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On the incompatibility of mass-ratio-spin correlation and IID component spins

Will M. Farr [1,2] and Ben Farr [3]

[1] Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794, USA
[2] Center for Computational Astrophysics, Flatiron Institute, 162 Fifth Avenue, New York, NY 10010, USA
[3] Institute for Fundamental Science, Department of Physics, University of Oregon, Eugene, OR 97403, USA

ABSTRACT

A correlation between spin and mass ratio is a feature of the population of merging compact binaries that has been identified at high statistical significance. Here we note that such a feature is incompatible with the common assumption of independently and identically distributed component spin vectors. It is also incompatible with a relaxed assumption of component spin distributions that are different, but both independent of the mass ratio.

1. INTRODUCTION

While the spins of compact objects in merging binaries have the potential to reveal key aspects of binary formation and evolution, constraining spin properties of individual binaries is difficult. Through hierarchical inference, the many uncertain measurements of spin in binaries are combined across the probabilistic Gravitational-Wave Transient Catalog (GWTC) to infer the underlying astrophysical distribution of spins.

Attempting to constrain population details of all six degrees of freedom spreads thin what little information strain observations provide. Instead, assertive models and priors are used to better leverage this data, enabling interpretable constraints. Some degrees of freedom can be neglected as nuisances, such as azimuthal components [4] It is common for inference efforts to focus on spin magnitudes and tilts (the angle between the orbital and component angular momenta), as isotropy is a robust signature of dynamical formation. Furthermore, it is often assumed that the spins of the primary (most massive) and secondary (least massive) components are independent draws from the same distribution, i.e., are independently and identically distributed (IID). It is also often assumed that the distribution of component spins is independent of the mass ratio $q = m_2/m_1 \leq 1$.

Alternatively, inference efforts can focus on constraining combinations of component spins that are better constrained by strain observations. The best-constrained combination is the effective inspiral spin,

$$\chi_{\mathrm{eff}} = \frac{a_1 \cos\theta_1 + q\, a_2 \cos\theta_2}{1+q}, \tag{1}$$

where a_i are component spin magnitudes, θ_i are component spin tilts, and $q = m_2/m_1 \leq 1$ is the mass ratio.

T. A. Callister et al. (2021) was the first to report observational evidence of the binary black hole population exhibiting a joint distribution of χ_{eff} and mass-ratio q with correlated structure. That work found that the population average χ_{eff} at fixed q followed

$$E\left[\chi_{\mathrm{eff}} \mid q\right] \simeq 0.19^{+0.11}_{-0.09} - 0.46^{+0.29}_{-0.28}\left(q - 0.5\right). \tag{2}$$

Subsequent works have confirmed this relation, and tightened the constraints on the slope and intercept of this linear χ_{eff} - q relation (The LIGO Scientific Collaboration et al. 2021; R. Abbott et al. 2023).

Note that these relationships imply

$$E\left[\chi_{\mathrm{eff}} \mid q = 1\right] \simeq 0 \tag{3}$$

and

$$\left.\frac{\partial E\left[\chi_{\mathrm{eff}} \mid q\right]}{\partial q}\right|_{q=1} \simeq -0.5 < 0. \tag{4}$$

Email: wfarr@flatironinstitute.org, bfarr@uoregon.edu

[4] Though azimuthal separation between component spins may be astrophysically informative; see, e.g., D. Gerosa et al. (2013).

The purpose of this note is to make it clear that if the population does indeed have correlation between mass ratio and effective inspiral spin, then the component spins *cannot* be IID and independent of the mass ratio, as is commonly assumed in many analyses.

2. IMPLICATIONS FOR THE SPIN POPULATION DISTRIBUTION

Since the effective spin only depends on the component of spin aligned with the orbit, we'll define $\chi_{i,\hat{L}} = a_i \cos\theta_i$. Let's consider the conditional expectation value of the $\chi_{\rm eff}$ distribution,

$$E\left[\chi_{\rm eff} \mid q\right] = \frac{1}{1+q}\left(E\left[\chi_{1,\hat{L}}\right] + qE\left[\chi_{2,\hat{L}}\right]\right). \tag{5}$$

If the component spins are IID, then the expectation values on the right are equal, the $(1+q)$ terms cancel, and the conditional expectation value is independent of q; this is manifestly incompatible with Eq. (2).

What if the spin distributions for the two components are different, but independent of q? Then, evaluating Eq. (5) at $q = 1$, we have

$$E\left[\chi_{\rm eff} \mid q = 1\right] = \frac{E\left[\chi_{1,\hat{L}}\right] + E\left[\chi_{2,\hat{L}}\right]}{2}, \tag{6}$$

and the relations in Eq. (2) imply

$$E\left[\chi_{1,\hat{L}}\right] + E\left[\chi_{2,\hat{L}}\right] \simeq 0. \tag{7}$$

Taking a derivative of Eq. (5) with respect to q at $q = 1$ (and exploiting that the expectation values are independent of q because the distributions are independent of q) produces

$$\left.\frac{\partial E\left[\chi_{\rm eff} \mid q\right]}{\partial q}\right|_{q=1} = \frac{E\left[\chi_{2,\hat{L}}\right] - E\left[\chi_{1,\hat{L}}\right]}{4} \tag{8}$$

and the relations in Eq. (2) imply

$$E\left[\chi_{2,\hat{L}}\right] - E\left[\chi_{1,\hat{L}}\right] \simeq -2. \tag{9}$$

Taken together, Eqs. (7) and (9) imply

$$E\left[\chi_{1,\hat{L}}\right] \simeq 1, \tag{10}$$

and

$$E\left[\chi_{2,\hat{L}}\right] \simeq -1. \tag{11}$$

Such expected values for the spins are not (quite) *un-physical*, but they are very unusual, and unlikely to result from any astrophysical process that we are aware of; we conclude that even different spin distributions for the two components of the binary *independent of the mass ratio* q are incompatible with the observed $\chi_{\rm eff}$ - q correlation.

In short, the most likely conclusion is that the observed correlation between $\chi_{\rm eff}$ and q implies that the distributions of component spins are (1) different and (2) dependent on the mass ratio q (perhaps directly, or perhaps through some other dependence on mass parameters).

REFERENCES

Abbott, R., Abbott, T. D., Acernese, F., et al. 2023, Physical Review X, 13, 011048, doi: 10.1103/PhysRevX.13.011048

Callister, T. A., Haster, C.-J., Ng, K. K. Y., Vitale, S., & Farr, W. M. 2021, ApJL, 922, L5, doi: 10.3847/2041-8213/ac2ccc

Gerosa, D., Kesden, M., Berti, E., O'Shaughnessy, R., & Sperhake, U. 2013, PhRvD, 87, 104028, doi: 10.1103/PhysRevD.87.104028

The LIGO Scientific Collaboration, the Virgo Collaboration, the KAGRA Collaboration, et al. 2021, arXiv e-prints, arXiv:2111.03606, doi: 10.48550/arXiv.2111.03606