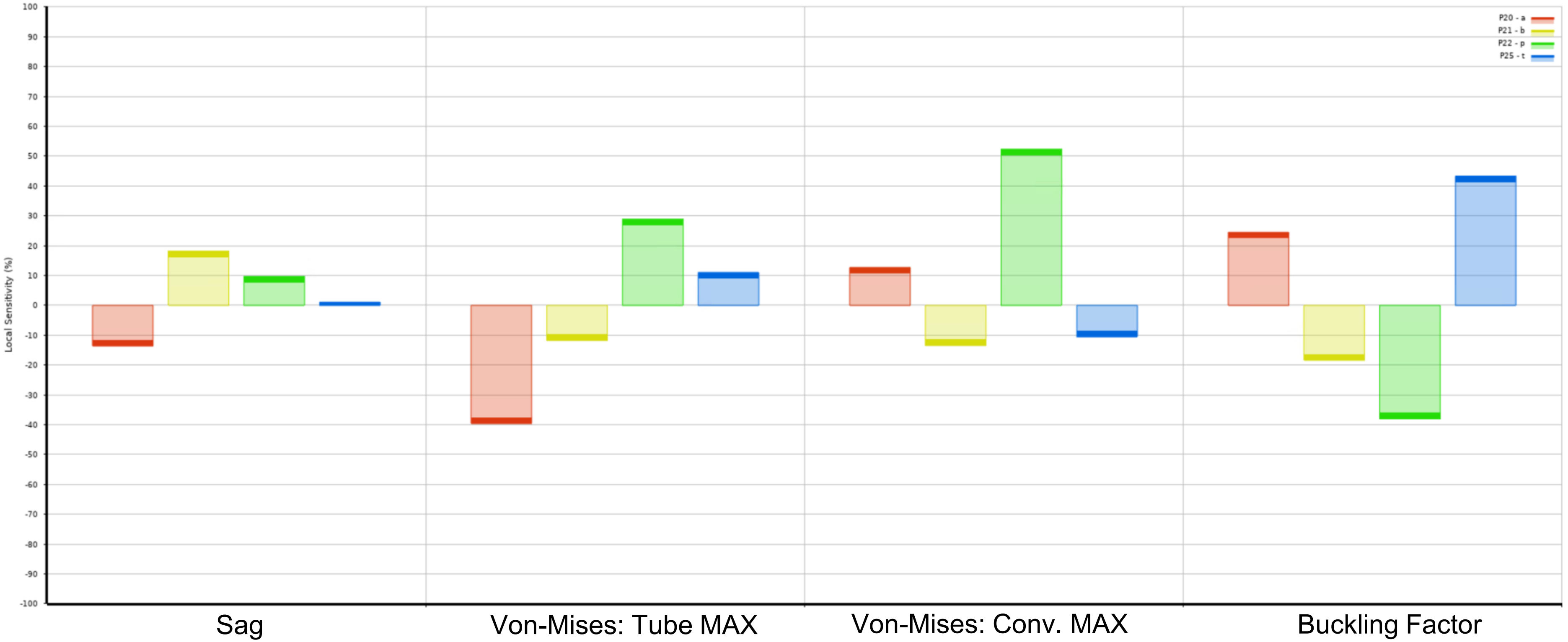
- Explores R^2 in normalized values: Verification Points (X) vs. Predicted Points (Y)
- A few points off-axis: still a good & solid model



Design Explorer (DX) Results

Local Sensitivity

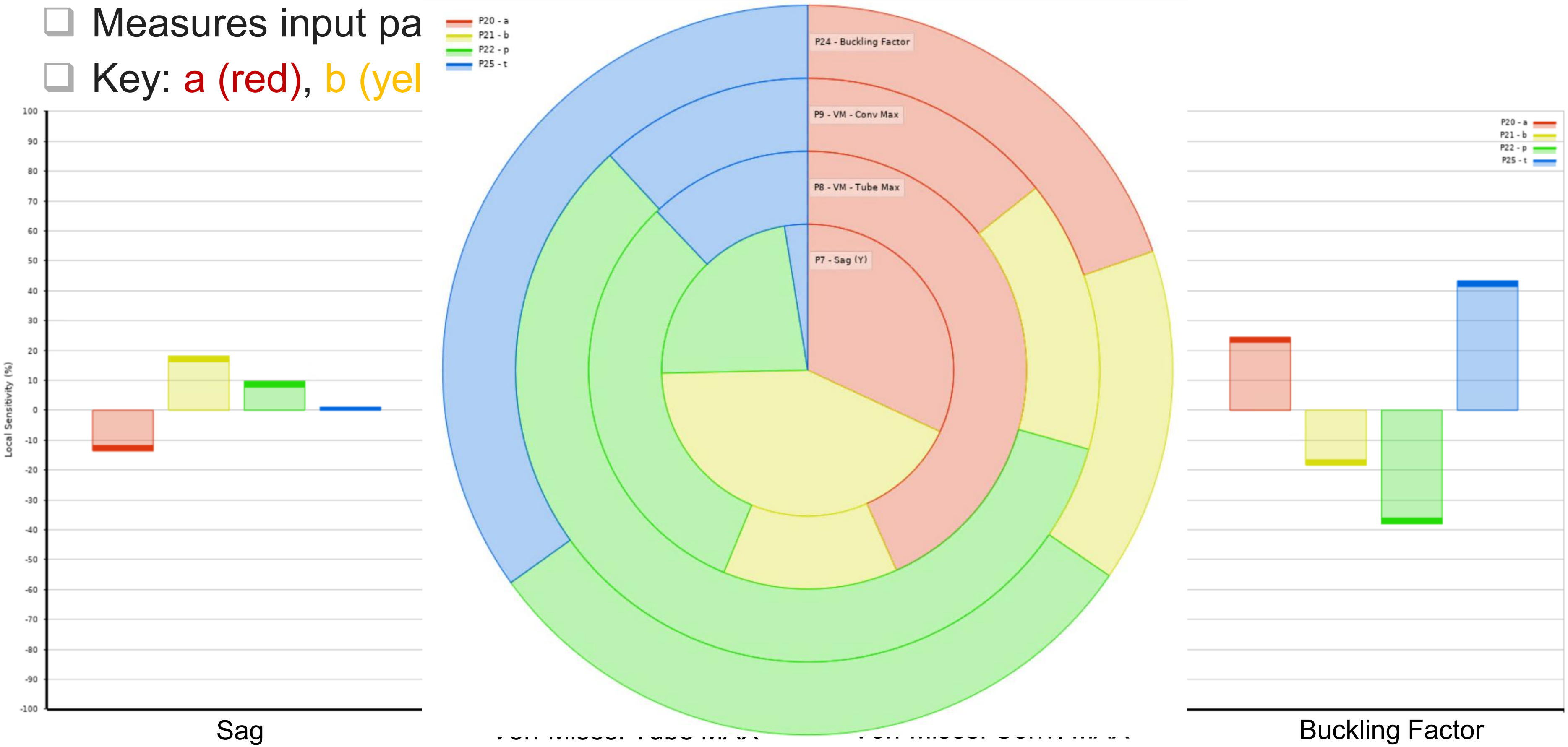
- Measures input parameters impact on output parameters
- Key: a (red), b (yellow), p (green), t (blue)



Design Explorer (DX) Results

Local Sensitivity

- Measures input pa
- Key: a (red), b (yel

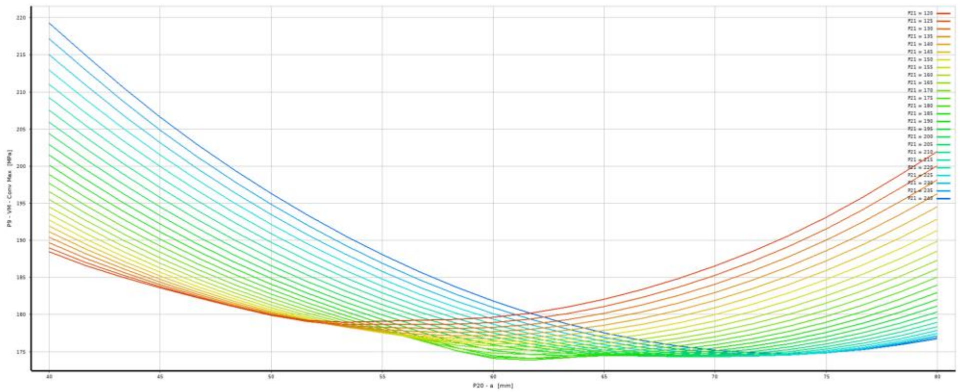


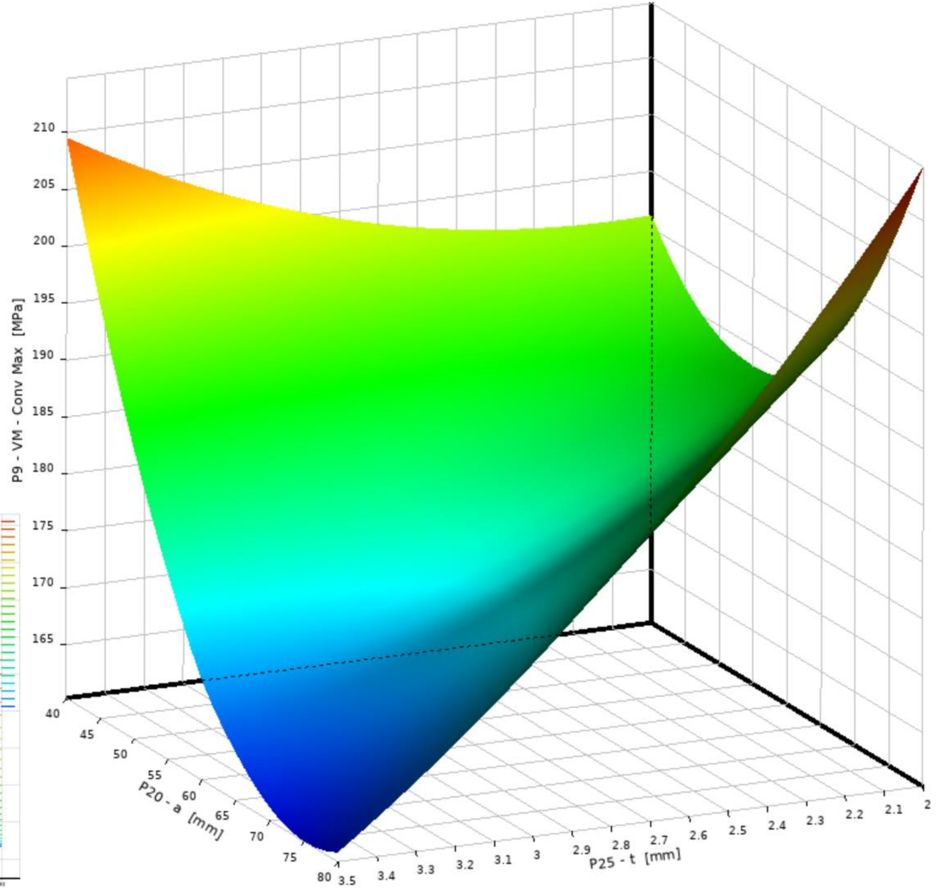
Design Explorer (DX) Results

Response Surfaces – 2D & 3D

- Visual display to understand the behavior of the input/output parameters

- Allows 2 input (x, y) vs. 1 output (z)
- Rest of inputs: modifiable values
- Also, 2D graphs w isolines

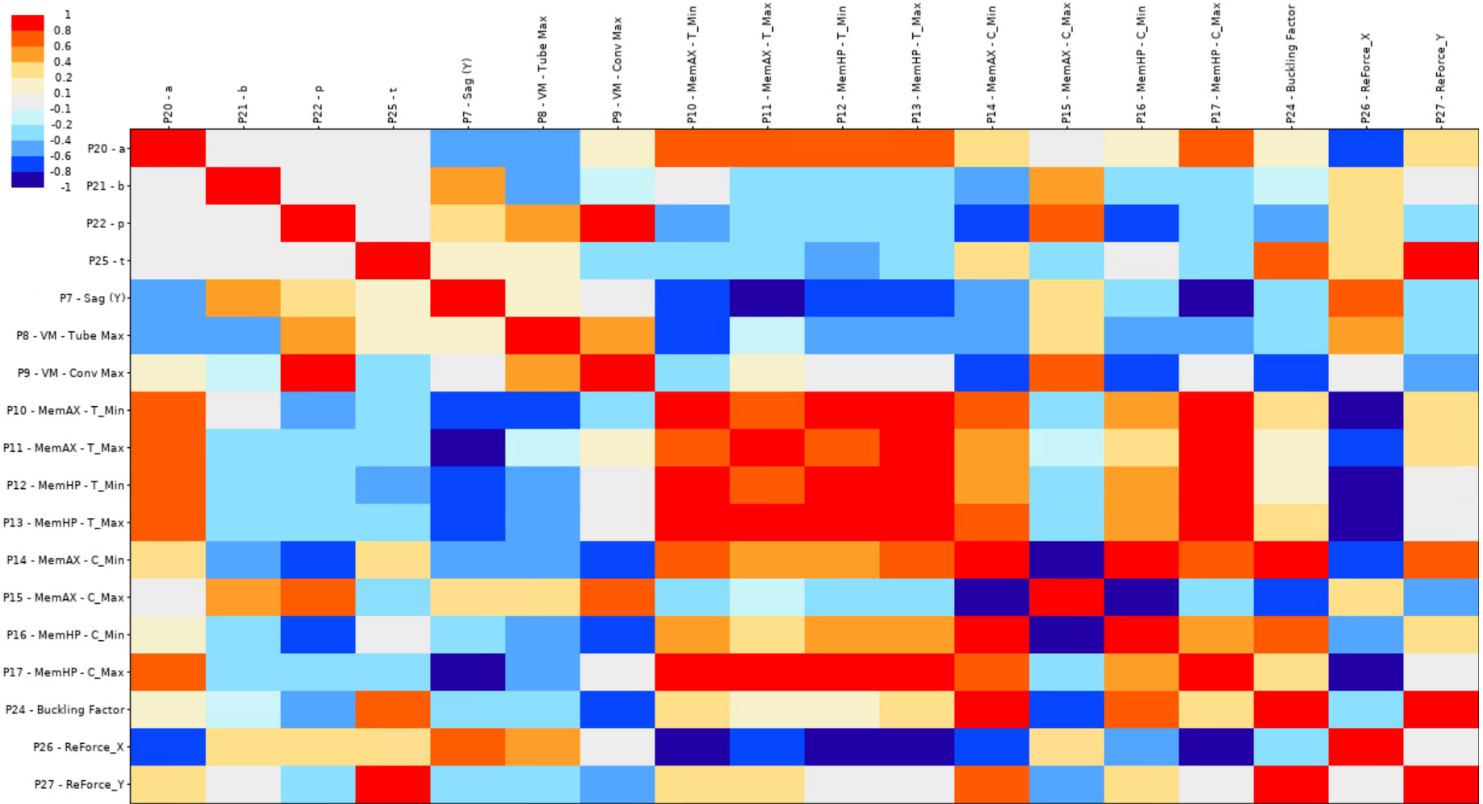




Design Explorer (DX) Results

Correlation Matrix

- Interaction between ALL parameters (output/output included! → Trade-offs)



DX Optimization Tool

- Design Points & Verification Points calculated + Results checked: OK!
- DX will cross the Design Space and come with a set of Candidate Points
- Set the target values for the desired parameters:
  - Maximize Buckling Factor, consider values only > 3: Rule of thumb for ASME cod.
  - Minimize t: set target of 2 [mm] as it is the lowest value for t
  - Von-Mises < 138 [Mpa]
  - Minimize Sag, consider values only < 10 [mm]

Table of Schematic F4: Optimization

	A	B	C	D	E	F	G	H	I
1	Name	Parameter	Objective			Constraint			
2			Type	Target	Tolerance	Type	Lower Bound	Upper Bound	Tolerance
3	Maximize P24; P24 >= 3	P24 - Buckling Factor	Maximize	10		Values >= Lower Bound	3		0.001
4	Minimize P25	P25 - t	Minimize	2		No Constraint			
5	Minimize P8; P8 <= 138 MPa	P8 - VM - Tube Max	Minimize	0		Values <= Upper Bound		138	0.001
6	Minimize P9; P9 <= 138 MPa	P9 - VM - Conv Max	Minimize	0		Values <= Upper Bound		138	0.001
7	Minimize P7; P7 >= -10 mm	P7 - Sag (Y)	Minimize	0		Values >= Lower Bound	-10		0.001
*		Select a Parameter							

DX Optimization Tool

- 3 candidate points are chosen: Verification can be requested as part of the process
- In this case, all 3 pivot around the same Design Space area (very close values)

7	⊟ Optimization Method			
8	MOGA	The MOGA method (Multi-Objective Genetic Algorithm) is a variant of the popular NSGA-II (Non-dominated Sorted Genetic Algorithm-II) based on controlled elitism concepts. It supports multiple objectives and constraints and aims at finding the global optimum.		
9	Configuration	Generate 100 samples initially, 100 samples per iteration and find 3 candidates in a maximum of 20 iterations.		
10	Status	Converged after 718 evaluations.		
11	⊟ Candidate Points			
12		Candidate Point 1	Candidate Point 2	Candidate Point 3
13	P20 - a (mm)	46.841	46.967	47.253
14	P21 - b (mm)	199.3	199.3	199.3
15	P22 - p (mm)	350.99	353.71	366.27
16	P25 - t (mm)	★ 2.5468	★ 2.5469	★ 2.5481
17	P7 - Sag (Y) (mm)	★★★ -9.8403	★★★ -9.8484	★★★ -9.7225
18	P8 - VM - Tube Max (MPa)	★★ 56.898	★★ 57.002	★★ 57.916
19	P9 - VM - Conv Max (MPa)	– 120.67	– 121.35	– 124.44
20	P24 - Buckling Factor	– 4.0941	– 4.0696	– 3.9314

- Verification Point:

- Sag: -9.87 [mm]
- VM Tube: 56.92 [MPa]
- VM Conv: 120.8 [MPa]
- Buckling: 4.4

Corrugated Summary

- On paper: a reasonable solution for a corrugated tube is possible
  - Most likely through ASME Method B: elastoplastic qualification
- 4 or 5 parameters are still manageable to get a good parametric model, suitable for optimization
- Final definition of values depends on engineering & tolerancing
  - t = 2.5468 [mm] is not a suitable value for manufacturing…
  - Post-processing needed after optimization
- Manufacturability of the corrugated beamtube to be studied by vendors as per RFI launched last month.