Robust Optics Simulation for Mode-Mismatch Reduction at LIGO Hanford
Final Report
T2500228

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Abstract: The Laser Interferometer Gravitational-wave Observatory (LIGO) Hanford site employs quantum squeezing to increase effective observational range by approximately 30 Mpc — a significant portion of LIGO's total 150 Mpc range. However, the quantum squeezing process may induce substantial mode-mismatch losses upon injection into the main interferometer beam. To combat this, two piezo-actuated curved mirrors exist near the squeezer output path to allow for precise control of beam propagation. In this work, we characterize the squeezed beam propagation along these actuated curved mirrors, creating a comprehensive model for extracting optimal voltage supply to each mirror for mode-mismatch minimization. Using experimental data from a diverted squeezer beam, we construct a macro-level model for beam parameter computation by supplied mirror voltage alone. Our model approach then yields both mode-mismatch quantification (by forward propagation) as well as mirror curvature estimations (by back propagation), ultimately establishing global characteristics for the beam and associated optics.

I. INTRODUCTION

Gravitational-wave detection as performed in the Laser Interferometer Gravitational-Wave Observatory (LIGO) involves the ultra-precise application of lasers to study extreme events in deep space.[1] Due to the sensitivity required of the light-based measurements, it is necessary that the quantum properties of light be substantively controlled. A technique that has proven successful in increasing the signal-to-noise ratio (SNR) of measurements is known as quantum squeezing,[2] which has yielded an approximate range increase of 30 Mpc out of LIGO's approximate 150 Mpc total range.

The idea and method of quantum squeezing is in the reshaping of the quantum noise. That is, the Heisenberg uncertainty principle, given in terms of the standard deviation of position σ_x and momentum σ_p as

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

with $\hbar$ the reduced Planck's constant, implies that increasing uncertainty in amplitude fluctuations allows phase uncertainty to be decreased (and vice versa). In quantum squeezing, this principle is applied to increase the SNR over large frequency ranges by decreasing quantum phase uncertainty.

Despite its successes, if the quantum squeezing system's beam has any mode-mismatch from the main interferometer laser, the net measurement noise does not optimally decrease due to unwanted additional noise. Mode-mismatch refers to a category of events: i.e. wavefront misalignment with mirror curvatures and/or non-ideal beam convergence. These phenomena can arise from incorrect distances for beam travel between optics, flawed curvatures of mirrors or lenses, manufacturing error, mirror tilt, faulty beam centering, etc.

There exist measures we can take to resolve the mode-mismatch, however. Mirrors which have adjustable curvatures via piezo-electronic transducers can manipulate the beam characteristics early in the path of travel, allowing us to fix potential mode-mismatch provided that we have a precise understanding of the beam interactions at each mirror. In this work, we create a comprehensive beam profiling Matlab model in which we may understand mode-mismatch between the squeezed beam and the main interferometer near the readout. Furthermore, applying principals of Gaussian optics, our simulation is able to extract precise estimates for mirror curvatures in-vacuum.

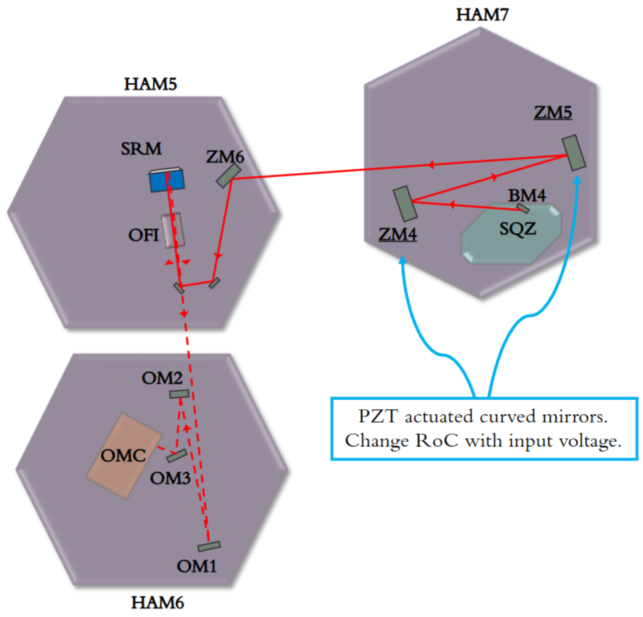
We collect and compare real mode-matching data to the simulation to confirm its accuracy. As the simulation only requests diverted beam path data to run, this work is translatable to other observatories which seek to understand mode-mismatch without disruptions to observation runs.

Throughout this work, the schematic of the beam path as shown in FIG. 1 will be our main spatial reference. As an overview: the squeezed beam exits the squeezer (SQZ), interacts with the actuated curved mirrors ZM4 and ZM5, passes through the Optical Faraday Isolator (OFI), reflects off the Signal Recycling Mirror (SRM) to pass through the OFI once more, then reflects off OM1 (curved mirror), OM2 (curved mirror), OM3 (flat mirror), ultimately reaching the Output Mode Cleaner (OMC).

All code and data referenced in this work can be found in the "alm beam simulation for SQZ" repository.[3]

II. APPROACH AND METHODOLOGY

LIGO Hanford's beam is sufficiently modeled by a Gaussian profile; that is, for some refractive index n and wavelength λ, the beam intensity profile evolves in the



PZT actuated curved mirrors. Change RoC with input voltage.

FIG. 1. Full beam path schematic from the SQZ output to the OMC.

radial and propagation directions (r, z respectively) as

$$I(r, z) = I_0 \left(\frac{w_0}{w(z)} \right)^2 \exp\left(-\frac{2r^2}{w(z)^2} \right)$$

where I_0 is the intensity at the center of the beam focus and w_0 is the beam radius at the focus. The beam radius as a function of propagation direction respects

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R} \right)^2}, \quad z_R = \frac{n\pi w_0^2}{\lambda}.$$

Notice that a Gaussian beam is characterized entirely when waist size and waist location are specified (assuming n and λ known). An immensely useful tool for beam characterization is the complex beam parameter q defined as [4]

$$q(z) = (z - z_0) + iz_R$$

where z_0 denotes the position of the beam waist. $q(z)$ thus contains all necessary information to understand the beam, when determined. When interacting with an optical component with known $ABCD$ matrix (ray transfer matrix), the output complex beam parameter is determined by

$$q_{out} = \frac{Aq + B}{Cq + D}. \tag{1}$$

In general, we may quantify the efficiency η of mode-matching via integrals over complex planar electric fields E_1, E_2 [5]

$$\eta = \frac{|\int E_1^* E_2 \, \mathrm{d}A|^2}{\int |E_1|^2 \, \mathrm{d}A \int |E_2|^2 \, \mathrm{d}A} \tag{2}$$

with optimal $\eta = 1$ occurring when $E_1 = E_2$. Crucially, the normalized electric field strength $u(r, z)$ for a circular Gaussian beam is determined from $q(z)$ alone, given as

$$u(r, z) = \frac{1}{q(z)} \exp\left(-ik \frac{r^2}{2q(z)} \right),$$

where $k = 2\pi n/\lambda$. Thus, in order to fully simulate our system, we obtain $q(z)$ at some arbitrary point and calculate the propagation using the ray transfer matrix of each component. Using the Matlab-based "a la mode" mode-matching simulation software,[6] we are able to input these parameters for efficient computation of mode overlap.

With the framework for component creation in a la mode, beam path creation reduces to finding component characteristics from documentation. We obtained precise values for component distances, radii of curvature and incidence angles (when applicable), and refractive indices, with certain parameters such as periscope height measured manually using a standard ruler. The sources of these metrics were logged in LIGO Document T2500228.[7]

We then collected the experimental beam q parameters on the SQZT7 diagnostic table (a remotely switchable diverter provides the beam; see FIG. 2). On SQZT7, both horizontal and vertical beam widths were measured using a NanoScan beam profiler at 5-7 consecutive points along the path. A metric ruler was used to measure the distances of each profile from the base of the periscope as in FIG. 2. Prior to collecting each dataset, the strain gauges for ZM4 and ZM5 were fixed to specified voltage readouts. That is, the 5-7 point profiling was performed for a total of 10 combinations of curvature radii for ZM4 and ZM5. The combinations were chosen to include extrema of the strain gauges to appropriately characterize the entire range of ZM4 and ZM5 curvatures.

All 10 datasets (associated to their strain gauge combination) were then fitted with an optimal q parameter using a la mode. The q parameters were interpolated using a low-order polynomial surface to model the beam as a continuous function of ZM4 and ZM5 strain gauge voltages (and thus a continuous function of ZM4 and ZM5 curvatures).

To understand mode-matching, this continuous function of q parameters was mathematically propagated forward through the beam path and into the OMC, wherein mode-matching was computed using (2). From this, mode-matching was represented as a continuous function of ZM4 and ZM5 strain gauges.

Since the exact curvature radii for ZM4 and ZM5 could not be known without costly in-vacuum measurements on HAM7, the interpolated q parameters were mathematically back-propagated to ZM5 for analysis. Using the ray

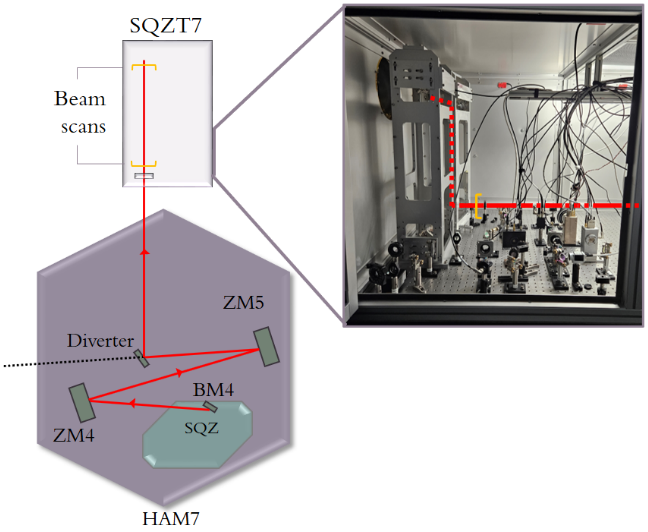


FIG. 2. Schematic of the diverted beam path to SQZT7.

transfer matrix method, the following relation holds:

$$\begin{bmatrix} q_s \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{R_4} & 1 \end{bmatrix} \begin{bmatrix} 1 & \ell \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{2}{R_5} & 1 \end{bmatrix} \begin{bmatrix} q_m \\ 1 \end{bmatrix}$$

$$\implies \begin{bmatrix} \frac{2}{R_4} \\ 1 \end{bmatrix} = \begin{bmatrix} 4q_m q_s + 4\ell q_m & 4(q_s - q_m + \ell) \\ 4\ell q_m q_s & 4\ell q_s - 4q_m q_s \end{bmatrix} \begin{bmatrix} \frac{2}{R_5} \\ 1 \end{bmatrix}$$

where ℓ is the distance between ZM4 and ZM5, q_m and q_s are the interpolated and seed q parameters respectively, and R_4 and R_5 are the curvature radii of ZM4 and ZM5 respectively. In order to solve the above overdetermined system, optical power of ZM4 was assumed to vary linearly with strain gauge voltage. Prior data available in LIGO Document E2100289 validates this assumption.[8]

The curvature R_5 was then calculated for all combinations of strain gauge voltages. In doing so, the curvature of the actuated mirrors could be estimated from the diverted beam path profiling alone.

III. RESULTS

The beam propagation for nominal ZM4 and ZM5 strain gauge values is shown in FIG. 3. The "q-manifold" — the continuous q parameters as a function of ZM4 and ZM5 strain gauge voltages — is displayed in FIG. 4. The use of color mapping is necessary as q is complex valued. Notice in particular that for fixed ZM5 voltage, the real part of the q parameter varies as a linear function (or rather, as a close approximate linear function) of ZM4 voltage, further validating our linearity assumption for the component. In order to know all pertinent characteristics of the beam, one only needs to evaluate the q-manifold with specified strain gauge values. To account for astigmatism, q-manifolds for both the horizontal and vertical beam widths were created for mode-matching analysis.

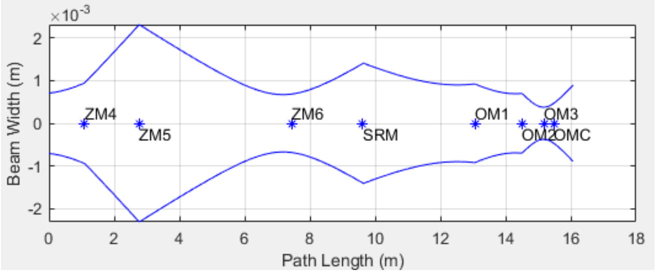


FIG. 3. Beam propagation simulated from the SQZ output to the OMC for strain gauge readouts of 6 V for ZM4 and -0.4 V for ZM5.

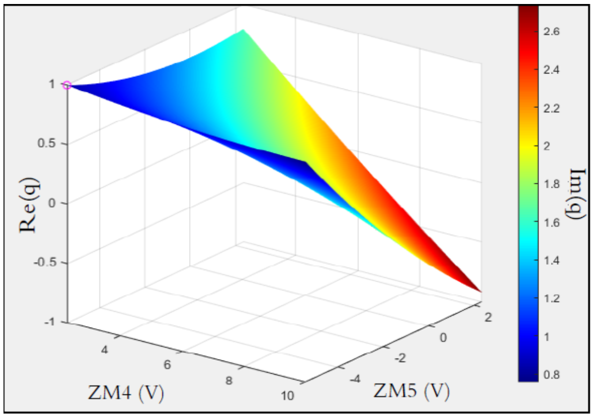


FIG. 4. Interpolated q parameters for horizontal width data over all supplied voltages to ZM4 and ZM5. The real part of q is plotted on the z-axis, while the imaginary part is mapped in color.

Forward propagation of the q-Manifold yields the "η-manifold" — the continuous function representing mode-matching values between the SQZ and OMC beams over all strain gauge readings. This surface is the geometric mean of the separate horizontal and vertical mode-matching surfaces computed from the horizontal and vertical q-manifolds.

Mode-matching scans at the OMC were performed to test the simulation's accuracy (see FIG. 5). While the simulation accurately predicts the value of η in the regions where high mode-match occurs, it underestimates η near suboptimal curvatures of ZM4 and ZM5. The simulated and experimental surfaces are similar in their relative profiles (i.e. both predict high and low η in approximately the same regions), implying a possible breakdown of the Gaussian beam assumption due to astigmatic behavior.

Finally, back-propagating the q-manifold allowed us to obtain highly satisfactory estimates for ZM5 from the assumed curvature of ZM4. In particular, since the seed beam before ZM4 has known q parameter, and since we assume a ZM4 curvature, the q parameter before ZM5 is determined by (1). Thus, the ZM5 curvature can be computed (also by (1), reorganized to solve for matrix element $C = -2/R_5$), and the simulation can propagate

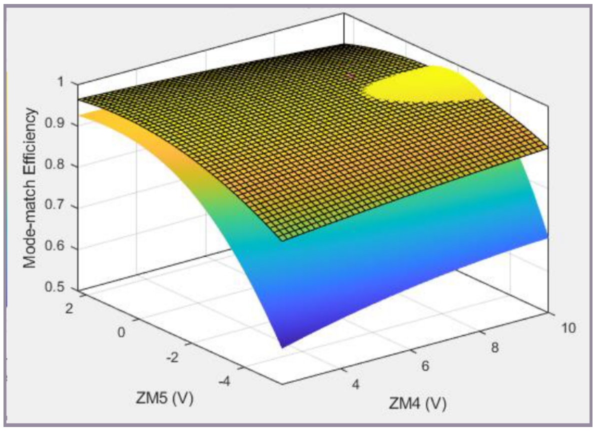
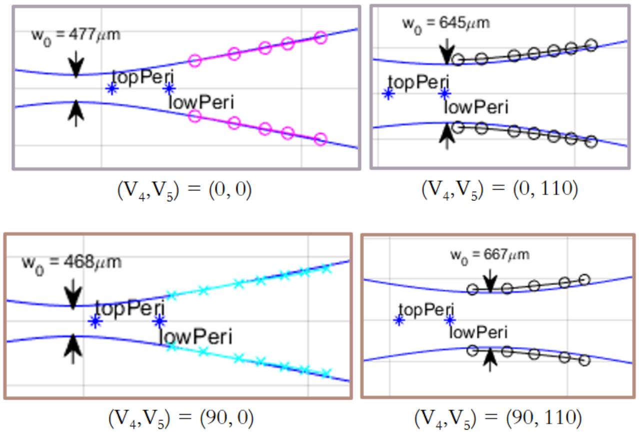


FIG. 5. Simulated mode-matching (smooth surface) and experimental mode-matching (gridded surface) overlaid.



$(V_4,V_5) = (0,0)$ $(V_4,V_5) = (0,110)$

$(V_4,V_5) = (90,0)$ $(V_4,V_5) = (90,110)$

FIG. 6. Simulated beam widths with estimated ZM4 and ZM5 curvatures (solid blue) against the experimental data used for each estimation. Each ordered pair has units in PZT voltage.

the beam from the SQZ to the OMC. FIG. 6 displays four such results from the estimates. We conclude that the simulation can indeed be used to find accurate mirror characteristics, avoiding the high financial costs and time consumption of performing measurements of components in vacuum.

IV. CONCLUSIONS

The beam simulation of the SQZ beam traveling toward the LIGO Hanford OMC that was created, in its full generality, performs particularly well in three specific areas: robust beam characterization with regards to the actuated mirror curvatures (the q-manifold), identifying broad mode-match trends (the η-manifold), and estimating curvature radii of otherwise inaccessible mirrors (q-manifold back-propagation). More precise computations are necessary to ensure the mode-matching values indeed match up with experiment, likely requiring a more detailed understanding of astigmatism caused by individual components. In general, the simulation is highly versatile, and can be translated to other optical setups (e.g. LIGO Livingston) so long as beam profiling is possible.

V. ACKNOWLEDGEMENTS

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VI. REFERENCES

[1] M. Pitkin et al. “Gravitational Wave Detection by Interferometry (Ground and Space)”. In: *Living Reviews in Relativity* 14 (July 2011). DOI: `10.12942/lrr-2011-5`. URL: `https://doi.org/10.12942/lrr-2011-5`.

[2] D. Ganapathy et al. “Broadband Quantum Enhancement of the LIGO Detectors with Frequency-Dependent Squeezing”. In: *Physical Review X* 13 (Oct. 2023). DOI: `10.1103/PhysRevX.13.041021`.

[3] L. Schrader. *alm beam simulation for SQZ*. `https://git.ligo.org/leendert.schrader/alm-beam-simulation-for-sqz`. 2025.

[4] A.E. Siegman. *Lasers*. University Science Books, 1986. ISBN: 0-935702-11-3.

[5] R. Paschotta. *Mode Matching*. RP Photonics Encyclopedia. 2005. DOI: `10.61835/x1d`. URL: `https://www.rp-photonics.com/mode_matching.html` (visited on 07/03/2025).

[6] N. Smith. *a la mode*. `https://github.com/nicolassmith/alm`. 2016.

[7] L. Schrader. *Leo Schrader SURF Project 2025 (DCC: T2500228)*. LIGO Document Control Center. 2025.

[8] C. Makarem. *A+ ISC PSAMS Characterization Data: ZM4 (SN1) (DCC: E2100289)*. LIGO Document Control Center. 2021.