Robust Optics Simulation for Mode-Mismatch Reduction at LIGO Hanford Interim Report #1

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I. BACKGROUND AND MOTIVATION

Gravitational-wave detection as performed in the Laser Interferometer Gravitational-Wave Observatory (LIGO) involves the ultra-precise application of lasers to study extreme events in deep space [1]. Due to the sensitivity required of the light-based measurements, it is necessary that the quantum properties of light be substantively controlled. A technique that has proven successful in increasing the signal-to-noise ratio (SNR) of measurements is known as quantum squeezing [2], which has yielded an approximate range increase of 30 Mpc out of LIGO's $\approx$ 150 Mpc total range. The idea and method of quantum squeezing is in the "re-allocation" of noise. That is, the Heisenberg uncertainty principle implies that increasing uncertainty in amplitude fluctuations (or radiation pressure noise) allows phase uncertainty (or shot noise) to be decreased, and vice versa. We are thus able to induce specific correlations that optimize the SNR in distinct frequency ranges.

Despite its successes, if the quantum squeezing system's beam has any mode-mismatch from the main interferometer laser, the net measurement noise does not optimally decrease due to unwanted signal loss. Mode-mismatch refers to a category of events: i.e. wavefront misalignment with mirror curvatures and/or non-ideal beam convergence. These phenomena can arise from incorrect distances for beam travel between optics, flawed curvatures of mirrors or lenses, manufacturing error, mirror tilt, faulty beam centering, etc.

There exist measures we can take to resolve the mode-mismatch, however. Mirrors which have adjustable curvatures via piezoelectronics can manipulate the beam characteristics early in the path of travel, allowing us to fix potential mode-mismatch provided that we have a precise understanding of the beam interactions at each mirror. If we gain this understanding and build a comprehensive strategy for adjustments, we extract as many benefits from the quantum squeezing as reasonably possible.

II. PROJECT SUMMARY

In order to properly adjust the variable-curvature mirrors according to mode-mismatch, we must first know how the beam propagates through the system in general. Simulations of beam travel are crucial to proactively correct mode-mismatching. LIGO Hanford's beam is sufficiently modeled by a Gaussian profile; that is, for some refractive index n and wavelength λ, the beam intensity profile evolves in the radial and propagation directions (r, z respectively) as

$$I(r,z) = I_0 \left(\frac{w_0}{w(z)}\right)^2 \exp\left(-\frac{2r^2}{w(z)^2}\right)$$

where I_0 is the intensity at the center of the beam at focus w_0, with the beam radius in general respecting

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}, \quad z_R = \frac{n\pi w_0^2}{\lambda}.$$

Notice that a Gaussian beam is characterized entirely by the waist size and location when n, λ known. An immensely useful tool for beam characterization is the complex beam parameter q defined as [3]

$$q(z) = (z - z_0) + iz_R$$

where z_0 denotes the waist position. $q(z)$ thus contains all necessary information to understand the beam when determined. When interacting with an optical component with known $ABCD$ matrix (ray transfer matrix), the output complex beam parameter is determined by

$$q_{out} = \frac{Aq+B}{Cq+D}.$$

In general, we may quantify the efficiency η of mode-matching by integrals over complex planar electric fields E_1, E_2 [4]:

$$\eta = \frac{|\int E_1^* E_2 \,\mathrm{d}A|^2}{\int |E_1|^2 \,\mathrm{d}A \int |E_2|^2 \,\mathrm{d}A}$$

with optimal $\eta = 1$ occurring when $E_1 = E_2$. Crucially, the complex electric field for a Gaussian beam is determinable from $q(z)$ alone. Thus, in order to fully simulate our system, we just need to obtain $q(z)$ at some arbitrary point and calculate the propagation through each component's ray transfer matrices. Using the Matlab-based "a la mode" mode-matching simulation software [5], we are able to input these parameters for efficient computation of mode overlap.

My work involves building such a simulation for the beam traveling from the quantum squeezer to the main interferometer and the output mode cleaner (OMC). I will then combine results with calibration measurements of the actuated curvature mirrors in order to build an optimization routine, predicting the necessary voltage supplied to the mirrors for ideal mode-matching. This will serve as a comprehensive strategy to reduce signal losses in a simple and non-invasive manner.

III. WORK DONE AND PROGRESS

The construction of the full beam propagation from the output of the quantum squeezer to the OMC has been complete. This includes the interactions with the actuated mirrors, optical Faraday isolator (OFI), signal recycling mirror (SRM), and various other optical components with set curvatures between the OFI and OMC. I have compiled a comprehensive list of relevant documentation related to length, radii of curvature, incidence angles, refractive indices, and beam path measurements for each mirror in the path to reliably track the sources of all data.

Moreover, I expanded the a la mode software to take in incidence angles as arguments to increase the simulation accuracy for curved mirror propagation. I dissected each optical component interaction in the simulation to find the overlap of the measured beam with the ideal mode, finding a lower limit of $\approx 98\%$, indicating a well-constructed simulation. Changing the curvature values of the actuated mirrors within their designated range allowed us to obtain an arbitrarily good — i.e. numerically perfect (100%) — match, further signifying that the investigation on precise mirror controls will be fruitful once established.

IV. PAST CHALLENGES

Two major hurdles appeared in the simulation of the propagation through the full squeezer-to-OMC path: obtaining appropriate measurements and modeling the SRM component. Understanding the precision of measurements and when they were collected was crucial to ensure the simulation's accuracy given the evolution of LIGO's optical setups over time. Ultimately, we had to obtain and document certain measurements such as mirror-to-mirror height in a periscope ourselves to complete the simulation.

As for modeling the SRM, the beam propagation in this component is somewhat complex compared to others, with a five-step interaction as simplified in the FIG. 1. Originally, the SRM was modeled in the beam path as a curved mirror interposed between two dielectrics; however, a la mode was found to add overlapping component matrices (not multiply, as is theoretically correct), ultimately yielding a 25.4% mode-match. Thus, I resorted to constructing a unique transfer matrix M_{SRM} for the component as the combined transfer matrices of all five interactions, i.e.

$$M_{SRM}\begin{bmatrix}q\\1\end{bmatrix}=M_5M_4M_3M_2M_1\begin{bmatrix}q\\1\end{bmatrix}=\begin{bmatrix}q_{out}\\1\end{bmatrix}.$$

Using, M_{SRM}, we obtained the desired $\eta \geq 98\%$ for this and subsequent components.

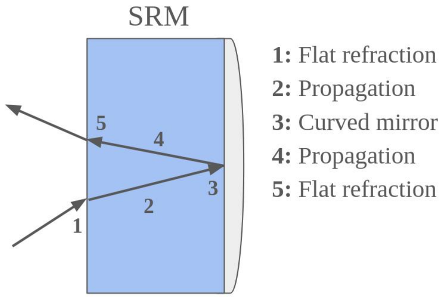

FIG. 1 Model of relevant optical interactions in the SRM.

V. POTENTIAL CHALLENGES

The actuated mirror curvatures were changed using the PZT and a strain gauge in order to measure the force-curvature relations in the past. However, when said testing was performed, the mirrors had a set preloading — a starting, baseline applied force — which was ultimately changed after all data was collected. That is, the exact PZT voltage-to-curvature relationships may no longer be reliable under the new preload. It is possible that the curvature changes can safely be assumed linear; despite this, we will still need to understand the effects of hysteresis (present in the original calibration data) to make the simulation as accurate as possible. Therefore, we will need some way to either adjust the old data or rigorously collect the new — both are likely to be highly involved in their own ways. Regardless, the progress made so far implies we may allot more time to studying the dynamics of the mirrors than originally expected as necessary.

VI. REFERENCES

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