Mapping and Correcting the Surface of the GQuEST End Mirrors

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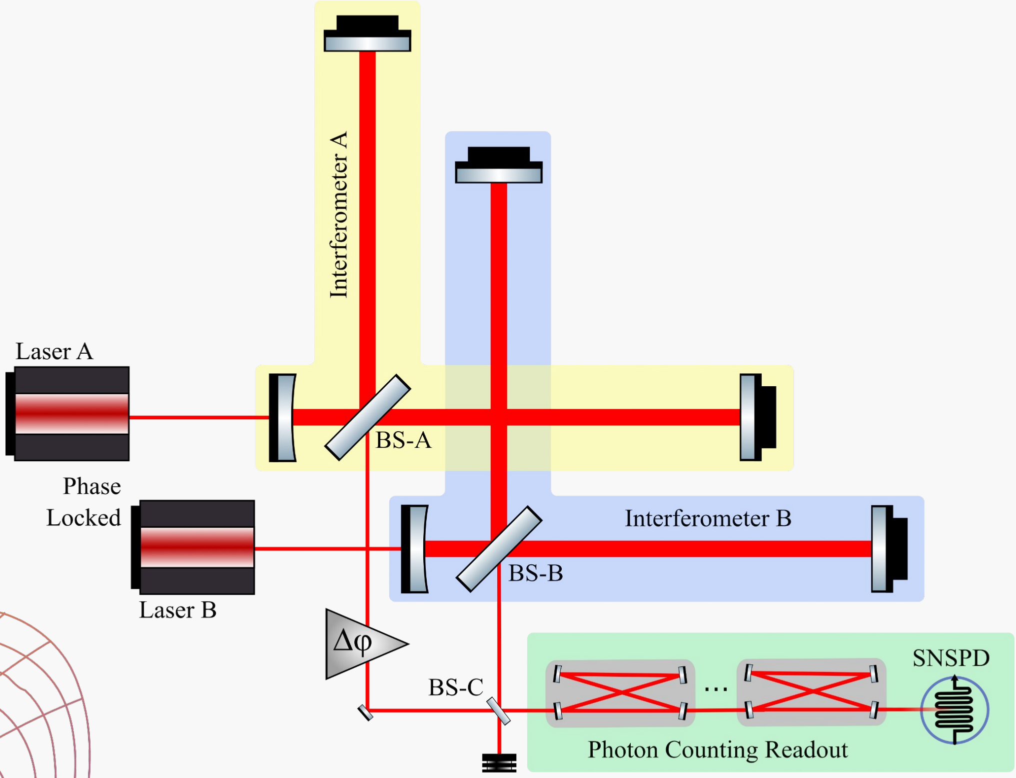
What am I going to talk about?

What is GQuEST/ Scientific Introduction → How to Correct Mirror Deformations → Achievements/ Advancements

What is GQuEST?

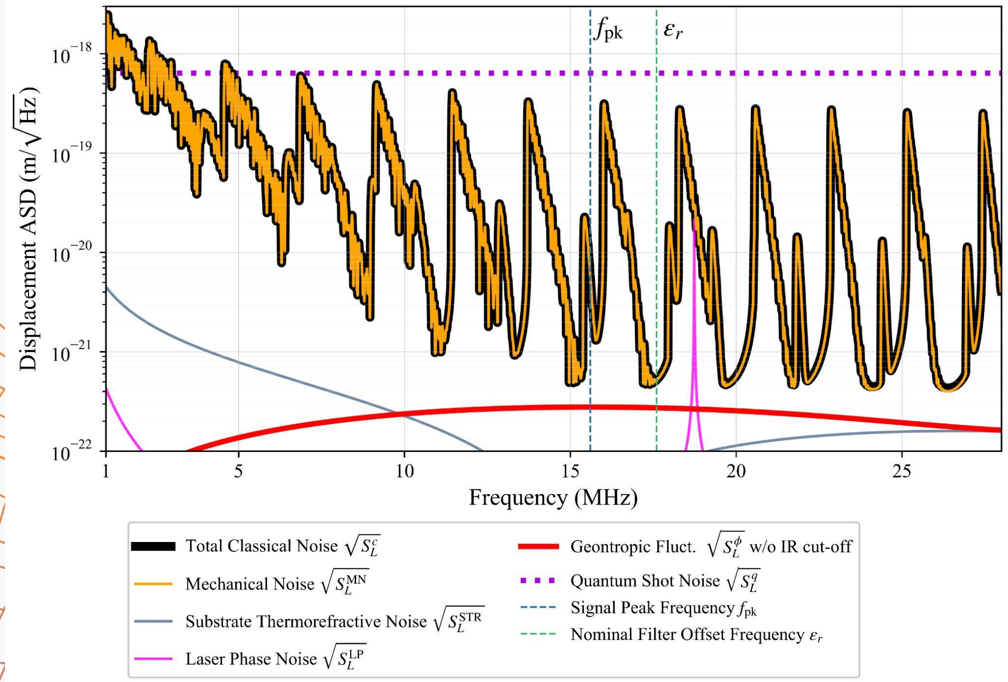
Gravity from the Quantum Entanglement of Space-Time

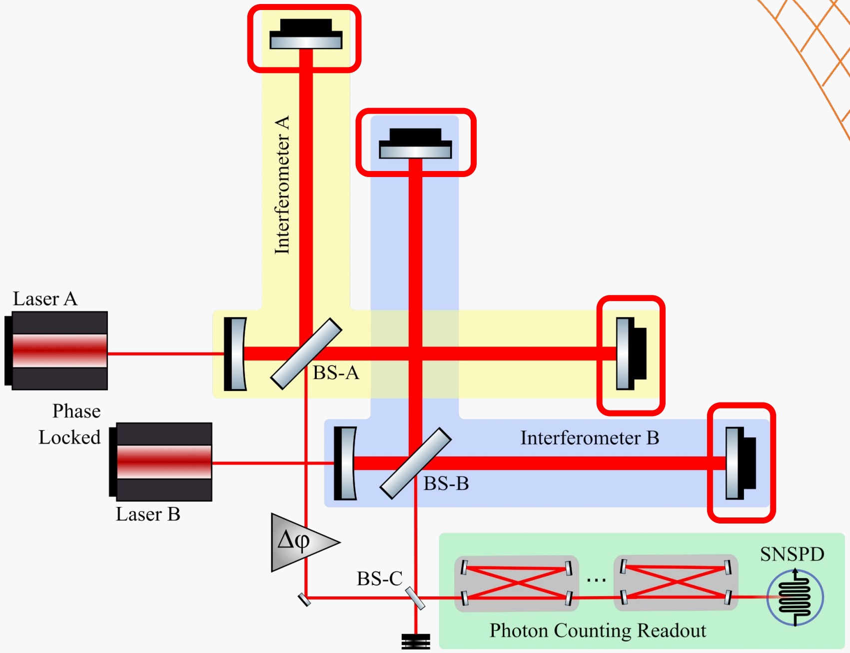


Erik Verlinde and Kathryn Zurek predict 'geontropic' fluctuations, stochastic fluctuations of space-time geometry induced by entropy.

GQuEST is a "table top" (7m) Michelson Interferometer designed to detect these **quantum gravity fluctuations**.

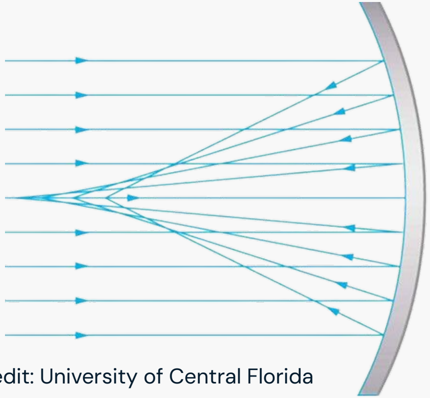
What are the advantages to GQuEST?





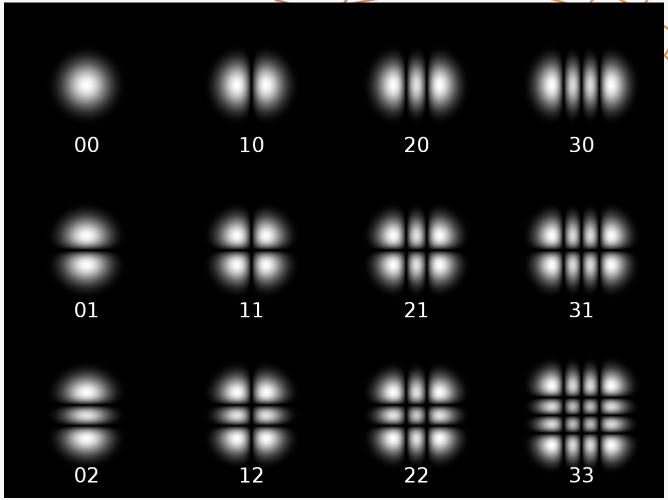
GQuEST filters the output light such that **only photons carrying the signal are detected** (some noise will still pass).

Scientific Introduction



Credit: University of Central Florida

- Lenses/Curved Mirrors have a focal length
- Inverse of focal length is focusing power (diopters)
- Diopters are nice because the more diopters the more aggressive the lens (focal length is the opposite)



Credit: Bob Mellish

- The Hermite Gauss basis is an orthogonal basis
- You can use it to decompose a coherent paraxial beam
- Coherent: Fixed phase relation
- Paraxial: Approximately parallel to the optical axis

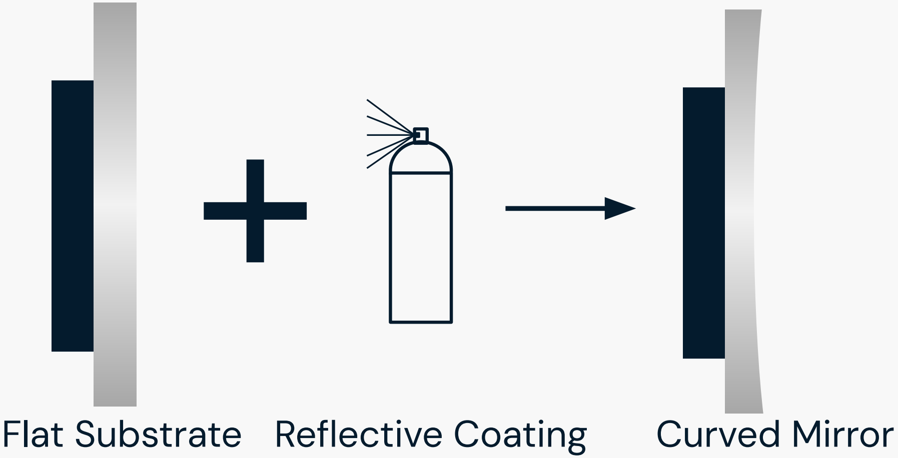
Noise in GQuEST

$$\bar{S}_L^{SMN}(\Omega) = \frac{16 k_B T h \varphi_s}{\pi^3 M_s w^2 \Omega}$$

Noise contribution from substrate
(mirror thickness - h)

$$r_{curv} \approx \frac{E_s h^2}{6\sigma_c h_c (1 - v_s)} = 7.6\text{m}$$

Unintentional curvature of mirror



- Because of photon counting, GQuEST is not limited by quantum shot noise but instead **limited by classical noise**.
- We mitigate the noise contribution by reducing the thickness of the end mirrors.
- This makes the **mirror more pliable**, and causes the mirror to bend when getting coated.
- The power, in diopters, lensed into higher order modes (HOM) is approximately,

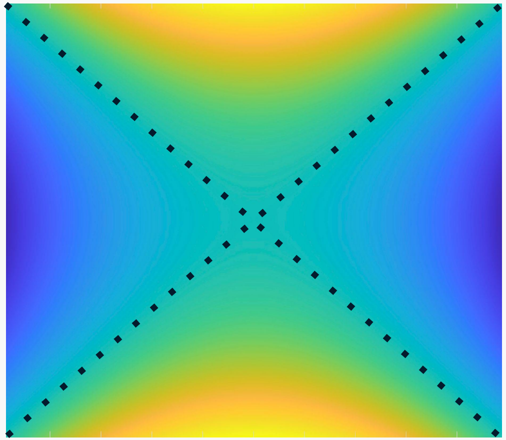
$$D = 2/r_{curv}$$

GOAL: Reduce quadratic deformations without inducing higher order deformations or surface roughness. Have a loss less than 50ppm.

HOMs and their Production

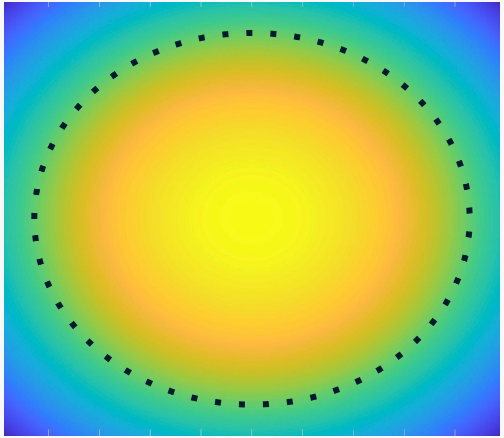
There are three main mirror deformations that we are concerned about. These are quadratic deformations. Linear and constant deformations can be corrected by a kinematic mount.

XMode



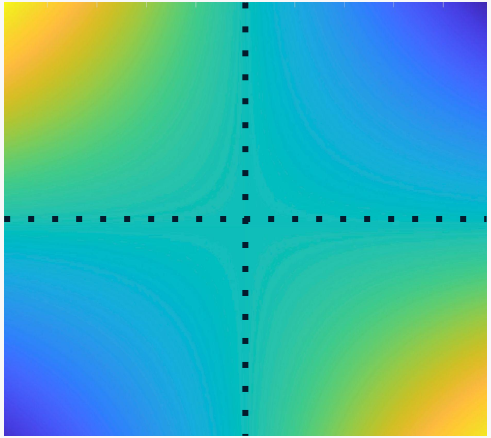
Leads to coupling into the HG_02 and HG_20

OMode



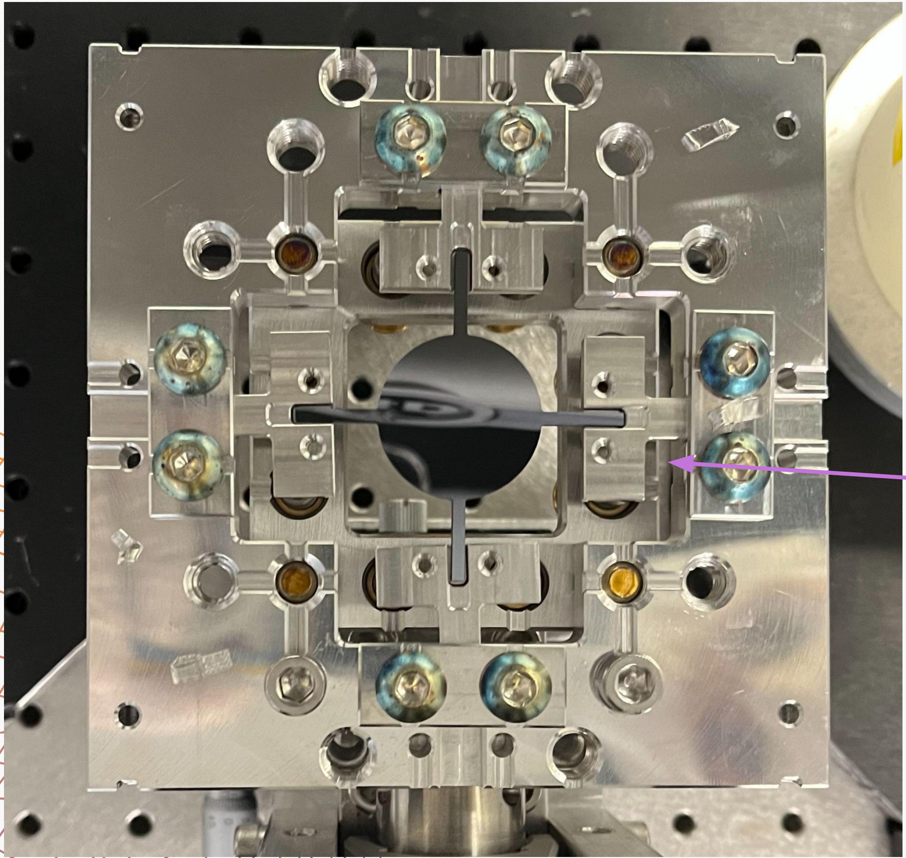
Leads to coupling into the HG_02 and HG_20

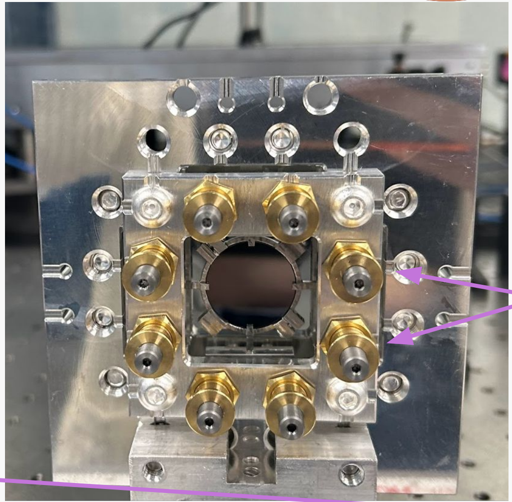
+Mode

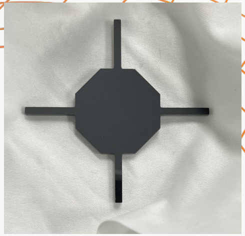


Leads to coupling into the HG_11

Adjustable Mirror Mount

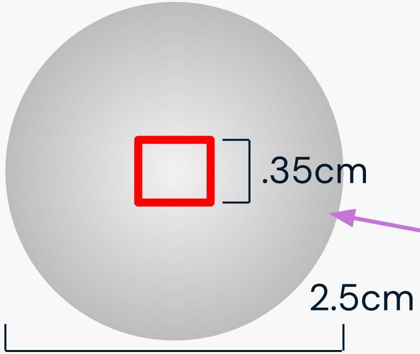






Adjustment Screws

Clamping Ts



NanoCam Imaging Area

Calculate Coupling to HOMs and Lensing Power

Calculating the coupling coefficient, we assume incoming HG_{00}

$$k_{nmn'm'} = \int \int u^*_{n'm'} e^{2ikz(x,y)} u_{nm}(x,y) dx dy$$

$$u_{nm}(x,y) = \left(\frac{1}{2^{n+m-1} n! m! \pi}\right)^{1/2} \left(\frac{1}{w_0}\right) \left(\frac{q_0}{q(z)}\right) \left(\frac{q_0 q^*(z)}{q^* q(z)}\right)^{(n+m)/2} H_n\left(\frac{\sqrt{2}x}{w(z)}\right) H_m\left(\frac{\sqrt{2}y}{w(z)}\right) e^{-ik\frac{(x^2+y^2)}{2q(z)}}$$

Lensing Power in Diopters (n+m=2)

$$D_{nm} \approx \frac{4\sqrt{2} k_{nm}}{k w^2}$$

$$D_{tot} = \sqrt{D_{02}^2 + D_{11}^2 + D_{20}^2}$$

Start

Take an image of the mirror with the optical profiler

Calculate coupling coefficients to HOMs and lensing power

x30

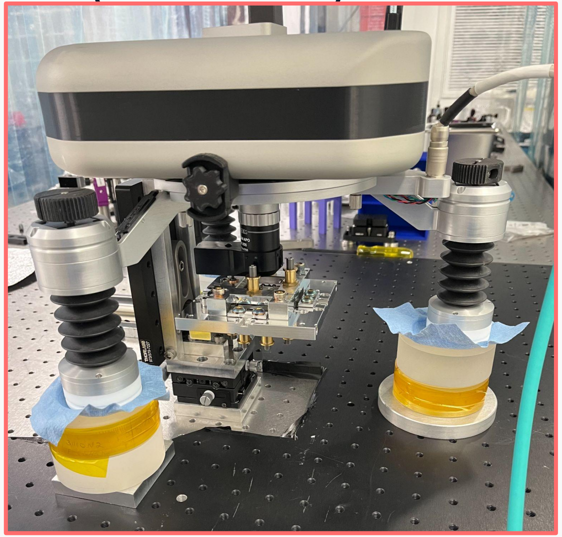
Is the mirror flat enough?

No

Adjust mirror surface with mirror mount

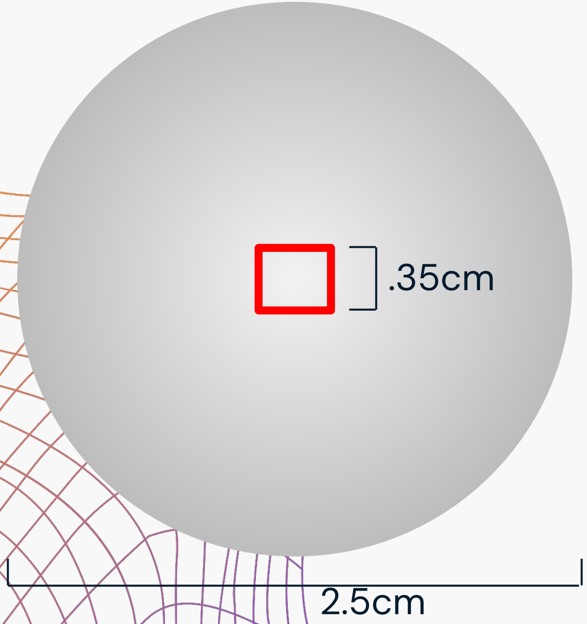
Yes

You’re done



Limited Field of View with the NanoCam

There is a major limitation with the NanoCam, with the current set of objectives we are only able to capture a **3.5mm by 4mm area**. The 2σ power radius of the laser on the mirror is expected to be around 3mm which means the area the NanoCam images **does not fully encompass the laser beam.**



<u>Issues with cropped domain</u>

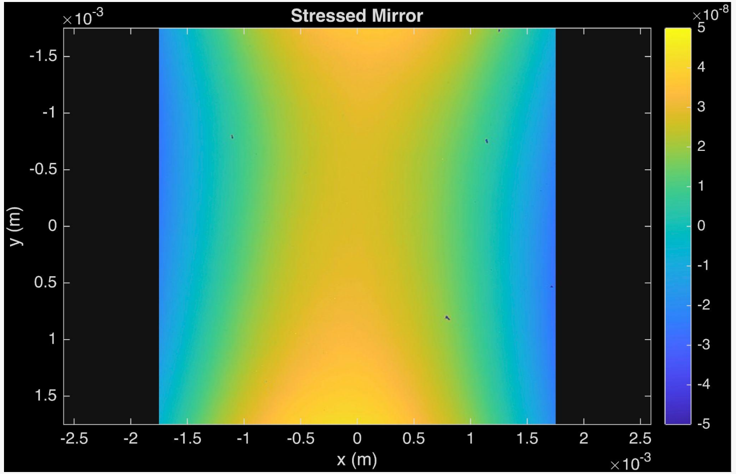
1. We are missing power
2. Hermite Polynomials are not orthogonal
3. Astigmatism with NanoCam

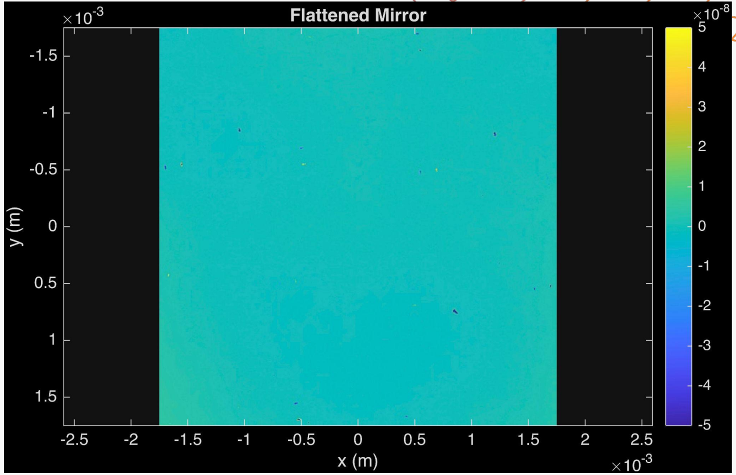
<u>Solutions</u>

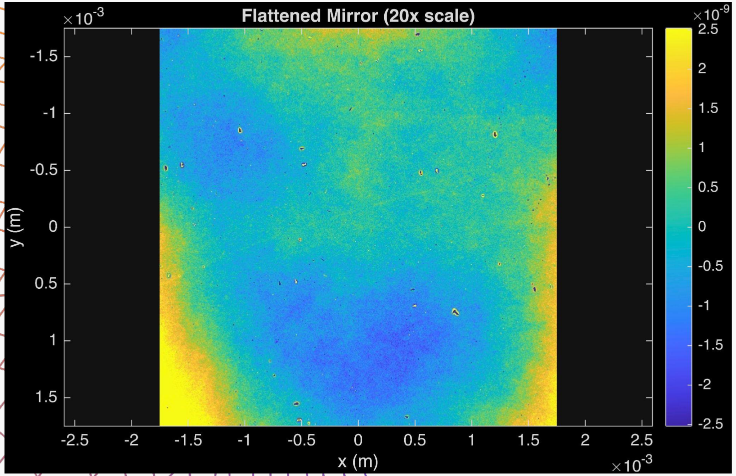
1. Use a least squares fit to approximate a larger mirror area, accounting for residuals
2. Convert from a clipped HG basis to the full HG basis using a matrix*
3. Developed a method to calculate the astigmatism without using a reference flat*

*cut for time, more info in appendix

Flattening the Octagonal Spoked Mirror



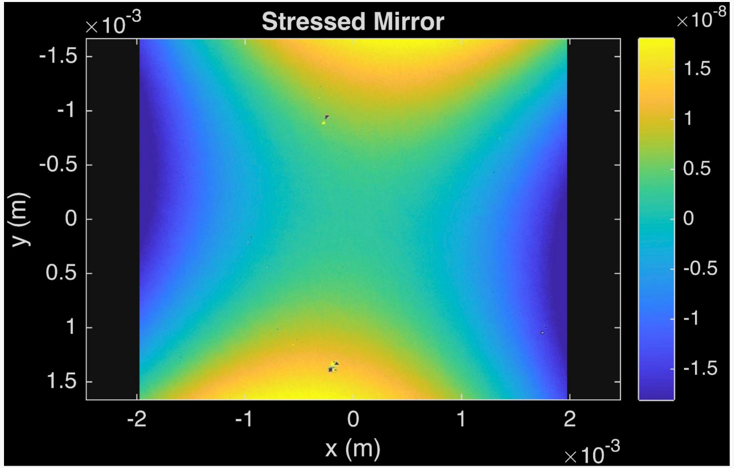


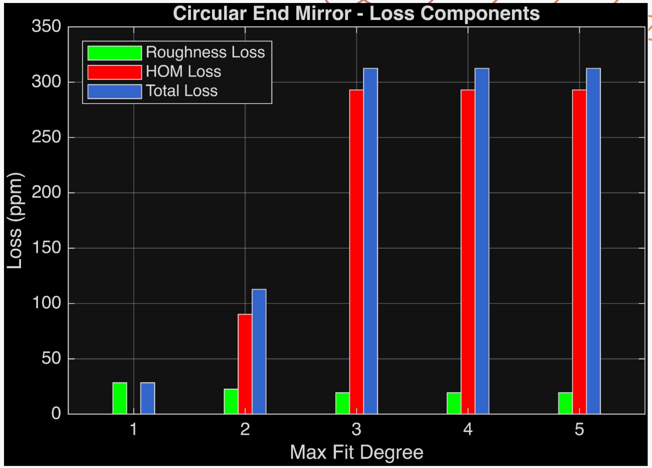


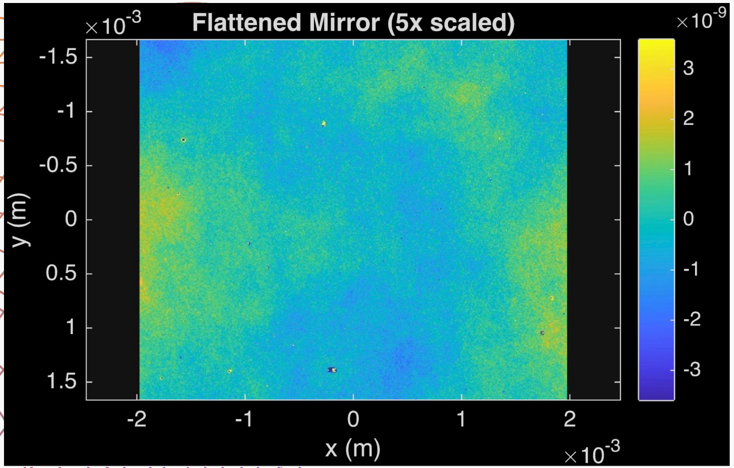
	Clamped Mirror	After Flattening
rms in m	2.23×10^{-8}	9.53×10^{-10}
weighted rms in m	1.90×10^{-8}	8.30×10^{-10}
diopters	0.058023	0.0012017
power scattered from residuals	38.5ppm	24.8ppm
total loss	185,453ppm	1005ppm

*below NanoCam astigmatism

Flattening the Circular Spoked Mirror

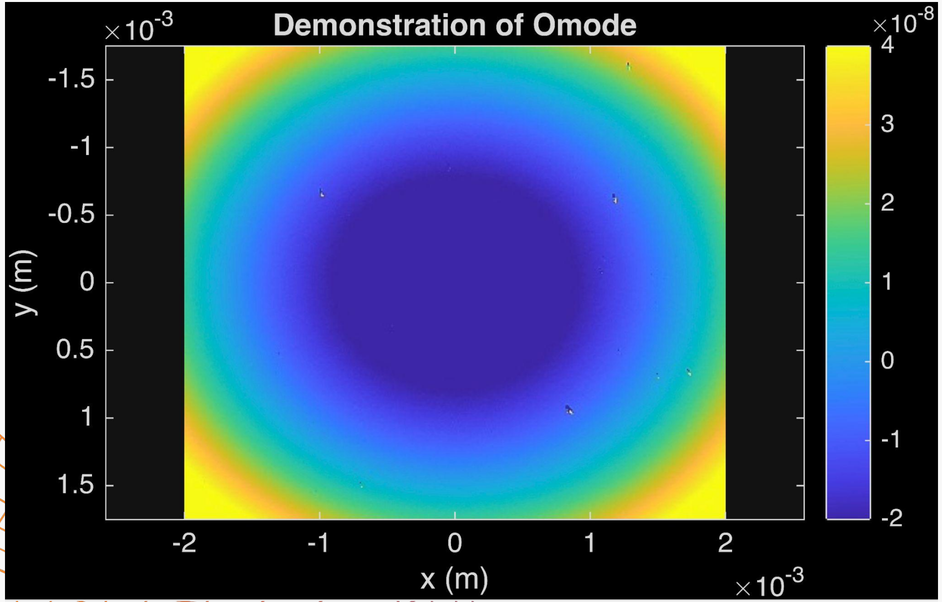


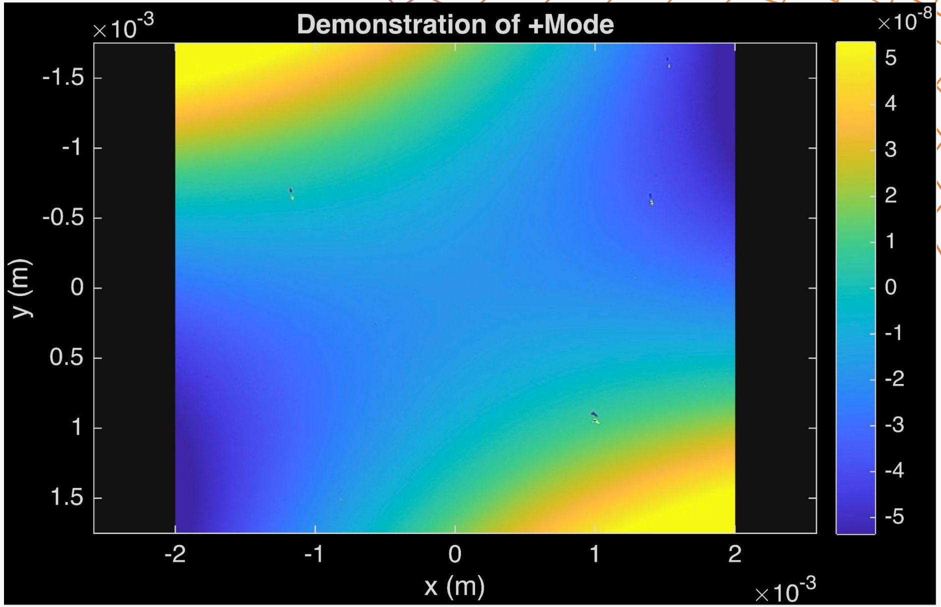




	Clamped Mirror	After Flattening
rms in m	9.07×10^{-9}	7.58×10^{-10}
weighted rms in m	7.73×10^{-9}	6.57×10^{-10}
diopters	0.03014	0.00051475
power scattered from residuals	235ppm	19ppm
total loss	43,461ppm	312ppm

Creating Omode and +Mode

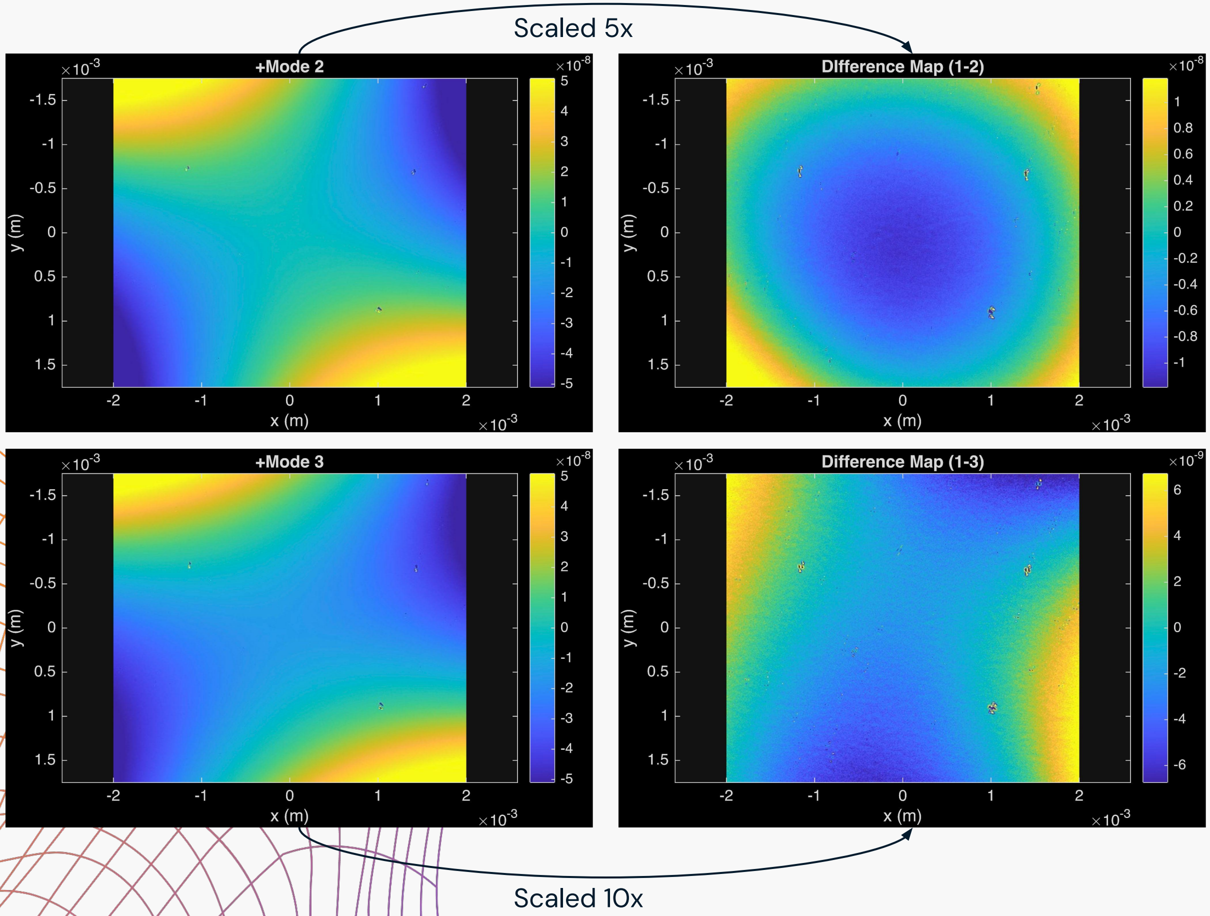


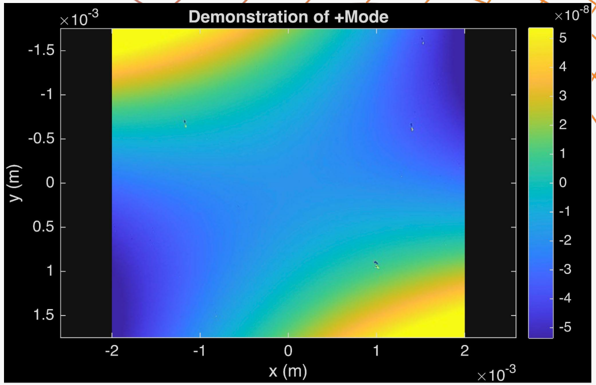


	OMode	+Mode
diopters	0.056	0.061
power coupled above n+m=2	0.0131	0.0455
weighted rms of the mirror surface	1.82×10^{-8} meters	2.91×10^{-8} meters
weighted rms of residuals of quadratic fit	7.59×10^{-10} meters	1.05×10^{-9} meters

*a third of what PSAMS at LIGO can do

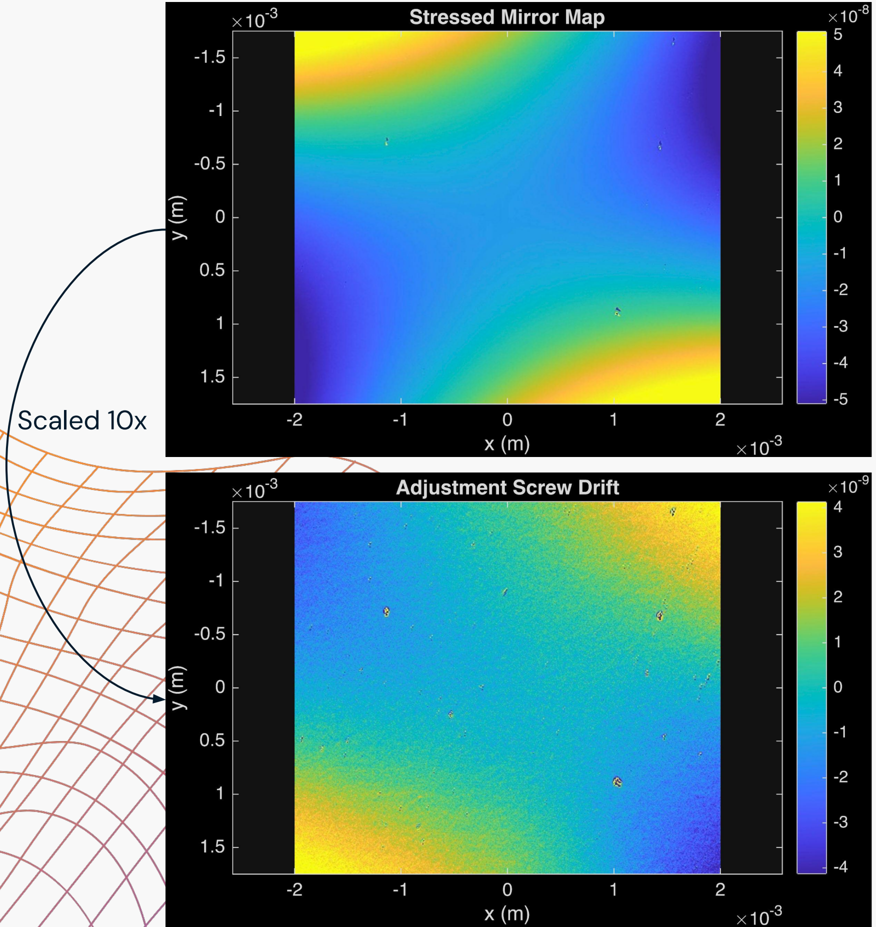
Repeatability

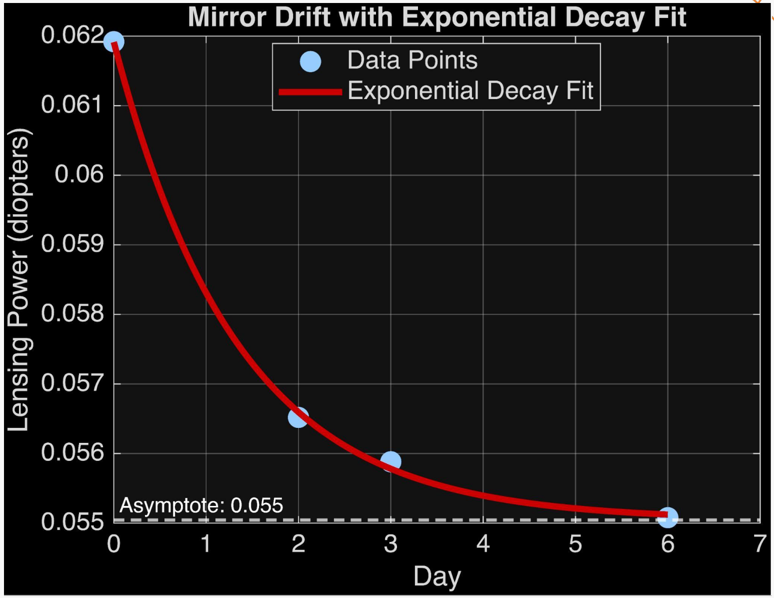




	Mean
Total lensing power (diopters)	$(6.33 \pm 0.05) \times 10^{-2}$
Lensing 20 (diopters)	$(2.39 \pm 0.5) \times 10^{-2}$
Lensing 11 (diopters)	$(4.46 \pm 0.3) \times 10^{-2}$
Lensing 02 (diopters)	$(3.74 \pm 0.5) \times 10^{-2}$

Mirror Drift Over Time





Roughly a 10% decrease in focusing power in the first three days and then significant leveling.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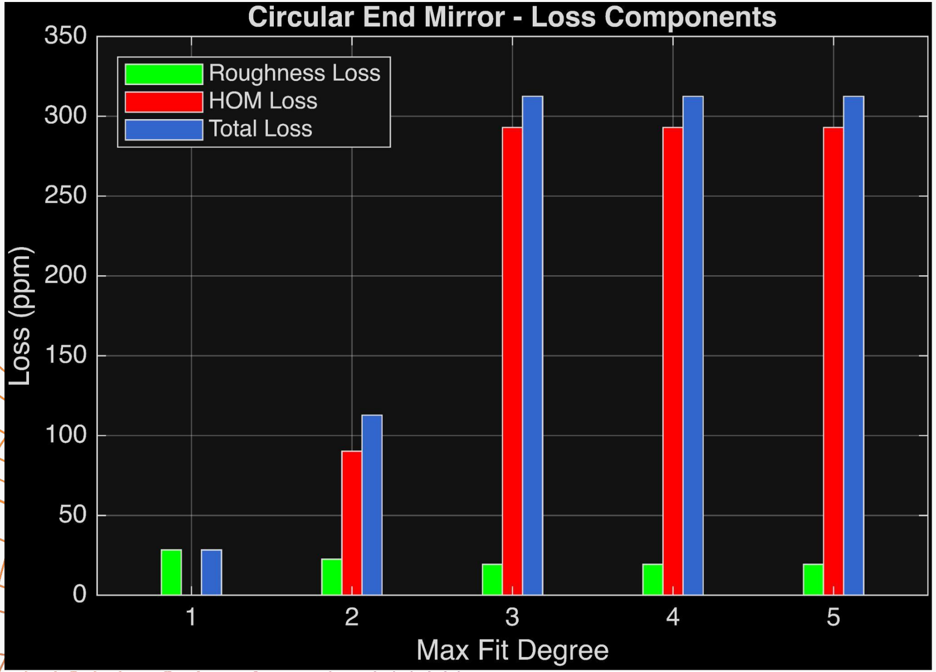
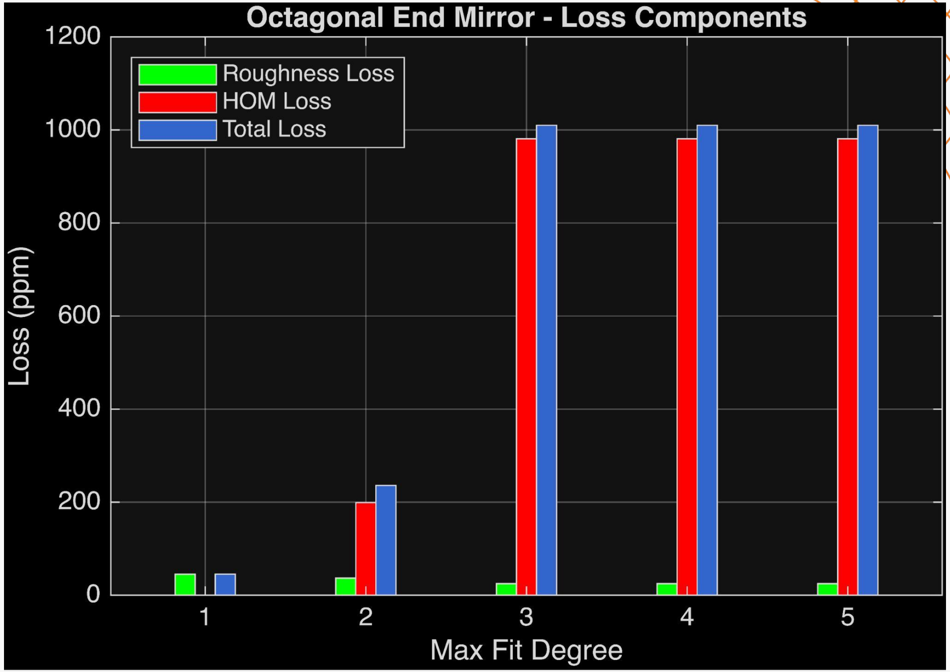
Thank you to Caltech, LIGO, NSF, Fermilab, the Department of Energy, and the Heising-Simons Foundation for making this opportunity possible. And a special thanks to Garilynn Billingsley for her advice and letting us use her 4D NanoCam (optical profiler).

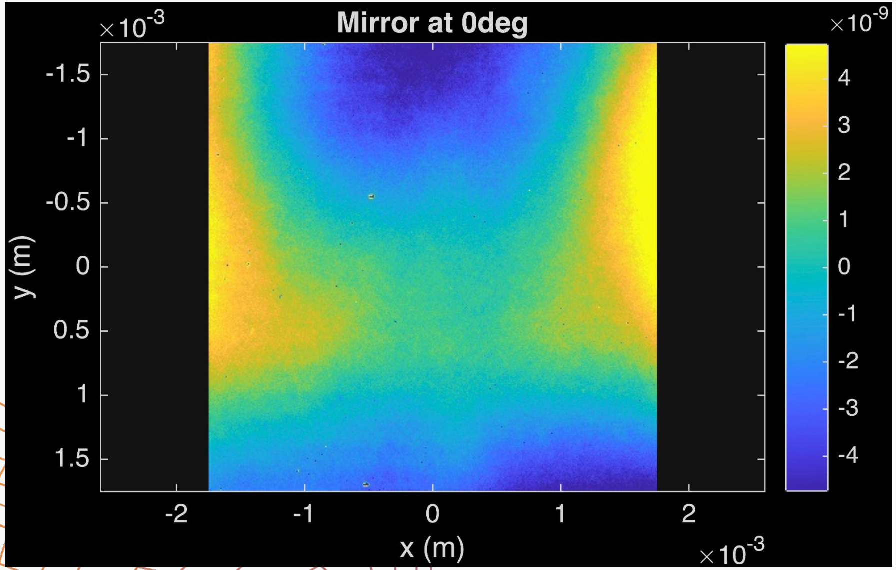
Questions?

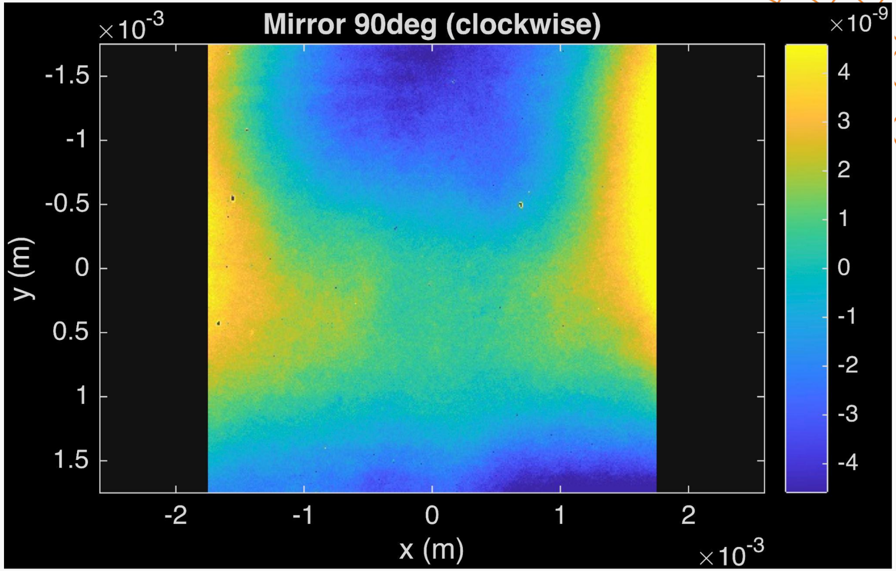
Loss Analysis at Different Fitting





NanoCam Astigmatism 1





$$Z_1 = m_1 + n$$

$$Z_2 = m_2 + n$$

$$Z_1 = m_1 + n$$
$$Z_2 = m_2 + n$$

NanoCam Astigmatism 2

$$\hat{r}^{-1} Z_2 = \hat{r}^{-1} m_2 + \hat{r}^{-1} n$$
$$Z_1 - \hat{r}^{-1} Z_2 = m_1 + n - \hat{r}^{-1} m_2 - \hat{r}^{-1} n$$
$$Z_1 - \hat{r}^{-1} Z_2 = n - \hat{r}^{-1} n$$

$$n = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix}, \quad \hat{r}^{-1} n = \begin{bmatrix} n_{13} & n_{23} & n_{33} \\ n_{12} & n_{22} & n_{32} \\ n_{11} & n_{21} & n_{31} \end{bmatrix}$$

$$n - \hat{r}^{-1} n = \begin{bmatrix} n_{11} - n_{13} & n_{12} - n_{23} & n_{13} - n_{33} \\ n_{21} - n_{12} & n_{22} - n_{22} & n_{23} - n_{32} \\ n_{31} - n_{11} & n_{32} - n_{21} & n_{33} - n_{31} \end{bmatrix}$$

NanoCam Astigmatism 3

$$Z_1 - \hat{r}^{-1} Z_2 = n - \hat{r}^{-1} n$$

$$n - \hat{r}^{-1} n = \begin{bmatrix} n_{11} - n_{13} & n_{12} - n_{23} & n_{13} - n_{33} \\ n_{21} - n_{12} & n_{22} - n_{22} & n_{23} - n_{32} \\ n_{31} - n_{11} & n_{32} - n_{21} & n_{33} - n_{31} \end{bmatrix}$$

$$\vec{Z} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vec{n}$$

The Matrix Approach 1

The current approach to work around the limited domain of the NanoCam is to **work in two Hermite Gauss bases: HG clipped basis and the HG full basis.**

$$\begin{bmatrix} \mathrm{HG}_{\text{clipped},1} \\ \mathrm{HG}_{\text{clipped},2} \\ \vdots \\ \mathrm{HG}_{\text{clipped},n} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1m} \\ m_{21} & m_{22} & \cdots & m_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{nm} \end{bmatrix} \cdot \begin{bmatrix} \mathrm{HG}_{\text{full},1} \\ \mathrm{HG}_{\text{full},2} \\ \vdots \\ \mathrm{HG}_{\text{full},m} \end{bmatrix}$$

$$M^{-1} HG_{\text{clipped}} = HG_{\text{full}}$$

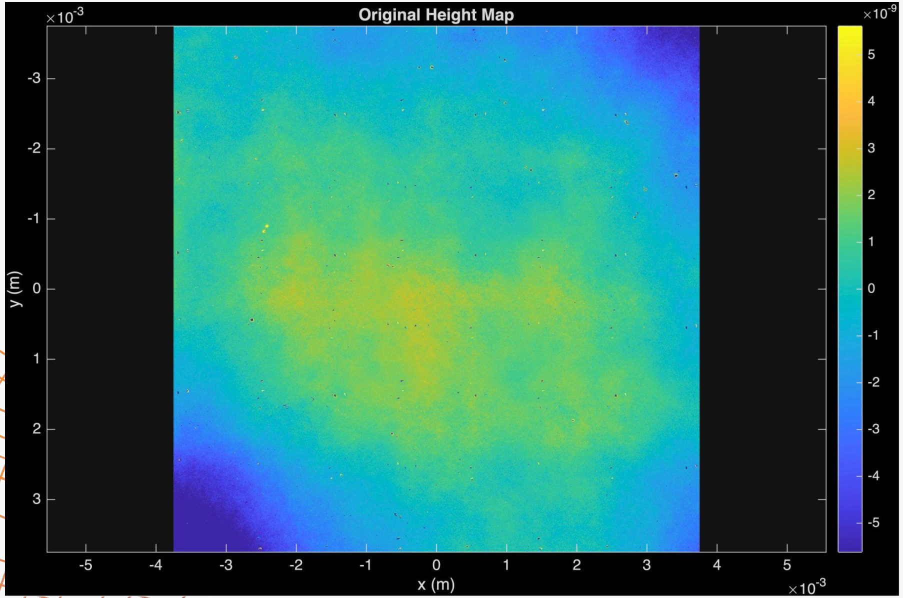
The Matrix Approach 1

Index	Hermite Gauss Mode
HG_1	HG_{00}
HG_2	HG_{10}
HG_3	HG_{01}
HG_4	HG_{20}
HG_5	HG_{11}
HG_6	HG_{02}
HG_7	HG_{30}
HG_8	HG_{21}
HG_9	HG_{12}
HG_{10}	HG_{03}

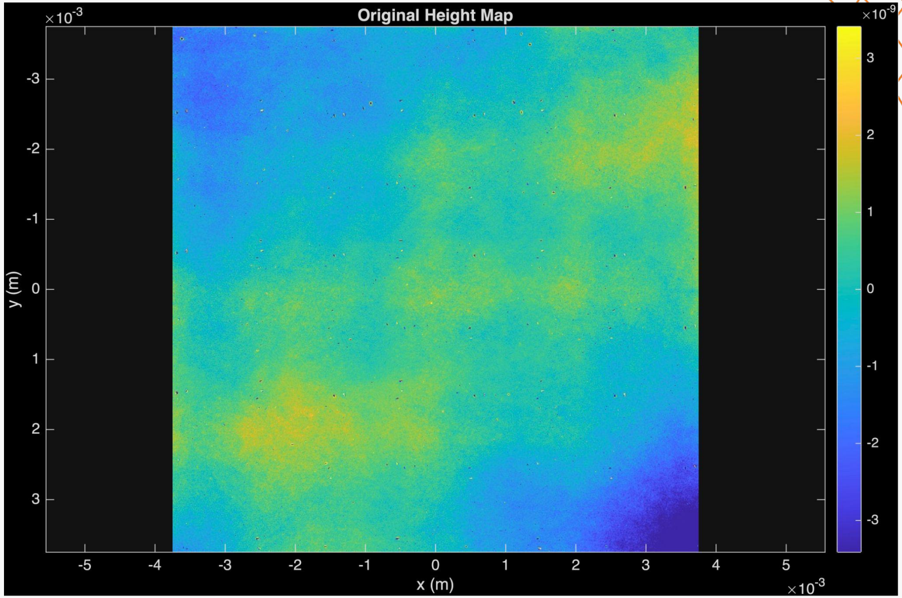
$$m_{ij} = \int\int u_i^*(x,y)u_j(x,y)dxdy$$

$$\begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1m} \\ m_{21} & m_{22} & \cdots & m_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{nm} \end{bmatrix}$$

Stitching Images Together



Circular Spoked Mirror



Octagonal Spoked Mirror

What NanoCam looks like