Sensor Noise Coupling to Sensor Output

Brian Lantz

Aug 13, 2025, LIGO-T2500279-v1

1 Introduction

This is a simple calculation to show how the noise of various sensors in a multi-sensor feedback loop appears in the signals for those sensors. Figure 1 shows a simplified control sketch for a 3 sensor loop. The plant is P, K is the controller, there are 3 sensors, S_L, S_M, and S_H, representing a low-frequency, a mid-frequency, and a high-frequency sensor. We define these sensors to be noisy, but perfectly calibrated. We set the calibration be 1. We ignore the sensor blocks for the rest of the calculation. Details of the actual readout chain can be found in D1001575, where the calibration before the complimentary blends is 1 count/ nm, so there are some factors of 1e-9 in the code to get everything into meters. Again, for this simplified calculation, the sensor blocks are 1, so the units of the input-referred sensor noise and the sensor readout are the same. The sensors are in the drawing to make it easier to point to. The input referred noise of the sensors is n_L, n_M, and n_H. The sensors are combined with complementary filters F_L, F_M, and F_H. Because they are complementary, $F_L + F_M + F_H \equiv 1$. We also add a disturbance to the plant, d.

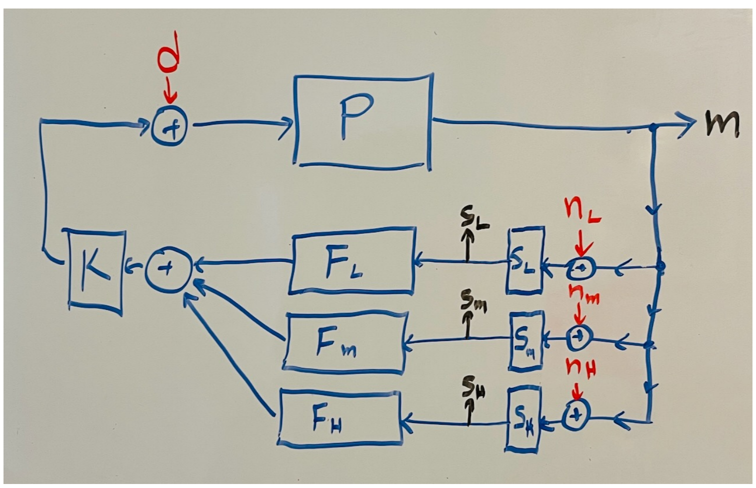

Figure 1: Sketch of simplified servo with 3 noisy sensors

There are four outputs of the calculation. The motion of the plant is m. The signals from the 3 sensors are s_L, s_M, and s_H. The signal is for a particular sensor is just the motion plus the noise of that sensor, e.g. $s_L = m + n_L$.

The motion of the plant is

$$\begin{aligned} m = P * d + P * K * (&F_L * S_L * (n_L + m) \\ &+F_M * S_M * (n_M + m) \\ &+F_H * S_H * (n_H + m)) \end{aligned} \tag{1}$$

2 Only 2 sensors

The first simplification is assume JUST 2 SENSORS. This is because the HAM-ISI has only 2 right now for most of the loops. We'll see that a two and three are qualitatively different in a moment.

With just 2 sensors, and the sensor response set to 1, the motion equation becomes

$$m = P * d + P * K * (F_L * (n_L + m) + F_H * (n_H + m)) \tag{2}$$

We rearrange, and since the blends are complementary, m becomes

$$m = P * d + P * K * m + P * K * (F_L * n_L + F_H * n_H) \tag{3}$$

The because the sensor are complementary, the open loop gain is just $G = P * K$, so we get

$$m = P * d + G * m + G(F_L * n_L + F_H * n_H) \tag{4}$$

$$m * (1 - G) = P * d + G(F_L * n_L + F_H * n_H) \tag{5}$$

$$m = \frac{P * d}{(1 - G)} + \frac{G}{(1 - G)}(F_L * n_L + F_H * n_H) \tag{6}$$

and when the loop gain G is large, this becomes

$$m \approx \frac{P * d}{(1 - G)} + -1 * (F_L * n_L + F_H * n_H) \tag{7}$$

So far this looks just like every other time I've done this. Now consider what the sensor signals look like. The low frequency sensor signal is

$$S_L = m + n_L \tag{8}$$

$$S_L = \frac{P * d}{(1 - G)} - (F_L * n_L + F_H * n_H) + n_L \tag{9}$$

$$S_L = \frac{P * d}{(1 - G)} + ((1 - F_L) * n_L - F_H * n_H) \tag{10}$$

$$S_L = \frac{P * d}{(1 - G)} + F_H * (n_L - n_H) \tag{11}$$

This is interesting - At the sensor output, the signal from the sensor noise is the noise of this sensor times THE OTHER blend filter. This seems a bit odd, but it makes sense; the "low frequency" sensor has noise at low and high frequency. The low frequency part is passed by the blend filter, and the servo control impresses that onto the table as real motion. The motion is equal and opposite the sensor noise such that the sum is nearly zero. Thus, the plant motion is moving as -1 times the low frequency sensor noise. When this motion is added to the sensor, it cancels the low frequency part of the noise at the sensor output.

Likewise, the signal from the the high-frequency sensor is

$$S_H = \frac{P * d}{(1 - G)} + F_L * (n_H - n_L) \tag{12}$$

We notice several important things. First, disturbances of the plant appear in the same way at both sensors, but the sensor noise does not. The difference is because of where the noise enters the loop - the plant disturbance is common to the sensors, but the sensor noise is not - so maybe it isn't so surprising that the the disturbance appears in a common way, but the sensor noise does not. Second, the shape of the noise is different in the two sensor outputs, and it is shaped by the other filter. For spectral measurements, where the noises are independent and the phase is lost, effectively the noise in the two sensors is indistinguishable. This isn't strictly true (there is a sign difference), but I'm not sure how to take advantage of that.

For spectral measurements, we need to replace the instantaneous noise with the average spectra, so equation 11 becomes

$$S_L = \frac{P * d}{(1 - G)} + F_H * \sqrt{\tilde{n}_L^2 + \tilde{n}_H^2} \tag{13}$$

This is interesting because it means that you can't figure out which sensor is causing the noise by simply changing the blend filter while watching the in-loop sensor output.

However, with a witness sensor, you can (in principle) tell see the difference, because the noises are filtered by different filters. This could be a powerful way to help track down issues, especially if one has a good rotation sensor.

However, for 3 sensors, the noise may be distinguishable. As we'll see, this is because the complement of one blend is the sum of the other 2.

3 Real motion vs. sensor readout

A great deal of care needs to be taken when one interprets the sensor readouts if the loops are dominated by sensor noise. For the ISI, we often use the GS-13 readout as the monitor of the platform motion. When the motion of the platform is limited by sensor noise, this will not be accurate (as we all know). For example, for the rx and ry loops, the signals are often completely dominated by sensor noise, so it's useful to compare the sensor signal to the actual motion. We begin by setting the external disturbance d to zero (it's a linear system, so we can put it back in later if we want). Equation 7 becomes

$$m \approx -F_L * n_L - \boldsymbol{F_H} * n_H \tag{14}$$

However, the equation for the measured signal from the high frequency sensor (equation 12) becomes

$$S_H \approx -F_L * n_L + \boldsymbol{F_L} * n_H \tag{15}$$

The difference is shown in bold. For the high-pass sensor, the real platform motion includes the high-pass filter times the noise of the high-frequency sensor, but the in-loop measurement shows that noise very differently. For example, typically when you look at the signal for the rx and ry GS-13s, the signal matches the sensor noise at low frequency, even though the real platform motion is well below that.

Above the blend frequency, it is often true that the noise of the low-frequency sensor dominates the platform motion. At those frequencies F_L will also be less than F_H, so at those frequencies, the GS-13 signal will be a good representation of the platform motion. Perhaps this is not surprising - if the platform is dominated by CPS noise above the blend frequency, then the GS-13 can measure that motion.

4 Excess Noise

All the HAM-ISI seem to have excess noise around the microseism when the ground motion is large. This seems to be some sort of 'sensor noise' in that it is signal in some sensor which does not seem to come from a plant disturbance like d. This could be ground rotation being sensed by the CPS - this appears as a 'noise' in the CPS sensor. This could be a tilt coupling to the GS-13s, or it could be something else like magnetic coupling or non-nonlinearities.

It is straightforward to calculated the equivalent open-loop noise, but you can't tell which sensor is responsible with only 2 in-loop sensors. In equation 13, we saw that you can't figure out which sensor is causing the noise by simply changing the blend filter. If you assign the signal to sensor noise, then eqn. 13 becomes

$$S_L \approx F_H * \sqrt{\tilde{n}_L^2 + \tilde{n}_H^2} \tag{16}$$

$$S_L/F_H \approx \sqrt{\tilde{n}_L^2 + \tilde{n}_H^2}, \text{ and} \tag{17}$$

$$S_H/F_L \approx \sqrt{\tilde{n}_L^2 + \tilde{n}_H^2} \tag{18}$$

This shows several things: you can't tell which sensor is causing the noise, the equivalent open loop noise is the same - for either sensor, and changing the blend filter doesn't tell you anything new.

However - the actual table motion is changing, of course - so a witness sensor, or coupling into other directions (ie tilt-horizontal coupling) **will** be changing.

5 Three or more sensors

Now consider the case with 3 sensors. We assume correctly calibrated sensors with complementary filters, so equation 1 becomes

$$\begin{aligned} m = P * d + P * K * (F_L * (n_L + m) \\ + F_M * (n_M + m) \\ + F_H * (n_H + m)) \end{aligned} \tag{19}$$

and equation 7 becomes

$$m \approx \frac{P * d}{(1 - G)} - 1 * (F_L * n_L + F_M * n_M + F_H * n_H) \tag{20}$$

and the signal seen by s_L becomes

$$S_L = m + n_L \tag{21}$$

$$S_L = \frac{P * d}{(1 - G)} - (F_L * n_L + F_M * n_M + F_H * n_H) + n_L \tag{22}$$

$$S_L = \frac{P * d}{(1 - G)} + ((1 - F_L) * n_L - F_M * n_M - F_H * n_H) \tag{23}$$

$$S_L = \frac{P * d}{(1 - G)} + ((F_M + F_H) * n_L - F_M * n_M - F_H * n_H) \tag{24}$$

When we compare this to equation 11 we see that the noise of the sensors appears at the sensor output with different filters. This is different than the case with 2 sensors. This is because, with a 3 sensor blend, the complement for the low-pass F_L is different than F_H.

6 How to use this to your advantage

This section is left as an exercise to the reader. It seems like this could be useful for stage 1 of the BSC ISI, or with the addition of a CRS to the HAM ISI or stage 2 of the BSC ISI. Using the one of the sensors out-of-loop as a witness also seems like a good idea.