LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
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CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Technical Note **LIGO-TXXXXXXX-v1** 05/15/2025

2025 LIGO SURF Project Proposal: Mapping and Correcting the Surface of the GQuEST End Mirrors

Rafael C. Volkamer-Pastor

California Institute of Technology
LIGO Project, MS 18-34
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project, Room NW17-161
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
Route 10, Mile Marker 2
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137
E-mail: info@ligo.caltech.edu

LIGO Livingston Observatory
19100 LIGO Lane
Livingston, LA 70754
Phone (225) 686-3100
Fax (225) 686-7189
E-mail: info@ligo.caltech.edu

`http://www.ligo.caltech.edu/`

1 Introduction

The study of quantum gravity aims to find a joint theory of gravitation and quantum mechanics. Substantial theoretical efforts have been made toward quantum gravity. Erik Verlinde and Kathryn Zurek predict 'geontropic' fluctuations, stochastic fluctuations of space-time geometry induced by entropy [1]. GQuEST (Gravity from the Quantum Entanglement of Space-Time), an experiment in the McCuller Lab at Caltech, aims to test this theory by analyzing the phase of light. When light propagates through a space with these fluctuations, it is predicted to accumulate a phase difference relative to light traveling without the fluctuations. The goal of GQuEST is to detect these geontropic fluctuations using a tabletop Michelson interferometer, refer to Fig 1.

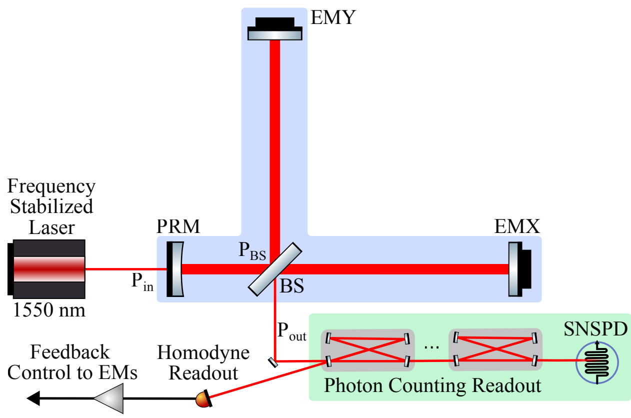

Figure 1: Michelson Interferometer used in GQuEST setup [2].

Geontropic fluctuations are characteristically of very low amplitude, roughly 7 orders of magnitude below the quantum shot noise of a lab scale, high power interferometer. Hence, to get strong results (5σ significance test), experiments are expected to take a run time of around 6 months [2]. Although feasible, this duration can be reduced by using the new photon-counting technique proposed by GQuEST. The method aims to filter the output light such that only photons carrying the signal are detected; however, some noise will still pass. An advantage to this new method is that it is not quantum limited, unlike the traditional homodyne readout of interferometers. This drastically reduces the total noise to 1 order of magnitude above the signal (compared to the previous 7), causing classical noise, for example vibrations in optical components, to become the new bottleneck. The photon counting method is expected to get strong results (5σ significance test) in weeks, compared to 6 months with traditional homodyne readout [2].

2 Objective

This project focuses on the consequences of reducing classical noise for the GQuEST end mirrors and the resulting necessary modifications. The contribution to the noise floor from the substrate of the mirror can be approximated by the following analytical equation,

$$\bar{S}_L^{SMN}(\Omega) = \frac{16 k_B T h \varphi_s}{\pi^3 M_s w^2 \Omega} \tag{1}$$

where k_B is Boltzmann's constant, T is temperature, h is mirror thickness, φ_s is the mechanical loss angle of substrate, M_s is the P-wave modulus of substrate, w is beam width (2σ radius in intensity), and Ω is measurement angular frequency [2]. It is easier to modify the mirror thickness than the temperature of the interferometer to reduce noise. Hence, GQuEST plans to utilize 2 mm silicon mirror, making the noise floor from the substrate below the coating thermal noise floor which we have less control over.

However, minimizing the mirror thickness causes the mirror to be more pliable. The GQuEST mirrors are made out of a substrate with a highly reflective Bragg reflection coating, similar to LIGO's mirrors. When the coating is applied it curves the face of the mirror as described,

$$r_{curv} \approx \frac{E_s h^2}{6\sigma_c h_c (1 - v_s)} \tag{2}$$

where E_s is Young's modulus of the substrate (the stiffness of the mirror), h is mirror thickness, σ_c is the coating stress, h_c is the thickness of the coating, and v_s is the Poisson ratio for silicon. With the current proposed parameters for GQuEST, $r_{curv} \approx 7.6$m [2]. Hence, it is clear that although minimizing the mirror thickness (h) is beneficial for reducing classical noise it results in a sharper mirror curve, the consequences of which I will further explain in the following paragraph.

When light reflects off of a curved mirror surface some of the light is converted into higher-order modes (HOMs). If the mirrors are not curved identically the HOMs will not all destructively interfere and hence produce a contrast defect, which causes extra light to escape the interferometer, increasing the 'bad' photon rate (photons passing the filter due to imperfections and not geontropic fluctuations). The optical power loss of these modes can be estimated to be roughly $D = 2/r_{curv}$. Specifically, a difference in the end-mirror curvature on the X and Y arms of the interferometer would scatter the majority of the light into the (Hermite-Gauss) HG_{20} and HG_{02} modes. The current hope is that these will be the only significant modes we need to correct for.

The amplitude coefficients of these modes would be

$$K_{20} \approx \frac{k D_x w^2}{4\sqrt{2}},\ K_{02} \approx \frac{k D_y w^2}{4\sqrt{2}} \tag{3}$$

where $D_x = D_x^{EMX} - D_x^{EMY}$ and $D_y = D_y^{EMX} - D_y^{EMY}$. This scattered light creates a contrast defect of $\Lambda_{CD} = K_{20}^2 + K_{02}^2$. Therefore, to achieve a significantly low contrast defect ($\Lambda_{CD} < 10$ppm) total curvature mismatch needs to be $D_{tot} = \sqrt{D_x^2 + D_y^2} < 3\times 10^{-4}$ diopters. With the current proposed parameters for GQuEST the mirror mismatch is expected to be

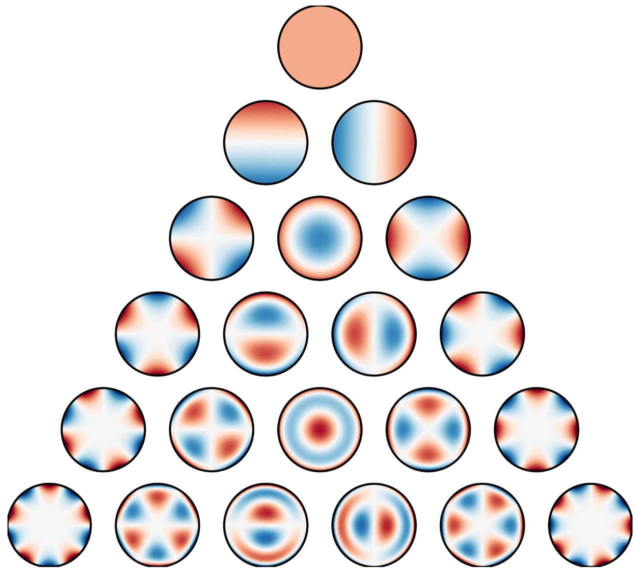

Figure 2: Visualization of the Zernike Polynomials made by Nschloe [3]. The Zernike polynomials provide an orthogonal basis on the unit disk. The top most mode refers to a mirrors position on the z-axis. The two modes in the second row refer to the tip and tilt of the mirror. The defocus mode is the middle mode of the third row. The astigmatic modes are the leftmost and rightmost mode in the third row.

roughly 0.26 diopters, indicating that there will need to be some curvature correction method [2].

We will initially compensate by coating the back of the mirror with an anti-reflective coating of custom thickness to cancel as much of the induced curvature as possible. An additional approach is to try to match the curvature of the two end mirrors in hopes to cancel out the HOMs. However, it likely we can't find perfect pairs and hence should not rely on their destructive interference. Instead, the remainder of the curvature difference will be adjusted through a custom mirror mount that is designed to twist and bend the mirror to counteract the curvature induced by the coating stress.

The goal of this project is to develop and test different methods to correct the curved surface of the GQuEST end mirrors to enable the photon counting method as a tool to study quantum gravity. Specifically, the focus of this summer will be to generate pure and combinations of HOMs under the assumption that if we learn how to generate HOMs in a consistent fashion then we will be able to make an inverse adjustment to remove them in the future. The success of this overarching goal is determined by our ability to generate individual and complex mixtures of both astigmatic modes and the defocus mode, a visual representation can be seen in Fig 2. This work builds on a previous project by LIGO SURF student Erin McGee, who worked under the mentorship of Daniel Grass during the summer of 2024; the current project will serve as a continuation of her efforts and will also be mentored by Grass.

Some intermediate objectives include becoming familiar with the mirror mounts seen in Fig 3,

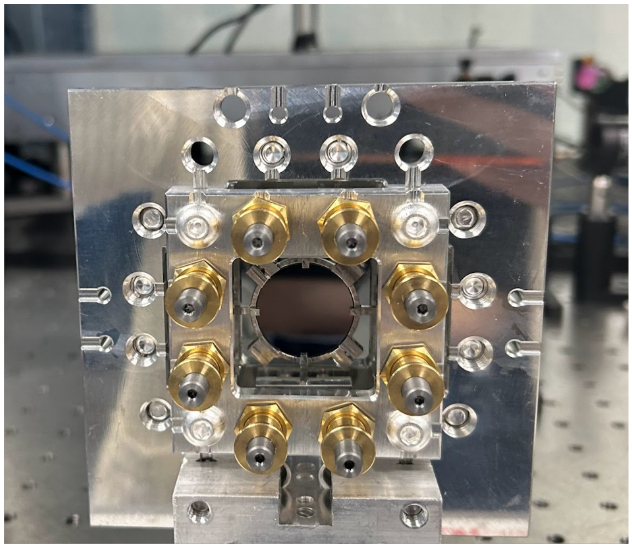

Figure 3: Image of the GQuEST mirror mount.

developing an adjustment to the mount to generate the defocus mode, testing the maximum stress threshold, and developing a simulation to predict what modes different mirror surfaces will generate.

3 Approach

This project will use a small scale Fizeau interferometer as its primary measurement apparatus. Fizeau interferometers are a common way to measure the surface of optical components, and work by comparing the measured optic to a reference flat. This allows us to see the curvature of the unstressed mirror and how the mirror surface changes due to adjusting the mirror mount. Through simulation we will be able to predict what modes the adjusted mirror surface will create. From this we can create a feedback loop. First, we adjust the surface of the mirror using the mount. Second, we take surface measurements with the Fizeau interferometer. We can extract this surface data and run it through a simulation to see what HOMs we expect the adjusted surface will generate. From our results in the simulation we restart the process and readjust the mirror mount as needed. We can repeat this process until we find consistent ways to generate different HOMs.

A key component of the project will be to build a simulation to predict the HOMs that different mirror surfaces will generate. This simulation will likely be developed in Python or MATLAB. This can be done by simply calculating the coupling coefficient of the beam. This tells us how much of the incoming beam is contained in the outgoing beam and can be done with the following,

$$k_{nmn'm'}(x,y) = \int\int u^*_{n'm'} e^{ik\phi(x,y)} u_{nm}(x,y)dxdy \tag{4}$$

where k is the wave number, ϕ is the surface of the mirror, and u_{nm} is a function of the beam and the relevant modes given by,

$$u_{nm}(x,y) = \left(\frac{1}{2^{n+m-1}n!m!\pi}\right)^{1/2}\left(\frac{1}{w_0}\right)\left(\frac{q_0}{q(z)}\right)\left(\frac{q_0 q^*(z)}{q^* q(z)}\right)^{(n+m)/2} \cdot H_n\left(\frac{\sqrt{2}x}{w(z)}\right)H_m\left(\frac{\sqrt{2}y}{w(z)}\right)e^{-ik\frac{(x^2+y^2)}{2q(z)}} \tag{5}$$

where n and m refer to the Hermite–Gaussian modes HG_{nm}, w_0 is beam waist, $w(z)$ is the beam width, q is the complex beam parameter, k is the wave number, x and y are the transverse directions of the beam, z is the beam axis, and H is the Hermite Polynomial of corresponding order n or m [4]. This simulation will be necessary to verify that our mirror mount is influencing the wavefront in the desired fashion.

4 Timeline

Weeks	Milestones and Goals
1–2	- Orientation, safety training, and lab access setup. - Learn mirror mount mechanics and Fizeau interferometer. - Take baseline mirror surface measurements. - Start simulation development for HOM prediction. - Report Prep.
3–4	- Begin deforming mirror to generate the defocus mode. - Begin deforming mirror to generate the astigmatic modes. - Evaluate deformation precision and repeatability.
5–6	- Explore stress limits and generate complex mode mixtures. - Replace mirror with monolithic silicon and repeat tests. - Report Prep.
7–8	- Introduce coated monolithic silicon mirror. Assess coating impact. - Conduct repeatability studies across mirror types.
9–10	- Prepare final report and presentation.

Table 1: Expected Milestone timeline for the project. Note: Two interim reports will also be submitted during weeks 3 and 6.

References

[1]E. P. Verlinde and K. M. Zurek, "Observational signatures of quantum gravity in interferometers", Physics Letters B **822**, 136663 (2021).

[2] S. M. Vermeulen, T. Cullen, D. Grass, I. A. O. MacMillan, A. J. Ramirez, J. Wack, B. Korzh, V. S. H. Lee, K. M. Zurek, C. Stoughton, and L. McCuller, "Photon counting interferometry to detect geontropic space-time fluctuations with GQuEST", Physical Review X **15**, 011034 (2025).

[3] *Zernike polynomials*, in *Wikipedia*, Page Version ID: 1283251556 (Mar. 31, 2025).

[4] *Erin McGee 2024 SURF reports and presentation*, Foswiki, `https://wiki.mccullerlab.com/Main/ErinMcGee2024SURFReportsAndPresentation` (visited on 04/06/2025).