IMPRINTS OF THE FREQUENCY-DOMAIN SOURCE ON BLACK HOLE RINGDOWN

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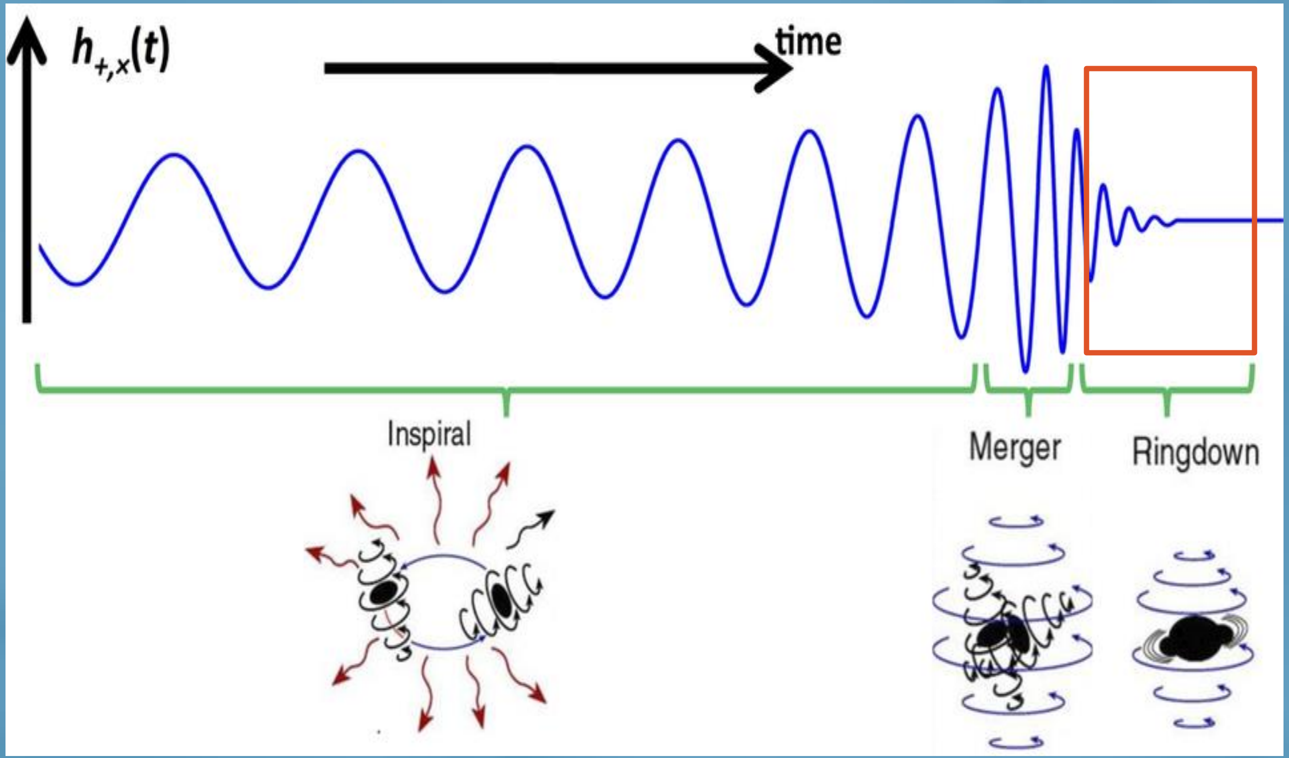
BACKGROUND



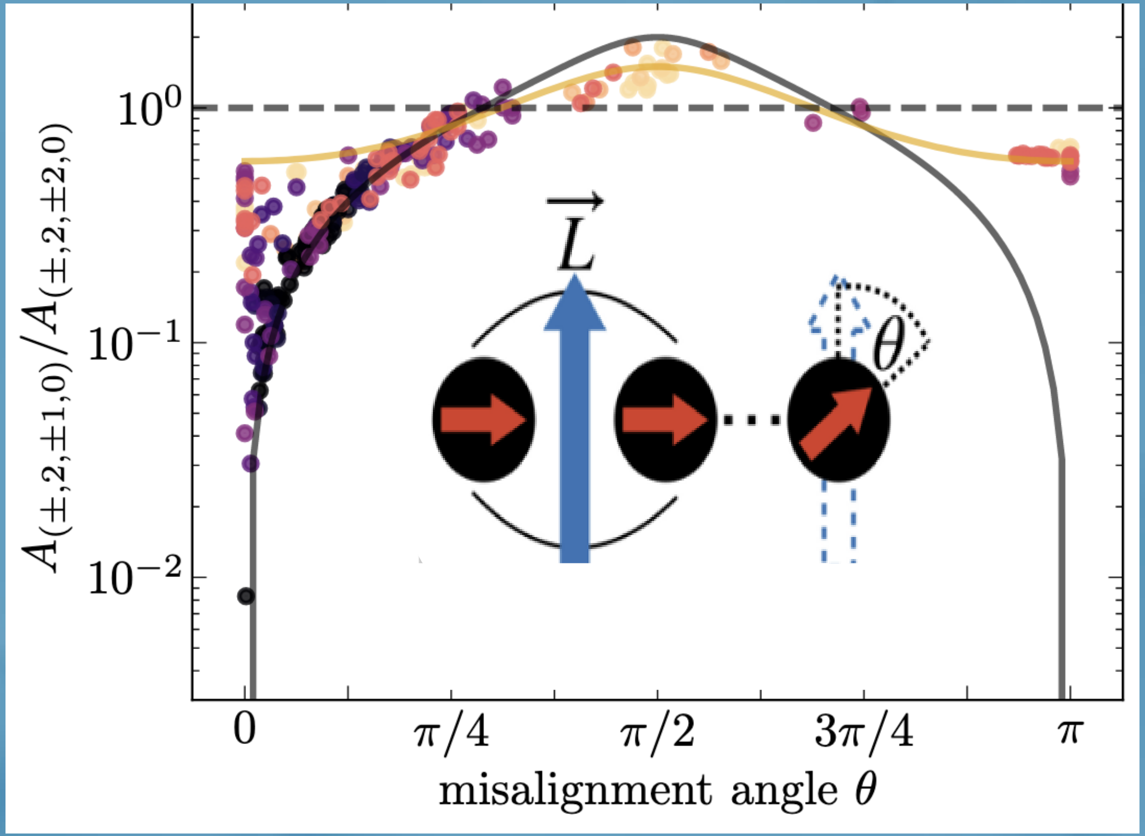
M. Favata/SXS/K. Thorne

- We treat the ringdown as a perturbation to the final black hole which allows us to use perturbation theory.
- Ringdown can be described as a sum of Quasinormal modes (QNMs).

$$h_{+,\times} \sim \sum_{\ell,m,n} A_{\ell mn}\, e^{-i\omega_{\ell mn} t}$$

$$\omega_{\ell mn} = \omega_R + i\omega_I$$

- According to the no hair theorem, the QNM frequencies are completely determined by the black hole's spin and mass.



Hengrui Zhu et al.

- Previous ringdown analysis has used QNM amplitudes to extract properties of a BBH system.

$$h_{+,\times} \sim \sum_{\ell,m,n} A_{\ell mn}\, e^{-i\omega_{\ell mn} t}$$

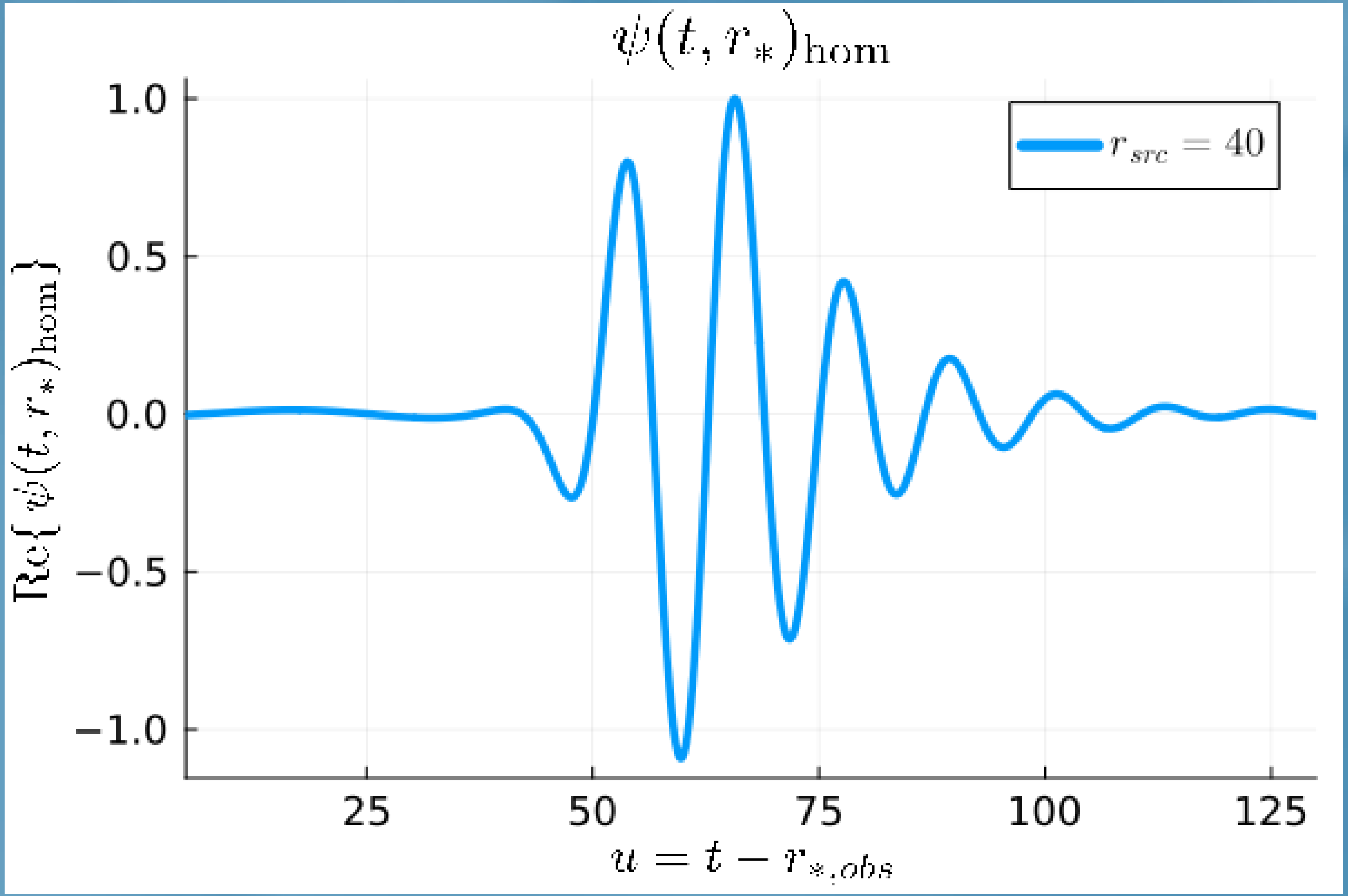
- Testing general relativity (GR) has also been a focus in these studies.

What can we infer about pre-merger physics from the ringdown?

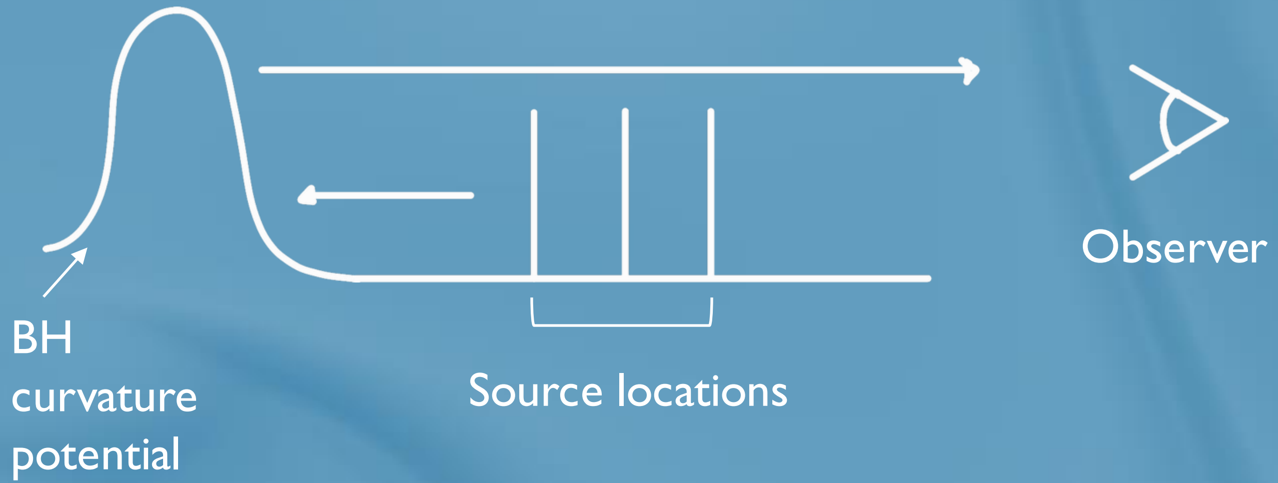
- Teukolsky equation describes how a perturbation to a rotating black hole evolves.

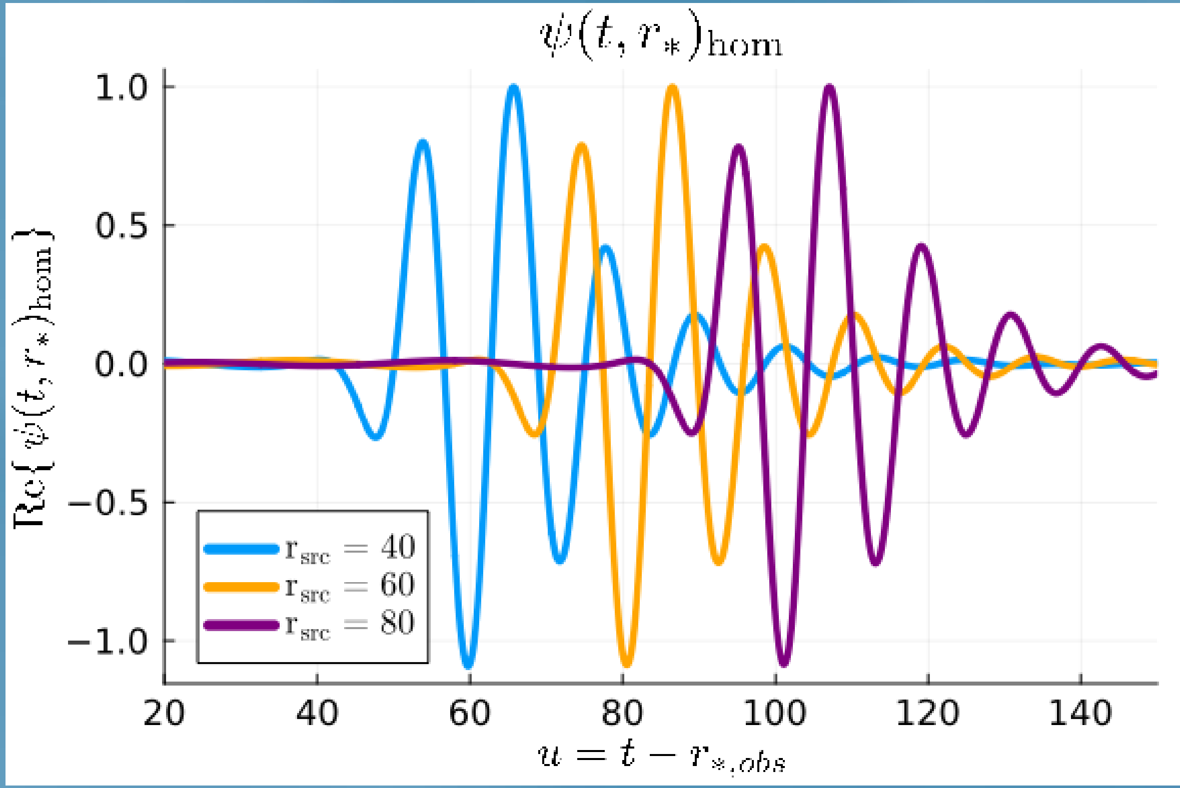
$$\frac{d^2\psi}{dr_*^2} + [\omega^2 - V(r)]\psi = 0$$

- We solve the radial homogeneous Teukolsky equation and then inverse Fourier transform to get the time domain solution.



- Move initial perturbation location → ringdown shifts by twice the amount we shift the location.

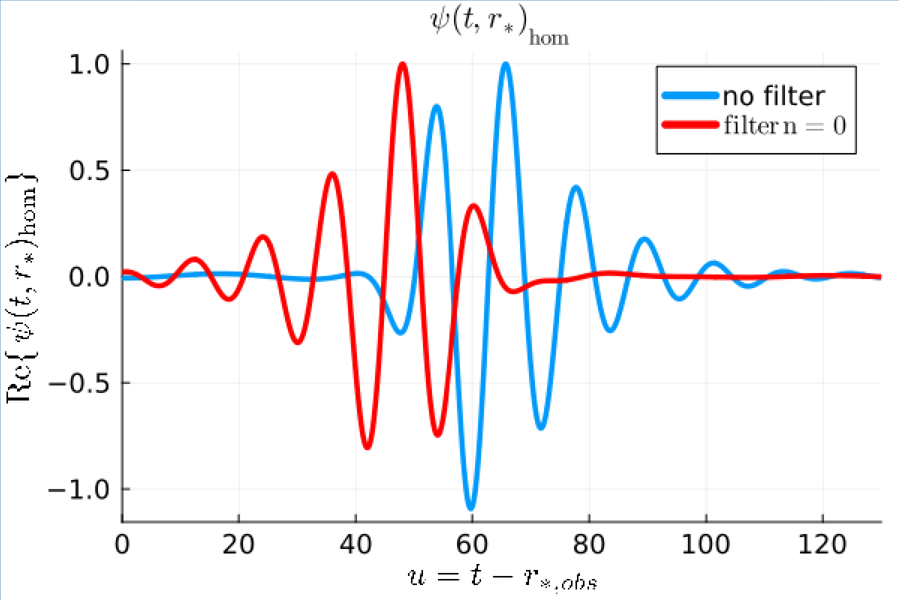


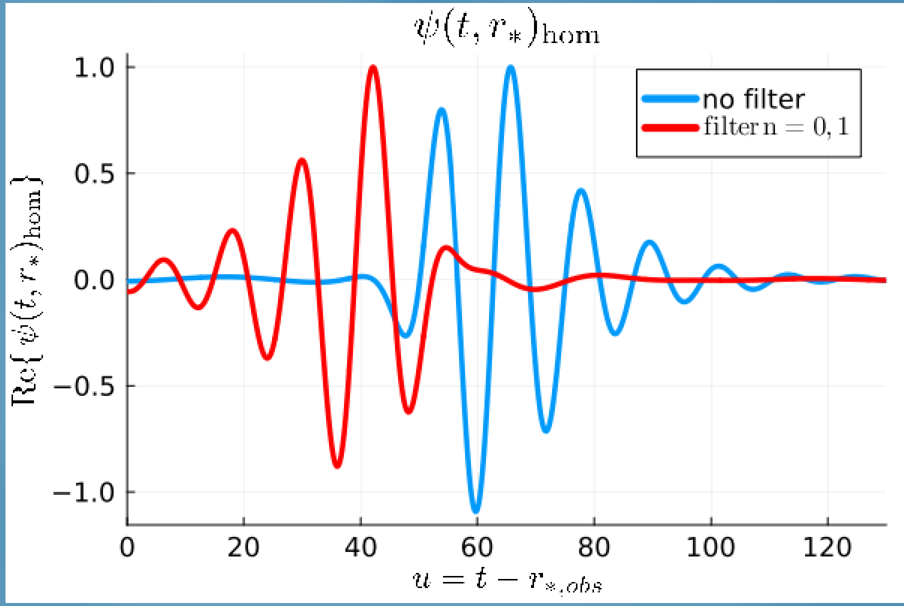


- QNM filter removes a chosen QNM (ω_{lmn}) from the ringdown signal.

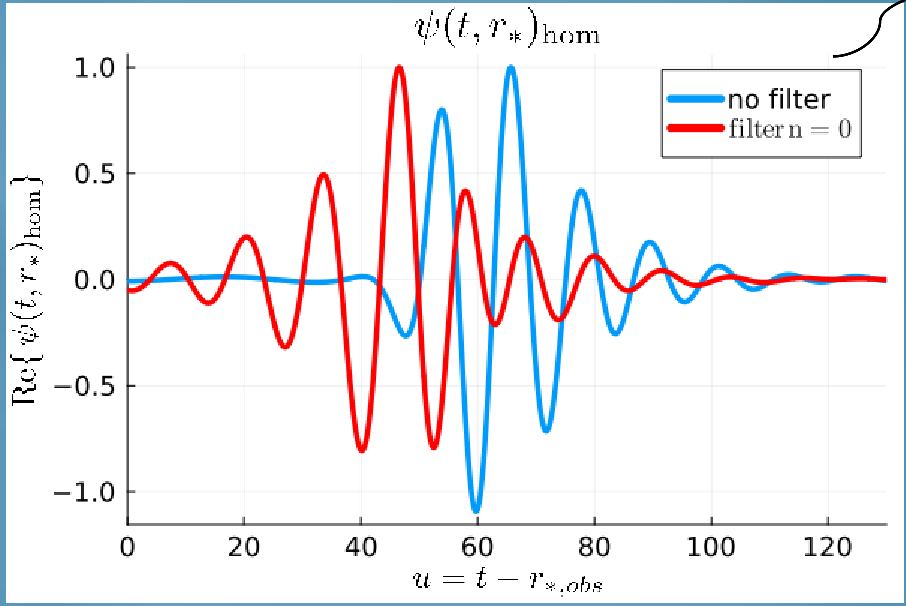
$$\mathcal{F}_{lmn} = \frac{\omega - \omega_{\ell mn}}{\omega - \omega^*_{\ell mn}}$$

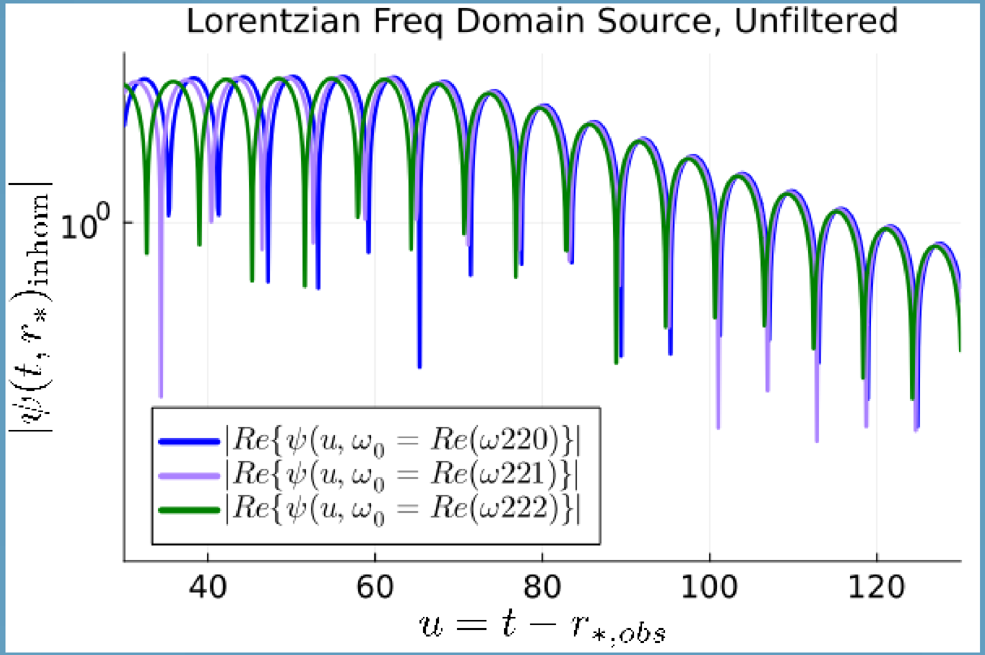
- Ringdown filters can be used as an alternative approach to testing GR.

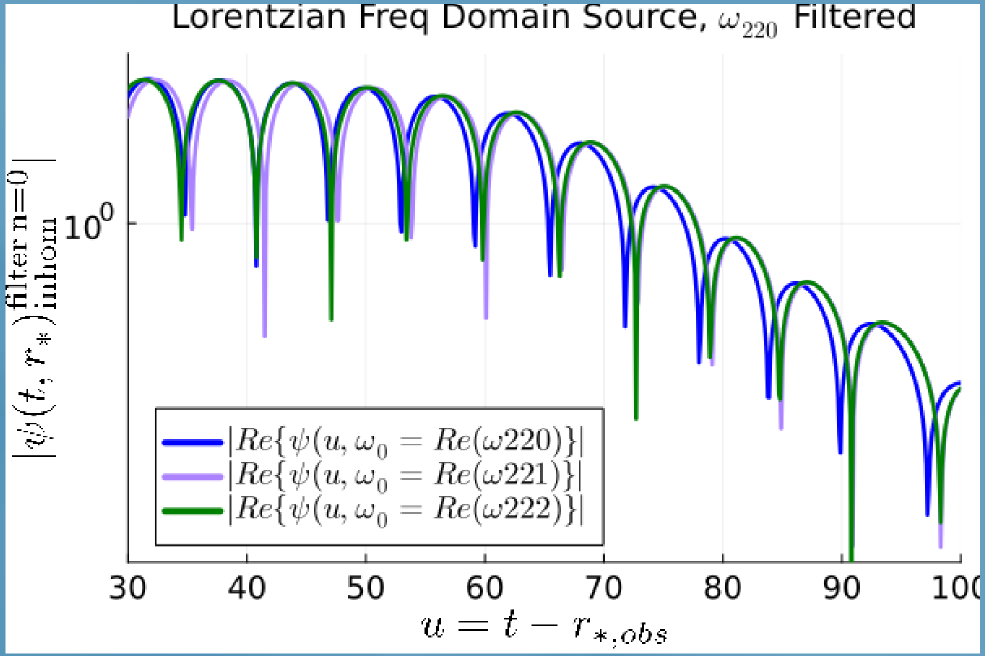




WRONG!







If we drive the source at Re($\omega_{\ell mn}$), can we see its imprint emphasized in the ringdown?"

→ It depends

$$\frac{d^2\psi}{dr_*^2} + [\omega^2 - V(r)]\psi = S$$

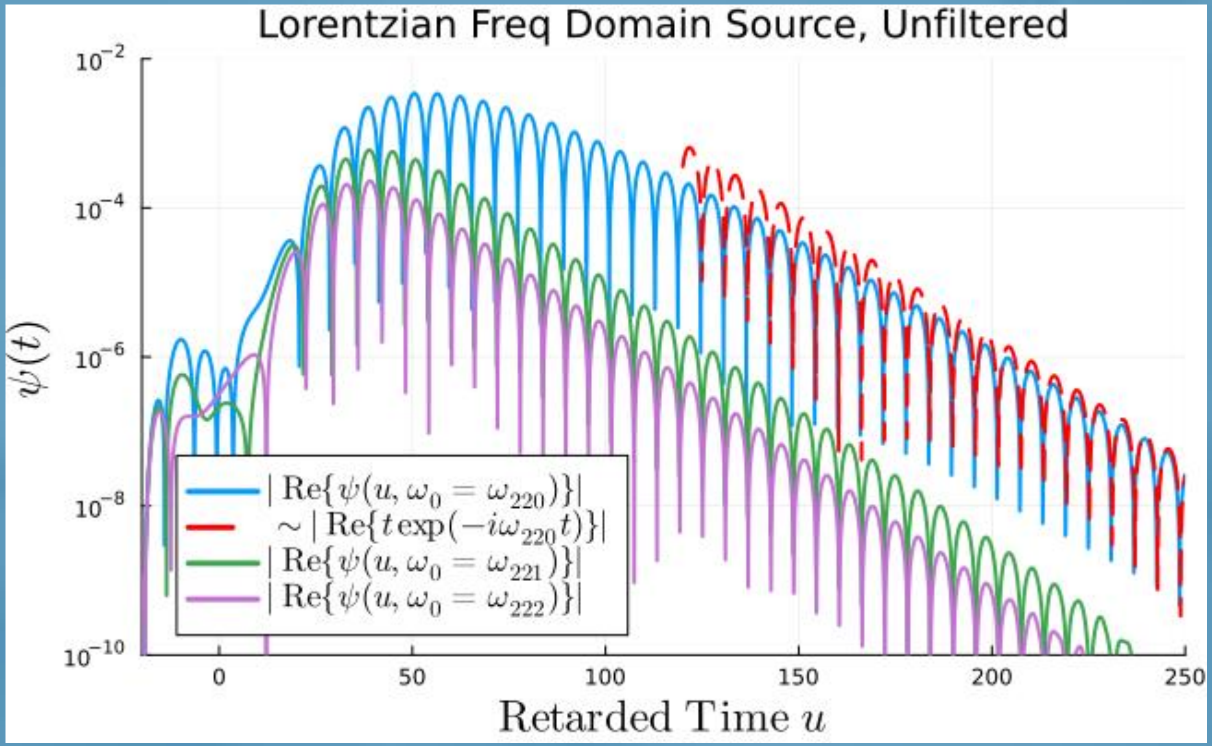
$S_{l'm'n'}(\omega_{lmn})$	ω_{220}	ω_{221}	ω_{222}
$S_{220}(\omega_{lmn})$	2.05356 - 0.0im	0.8497+ 0.0264im	0.6484 + 0.0150im
$S_{221}(\omega_{lmn})$	1.9527- 0.2297im	0.8419 - 0.0im	0.6458 + 0.0096im
$S_{222}(\omega_{lmn})$	1.7292 - 0.5580im	0.8306 - 0.0464im	0.6438 - 0.0im

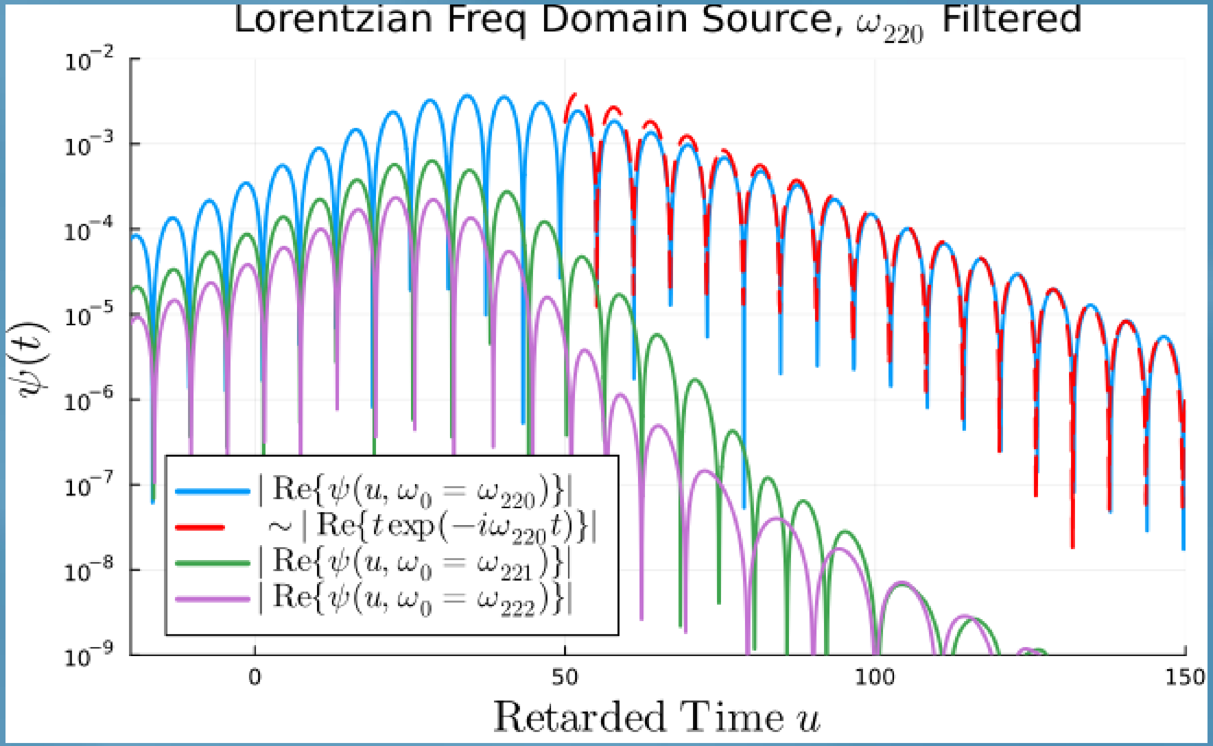
$$A_{\ell mn,\ell' m',n'} \sim \underbrace{E(\omega_{\ell mn})}_{\text{QNEF}}\underbrace{S_{l'm'n'}}_{\text{Source}}(\omega_{\ell mn})$$

$$S_{\ell' m' n'} \sim \frac{1}{1 + (\omega - \text{Re}(\omega_{\ell' m' n'}))^2}$$

$$S_{\ell' m' n'} \sim \frac{1}{1 + (\omega - \omega_{\ell' m' n'})^2}$$

- Driving source at the full QNM
- Source contamination





SUMMARY

- Constructed QNM filters to analyze ringdown.
- Looked at sources peaking at $\mathrm{Re}(\omega_{\ell mn})$ and at the full QNM frequency
- Found: no preferential excitation at $\mathrm{Re}(\omega_{\ell mn})$, but possible at full QNM
- Next steps: Extend this to plunge trajectories (more physical sources)

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