Robust Optics Simulation for Mode-Mismatch Reduction at LIGO Hanford
Interim Report #2

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Abstract: The Laser Interferometer Gravitational-wave Observatory (LIGO) Hanford site employs quantum squeezing to increase effective observational range by approximately 30 Mpc — a significant portion of LIGO's total 150 Mpc range. However, the quantum squeezing process may induce substantial mode-mismatch losses upon injection into the main interferometer beam. To combat this, two piezo-actuated curved mirrors exist near the squeezer output path to allow for precise control of beam propagation. In this work, we characterize the squeezed beam propagation along these actuated curved mirrors, creating a comprehensive model for extracting optimal voltage supply to each mirror for mode-mismatch minimization. Using experimental data from a diverted squeezer beam, we construct a macro-level model for beam parameter computation by supplied mirror voltage alone. Our model approach then yields both mode-mismatch quantification (by forward propagation) as well as mirror curvature estimations (by back propagation), ultimately establishing global characteristics for the beam and associated optics.

I. BACKGROUND AND MOTIVATION

Gravitational-wave detection as performed in the Laser Interferometer Gravitational-Wave Observatory (LIGO) involves the ultra-precise application of lasers to study extreme events in deep space [1]. Due to the sensitivity required of the light-based measurements, it is necessary that the quantum properties of light be substantively controlled. A technique that has proven successful in increasing the signal-to-noise ratio (SNR) of measurements is known as quantum squeezing [2], which has yielded an approximate range increase of 30 Mpc out of LIGO's $\approx$ 150 Mpc total range. The idea and method of quantum squeezing is in the "re-allocation" of noise. That is, the Heisenberg uncertainty principle implies that increasing uncertainty in amplitude fluctuations (or radiation pressure noise) allows phase uncertainty (or shot noise) to be decreased, and vice versa. We are thus able to induce specific correlations that optimize the SNR in distinct frequency ranges.

Despite its successes, if the quantum squeezing system's beam has any mode-mismatch from the main interferometer laser, the net measurement noise does not optimally decrease due to unwanted signal loss. Mode-mismatch refers to a category of events: i.e. wavefront misalignment with mirror curvatures and/or non-ideal beam convergence. These phenomena can arise from incorrect distances for beam travel between optics, flawed curvatures of mirrors or lenses, manufacturing error, mirror tilt, faulty beam centering, etc.

There exist measures we can take to resolve the mode-mismatch, however. Mirrors which have adjustable curvatures via piezoelectronics can manipulate the beam characteristics early in the path of travel, allowing us to fix potential mode-mismatch provided that we have a precise understanding of the beam interactions at each mirror. If we gain this understanding and build a comprehensive strategy for adjustments, we extract as many benefits from the quantum squeezing as reasonably possible.

II. PROJECT SUMMARY

In order to properly adjust the variable-curvature mirrors according to mode-mismatch, we must first know how the beam propagates through the system in general. Simulations of beam travel are crucial to proactively correct mode-mismatching. LIGO Hanford's beam is sufficiently modeled by a Gaussian profile; that is, for some refractive index n and wavelength λ, the beam intensity profile evolves in the radial and propagation directions (r, z respectively) as

$$I(r,z) = I_0 \left(\frac{w_0}{w(z)}\right)^2 \exp\left(-\frac{2r^2}{w(z)^2}\right)$$

where I_0 is the intensity at the center of the beam at focus w_0, with the beam radius in general respecting

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}, \quad z_R = \frac{n\pi w_0^2}{\lambda}.$$

Notice that a Gaussian beam is characterized entirely by the waist size and location when n, λ known. An immensely useful tool for beam characterization is the complex beam parameter q defined as [3]

$$q(z) = (z - z_0) + iz_R$$

where z_0 denotes the waist position. $q(z)$ thus contains all necessary information to understand the beam when determined. When interacting with an optical component with known $ABCD$ matrix (ray transfer matrix), the output complex beam parameter is determined by

$$q_{out} = \frac{Aq+B}{Cq+D}.$$

In general, we may quantify the efficiency η of mode-matching by integrals over complex planar electric fields E_1, E_2 [4]:

$$\eta = \frac{|\int E_1^* E_2 \, \mathrm{d}A|^2}{\int |E_1|^2 \, \mathrm{d}A \int |E_2|^2 \, \mathrm{d}A}$$

with optimal $\eta = 1$ occurring when $E_1 = E_2$. Crucially, the complex electric field for a Gaussian beam is determinable from $q(z)$ alone. Thus, in order to fully simulate our system, we just need to obtain $q(z)$ at some arbitrary point and calculate the propagation through each component's ray transfer matrix. Using the Matlab-based "a la mode" mode-matching simulation software [5], we are able to input these parameters for efficient computation of mode overlap.

My work involves building such a simulation for the beam traveling from the quantum squeezer to the main interferometer and the output mode cleaner (OMC). I will then combine results with calibration measurements of the actuated curvature mirrors in order to build an optimization routine, predicting the necessary voltage supplied to the mirrors for ideal mode-matching. This will serve as a comprehensive strategy to reduce signal losses in a simple and non-invasive manner.

III. WORK DONE AND PROGRESS

I began by completing the preliminary beam propagation model from the output of the quantum squeezer to the OMC (FIG. 2). This included implementing the interactions with the actuated mirrors, optical Faraday isolator (OFI), signal recycling mirror (SRM), and various other optical components with set curvatures between the OFI and OMC. I have compiled a comprehensive list of relevant documentation related to length, radii of curvature, incidence angles, refractive indices, and beam path measurements for each mirror in the path to reliably track the sources of all data.

Moreover, I expanded the a la mode software to take in incidence angles as arguments to increase the simulation accuracy for curved mirror propagation. I dissected each optical component interaction in the simulation to find the overlap of the measured beam with the ideal mode, finding a lower limit of ≈ 98%, indicating a well-constructed simulation. Changing the curvature values of the actuated mirrors within their designated range allowed us to obtain an arbitrarily good — i.e. numerically perfect (100%) — match, further signifying that the investigation on precise mirror controls will be fruitful once established.

The actuated mirror curvatures were changed using the PZT (piezo-electronic device) and a strain gauge in order to measure the force-curvature relations in the past. When said testing was performed, the mirrors had a set preloading — a starting, baseline applied force — which was ultimately changed after all data was collected. I used this voltage-strain data for ZM4 and ZM5 and interpolated voltage-to-curvature equations from the new preloading, granting the user access to a "controller" that plotted beam propagation with respect to PZT voltages.

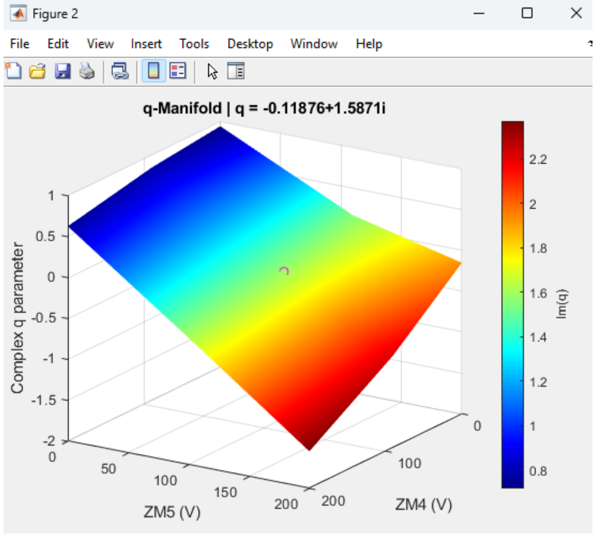


FIG. 1. Sample Matlab figure of the q manifold with the combination $V_{ZM4} = 90$ V, $V_{ZM5} = 110$ V circled in magenta.

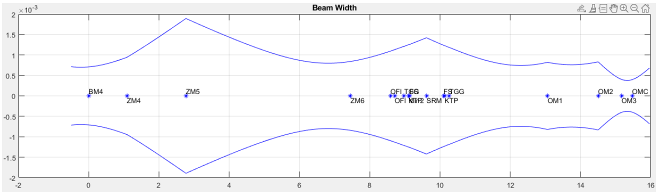
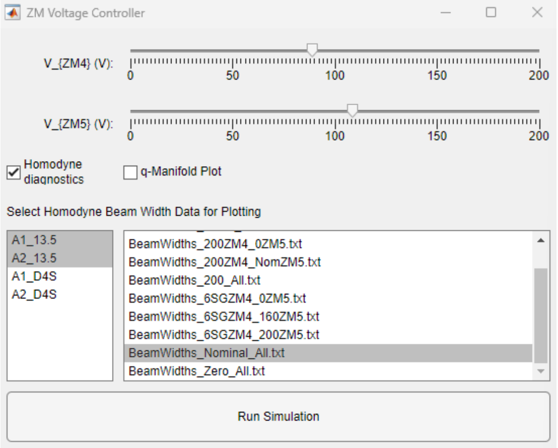


FIG. 2. Beam propagation simulated from the SQZ output to the OMC for $V_{ZM4} = 90$ V and $V_{ZM5} = 110$ V.

Once the beam travel could be predicted by input voltage, we collected precise beam width data on a diverted SQZ beam path. Data were collected for ten different characteristic combinations of PZT voltages (i.e. 0 V to ZM4 and 200 V to ZM5, 90 V to ZM4 and 110 V to ZM5, etc.). The beam width data was then fitted by a la mode's beam fitting protocol and back propagated to ZM5 to extract the q parameters for each voltage combination. I then constructed a "q manifold": a complex-valued surface which interpolates the q parameters as a continuous function of voltage input, thus providing a macro-scale approach for understanding the beam immediately after interacting with ZM4 and ZM5 (see FIG. 1).

By forward propagation of the q manifold, precise calculation of the mode-mismatch at the OMC becomes a matter of simply selecting PZT voltages. Assuming linear behavior of ZM5, I also back-propagated the q manifold to predict what curvature ZM5 must be — the curvature of ZM4 follows by knowing the properties of the seed beam and the q parameter calculated between ZM4 and ZM5.



☑ Homodyne diagnostics
☐ q-Manifold Plot

Select Homodyne Beam Width Data for Plotting

A1_13.5
A2_13.5
A1_D4S
A2_D4S

BeamWidths_200ZM4_0ZM5.txt
BeamWidths_200ZM4_NomZM5.txt
BeamWidths_200_All.txt
BeamWidths_6SGZM4_0ZM5.txt
BeamWidths_6SGZM4_160ZM5.txt
BeamWidths_6SGZM4_200ZM5.txt
BeamWidths_Nominal_All.txt
BeamWidths_Zero_All.txt

FIG. 3. Graphic user interface of current simulation version. See FIG. 4 for the output.

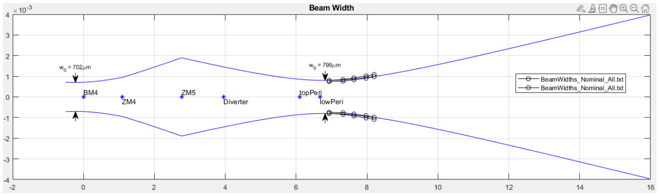


FIG. 4. Beam width plotted from the input of FIG. 3.

Furthermore, I built a modular graphic user interface capable of plotting any combination of beam width data; displaying the q manifold and the exact q parameter for the selected PZT voltages; and which allows visualization of either the diverted path or the main injection path (FIG. 3).

IV. PAST CHALLENGES

Understanding the accuracy of component measurements and when they were collected was crucial to ensure the simulation's accuracy given the evolution of LIGO's optical setups over time. Ultimately, we had to obtain certain measurements such as mirror-to-mirror height in a diverted path periscope ourselves to complete the simulation.

Modeling the SRM also posed certain issues: originally, the SRM was modeled in the beam path as a curved mirror interposed between two dielectrics; however, a la mode was found to add overlapping component matrices (not multiply, as is theoretically valid). I had to thus construct a unique transfer matrix for the component as the combined transfer matrices of all five interactions as in FIG. 5. General issues with construction of the user interface were persistent as well; though, this was resolved with sufficient documentation review.

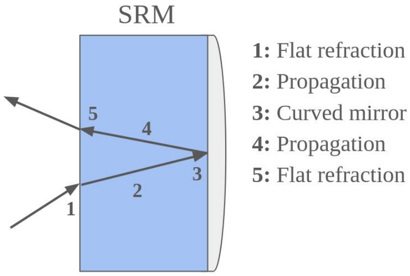


FIG. 5. Model of relevant optical interactions in the SRM.

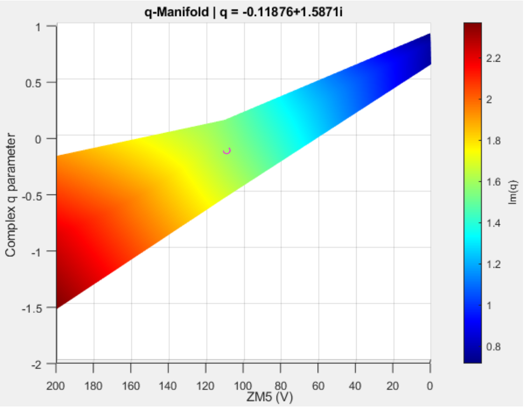


FIG. 6. Alternate view of q manifold; notice the presence of both linear and nonlinear behavior.

V. POTENTIAL CHALLENGES

An issue arises when plotting beam propagation for combinations of small V_{ZM4} and large V_{ZM5} — this is a consequence of our ZM5 voltage-strain linearity assumption failing, as seen in the q manifold's nonlinear behavior around $V_{ZM4} = 0$, $V_{ZM5} = 200$ (FIG. 6 provides a clearer view of this phenomena). We will thus need to incorporate a more complicated equation when assuming the curvature for ZM5 in the visualization process (this does not impact calculation of η).

Since we seek to optimize η, we will also make an "η manifold" assigning mode-matching efficiency to each q parameter of the q manifold. We foresee possible problems in extracting the maxima of this η manifold, as its structure is yet unknown to us: we may need some clever combination of surface gradients and numerical maximum extrapolation depending on the structure's complexity. For the moment, such combinations can only be left to conjecture.

VI. REFERENCES

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