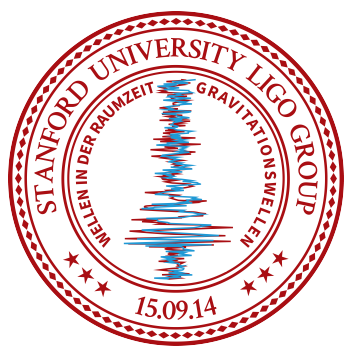




2 stage Reaction Chain

Brian Lantz, Dec 3 2024, G2402448-v1



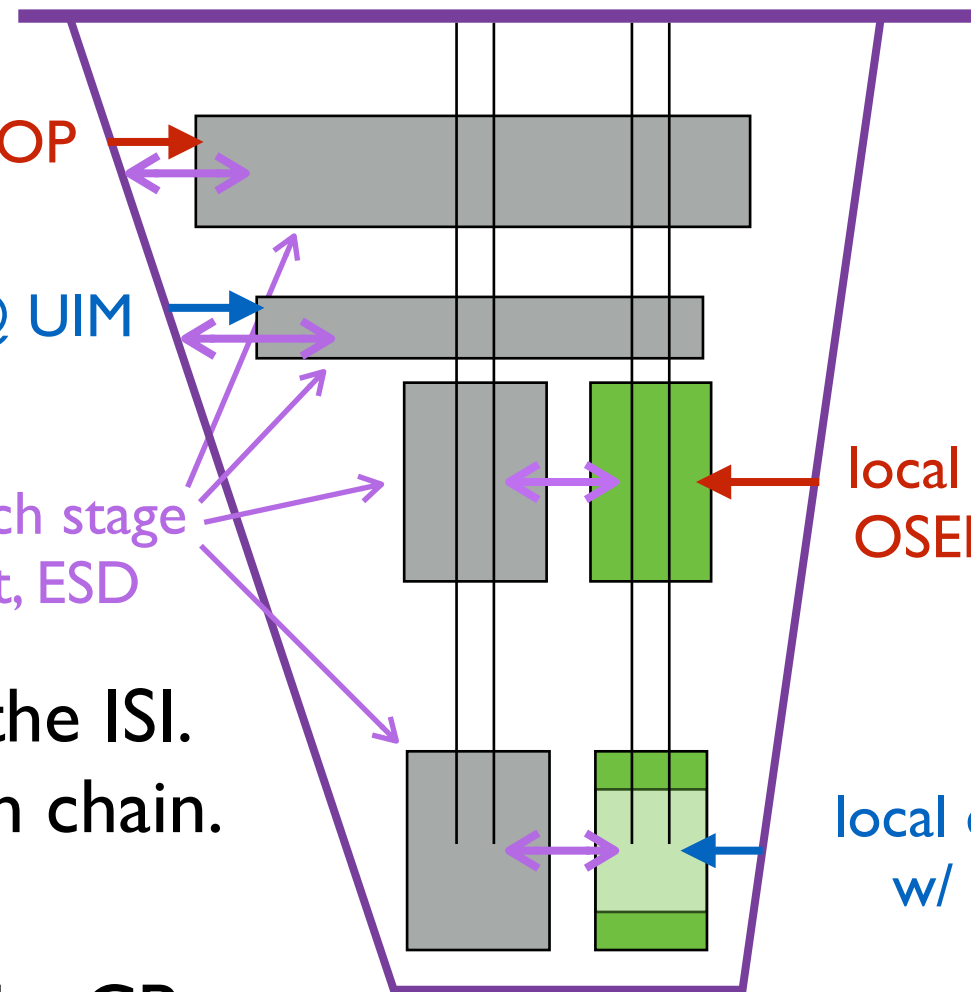
local damping w/ OSEM or better @ TOP

local damping w/ HoQI @ UIM

ISC drives at each stage
w/ coil-magnet, ESD

local damping w/
OSEM or better

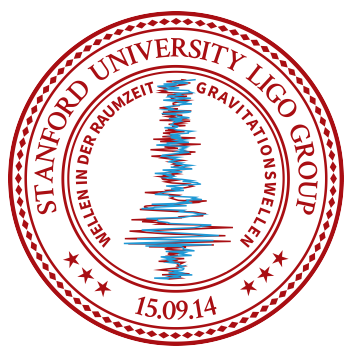
local damping
w/ cIFO?



Top 2 stages of the Quad react against the ISI.
PUM and optic react again the reaction chain.

Update:

- Isolation requirements were driven by CP.
spec's for ESD are very easy to meet.
- Want 2 stages as reaction masses to spare the ISI
- ITM & ETM chains match, both have big holes.
- Design is flexible, driven by ESD, clear aperture,
simple dynamics



2 stage Reaction Chain

Brian Lantz, Dec 3 2024, G2402448-v1



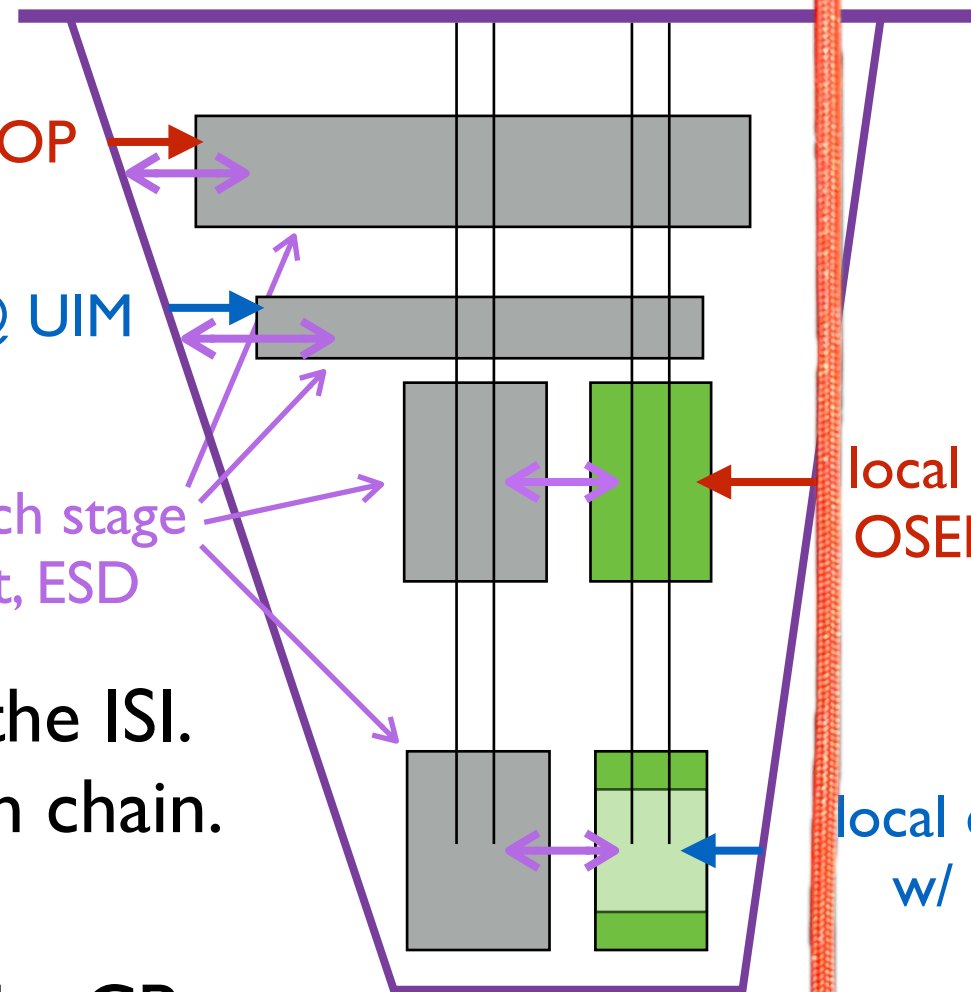
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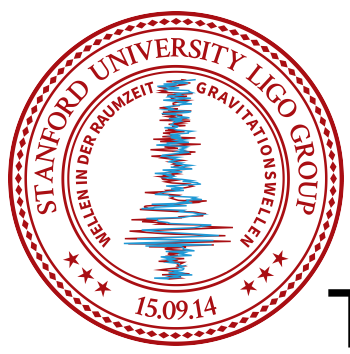
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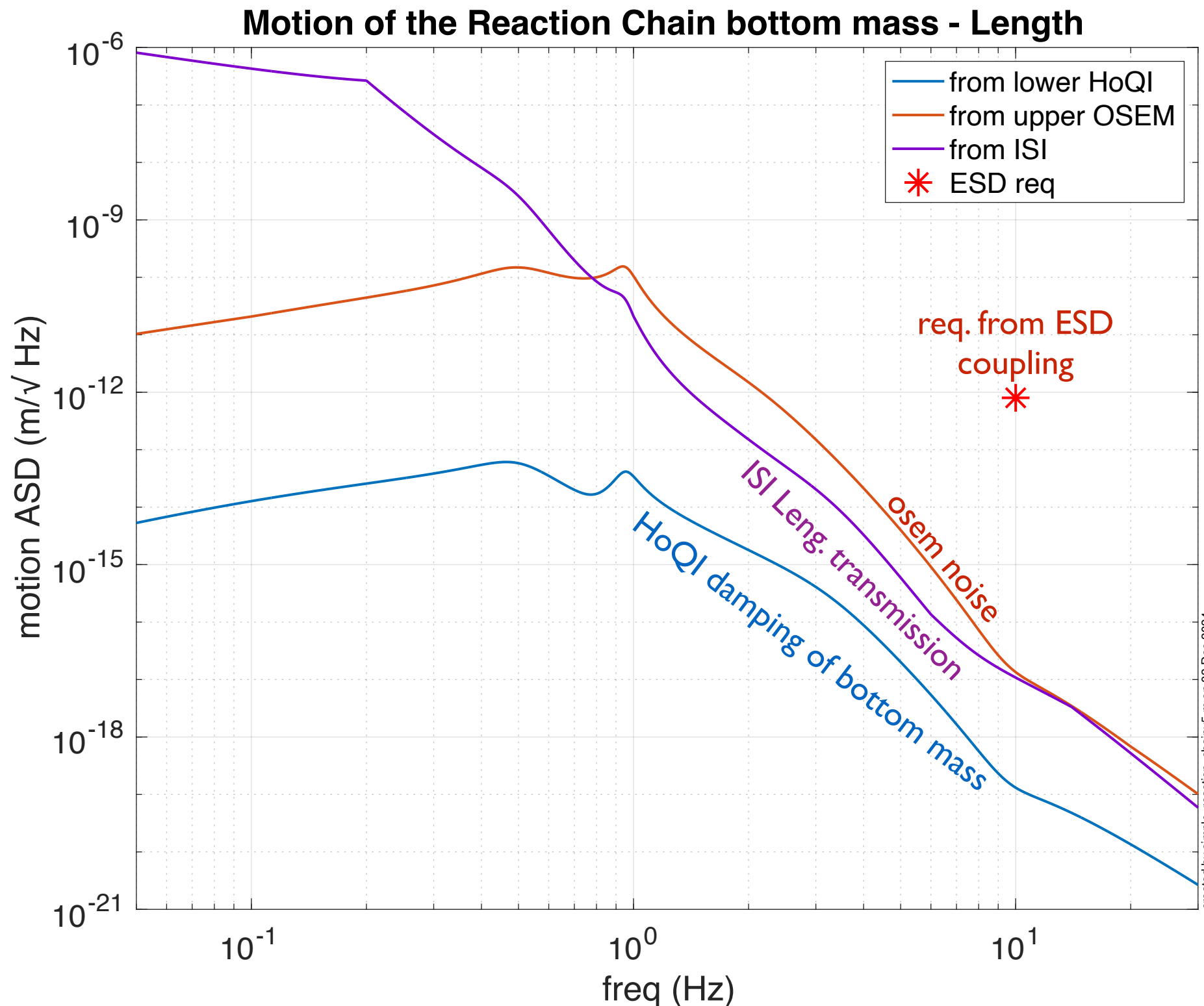
Update:

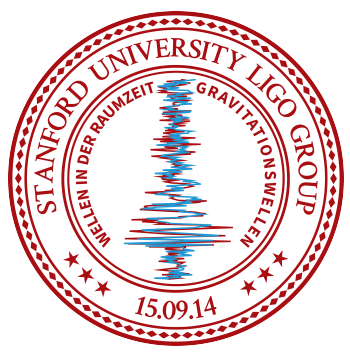
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simple dynamics



Reaction chain motion

The double reaction chain easily meets the 10 Hz ESD spec.
(ESD spec is $8e-13$ m/rtHz, ISI assumed to be $5e-13$ m/rtHz at 10 Hz)





What about the other DOFs



To meet the noise requirements for ESD coupling:

- coupling is so small that no isolation is needed for any DOF.
- will summarize calculation, please check that I'm not missing anything

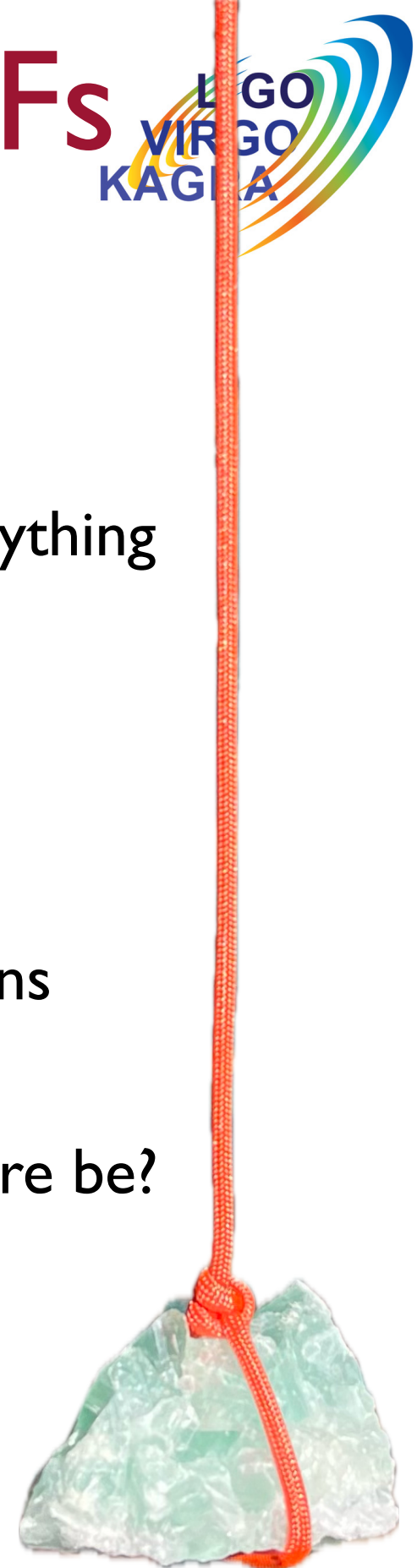
What drives the design of the reaction chain?

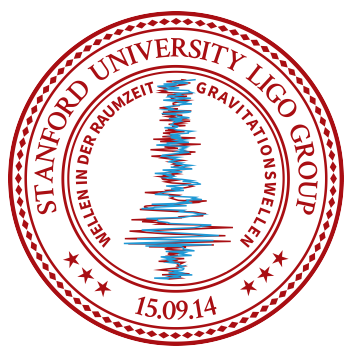
We need:

- We need a dynamic mass to isolate the ISI from the ISC drives
Drives on UIM are $\sim$ below 1 Hz, probably fine)
I would like a real simulation of this from real lock acquisitions
- We need RC tracking of that mass
- We need to control scattered light - how big should the aperture be?

We should try for:

- Let's make the thing easy to control, and OK for the ISI
- Keep all 12 modes below $\sim$ 4 Hz and well damped
 - limit is vertical springs supporting the lower stage
- Keep the same 4 wire suspension to decouple DOFs





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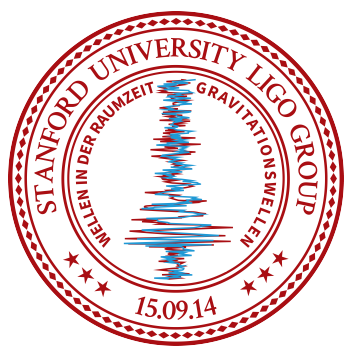
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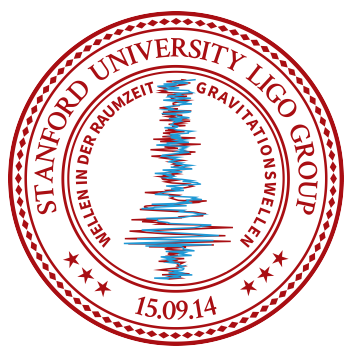
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- Keep the same 4 wire suspension to decouple DOFs



Mass of Reaction Mass?



- aLIGO AERM is 14 kg (Calum) = 35% of the test mass
 - Current ration seems to be fine, I suggest we keep it.
 - Yields a 35 kg reaction mass.
 - Given a hole size, it sets the thickness
-
- Lighter reaction mass = more RC motion from ISC drives
 - I don't know what the minimum mass is,
probably set by non-linearities in drive coupling

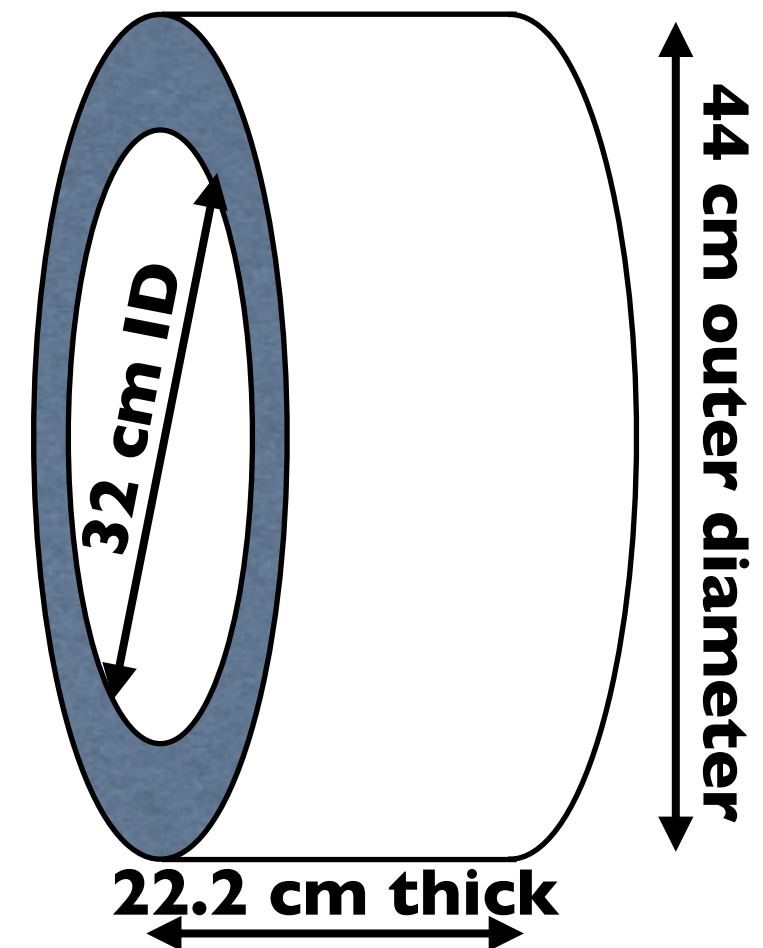


Size of Reaction Mass?

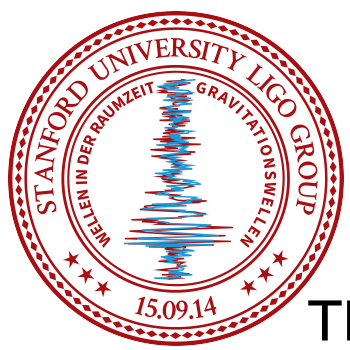
- 35 kg. Assume cylinder of fused silica
(fused silica is very convenient for the ESD)
- Set the OD to match the test mass (for cage dimension)
- What's the ID?
 - big hole reduces scattered light
 - small hole allows more ESD drive.
 - current ID is 22.25 cm.

Scatter is a problem now.

- 32 cm ID seems reasonable
- Do we have any data to guide this?



- 32 cm ID, 44 cm OD needs 22.2 cm thickness to make 35 kg.
- This gives front area 1.38 AERM, but Test Mass is 2.5x bigger
- ESD is only $1.38/2.5 = 55\%$ of current specific force



Coupling via ESD spring



TL;DR - scaling from aLIGO @ 400 V, ESD requires reaction mass motion < **8e-13** m/rtHz
With a 32 cm ID clear aperture and 44 cm OD, we can only scale the ESD by ~ 1.38

From aLIGO:

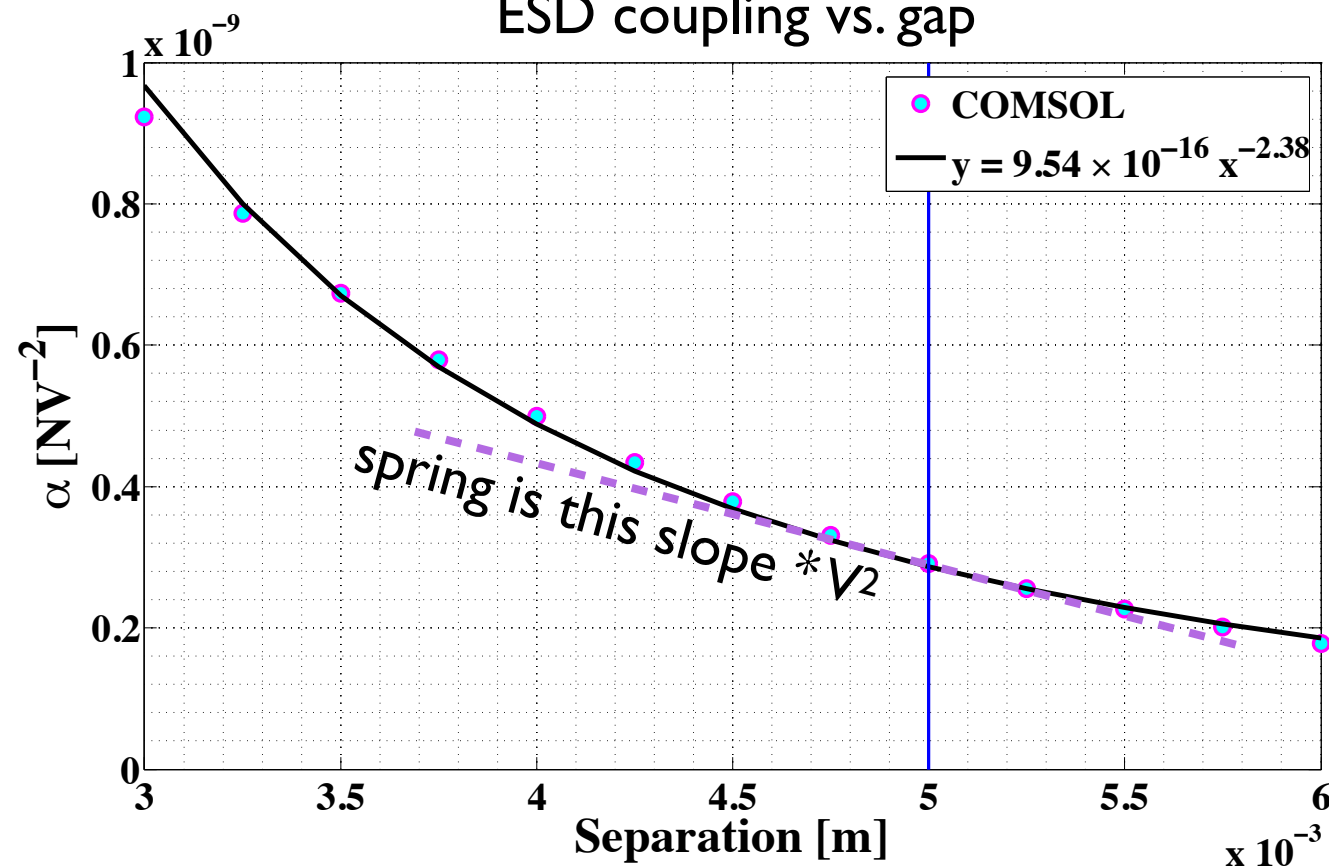
Effective spring rate of the ESD is proportional to the slope

$$da/dx = -2.38 * 9.54 * 10^{-16} * x^{-3.38} \text{ N/(m*V}^2\text{)}$$

at 5 mm, 400 V, (LLO only uses 100 V now)

$$k = dF/dx = -0.022 \text{ N/m (400 V) or } -0.0014 \text{ N/m (100 V).}$$

ESD coupling vs. gap



Scale to A# by guessing similar geometry and gap,

~~Use 2.5 more authority for the 100 kg mass.~~

Use 1.4 more authority for the 100 kg mass.

(based on available area: ID: 32 cm, OD: 44 cm)

Gives a conservative estimate of

$$k = -0.03 \text{ N/m for A\# ESD at 400 Volts}$$

- likely 30x smaller, $(400/73)^2$

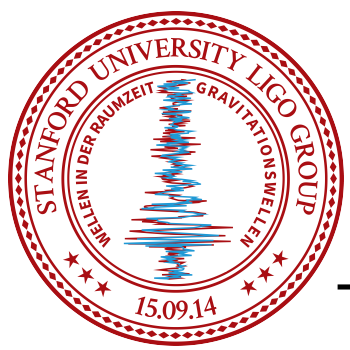
$$m * \omega^2 * X_{TM} = k * X_{RC}$$

at 10 Hz, 100 kg, k of 0.030, we get

$$X_{RC} = \mathbf{8e-13} \text{ m/rtHz}$$

(good ISIs are about 5e-13 m/rtHz at 10 Hz)

Figure 5.17: The ESD coupling coefficient α as a function of separation between the reaction mass and the test mass. Vertical line shows nominal separation corresponding to $2.9 \times 10^{-10} \text{ NV}^{-2}$. Least squares fitting reveals that dependence is stronger than quadratic.



Authority of ESD



TL;DR - Drive Noise calculation says this is OK for drive authority - IN Low Noise:
 $\alpha_{\text{LIGO ESD}} * 1.38 \text{ area} * 74 \text{ V bias} = 2.9\text{e-}8 \text{ N/V}$ (max coef for drive noise, G2401160)

From aLIGO: at 5 mm

$\alpha * \text{scaled Area} * \text{bias voltage} = 2.86\text{e-}10 \text{ N/V}^2 * 1.38 \text{ (area ratio)} * V_{\text{bias}} = 2.9\text{e-}8 \text{ N/V}$

$V_{\text{bias}} = 73.5 \text{ V}$

drive coupling = $2.9\text{e-}8 \text{ N/V}$. This is the max coupling from drive distribution calc.
 Set by noise in final stage driver electronics

ESD coupling vs. gap

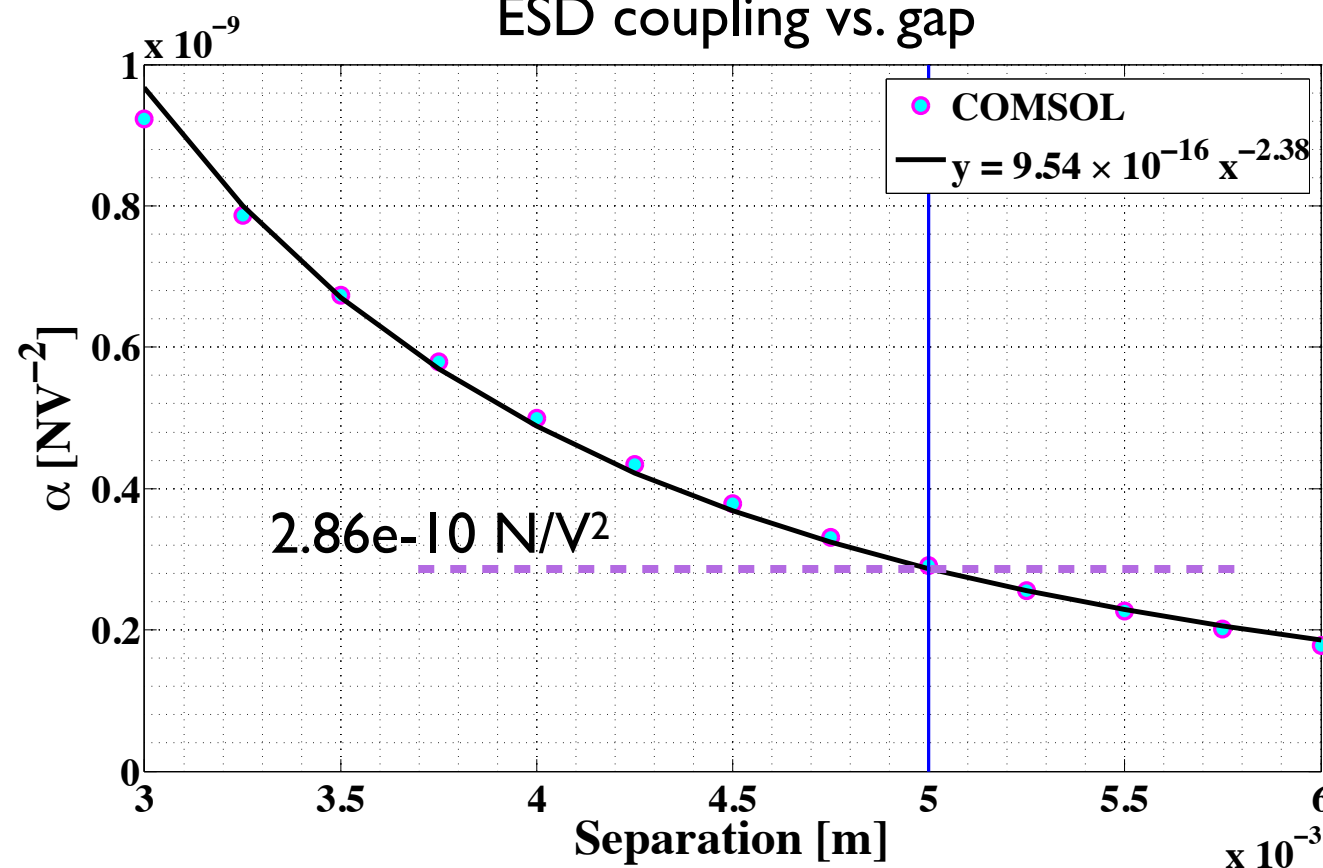
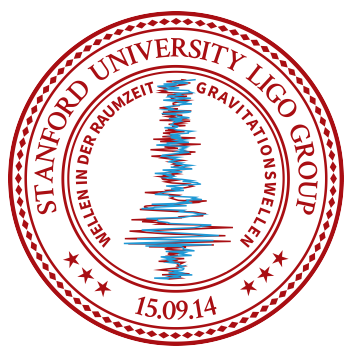


Figure 5.17: The ESD coupling coefficient α as a function of separation between the reaction mass and the test mass. Vertical line shows nominal separation corresponding to $2.9 \times 10^{-10} \text{ NV}^{-2}$. Least squares fitting reveals that dependence is stronger than quadratic.



Pitch & Yaw noise calc



TL;DR - scaling from aLIGO @ 400 V,

ESD requires reaction mass angle $< 1.9\text{e-11 rad/rtHz}$

(BSC-ISI yaw at 10 Hz ~ sensor noise limited ~ 2e-13 rad/rtHz ,

From ESD Length spring calculation :

use $k = 0.03 \text{ N/m}$

angular spring $= 4 * ((k/4) * \text{ESD_mean_rad}/\text{sqrt}(2)) = 0.03 \text{ N/m} * 19 \text{ cm}/\text{sqrt}(2) = 4\text{e-3 N/rad}$

$\text{angle_req_main_chain} * \text{moment} * \omega^2 = K_{\text{angular_spring}} * \text{angle_spec_for_reaction_chain}$

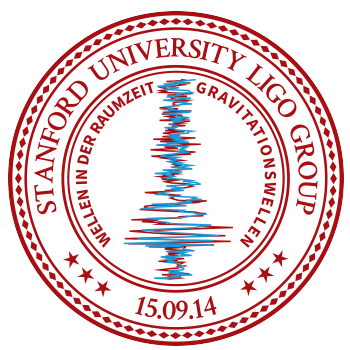
$\text{angle_spec_for_reaction_chain} = 1\text{e-17 rad/rtHz} * 1.9 \text{ mg} * \text{m}^2 * (2 \pi 10)^2 / 4\text{e03 N/rad}$

$\text{angle_spec_for_reaction_chain} = 1.9\text{e-11 rad/rtHz}$ - true for pitch and yaw.

No isolation is needed to meet the angular noise requirement.

The ISI angular motion is significantly below the reaction chain requirement.

What else are we looking for?



Vertical noise calc

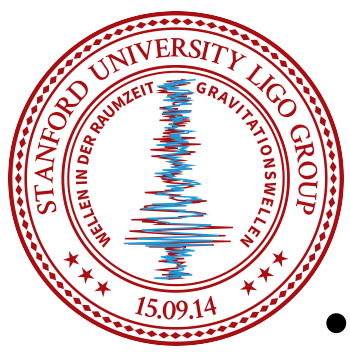


TL;DR - with scaling of $1e-3$ from horizontal
requires reaction mass vertical of $< 8e-10$ m/rtHz
(this is like the ground motion)

No isolation is needed to meet the vertical noise requirement from ESD spring coupling.

The ISI vertical motion is significantly below the reaction chain requirement.

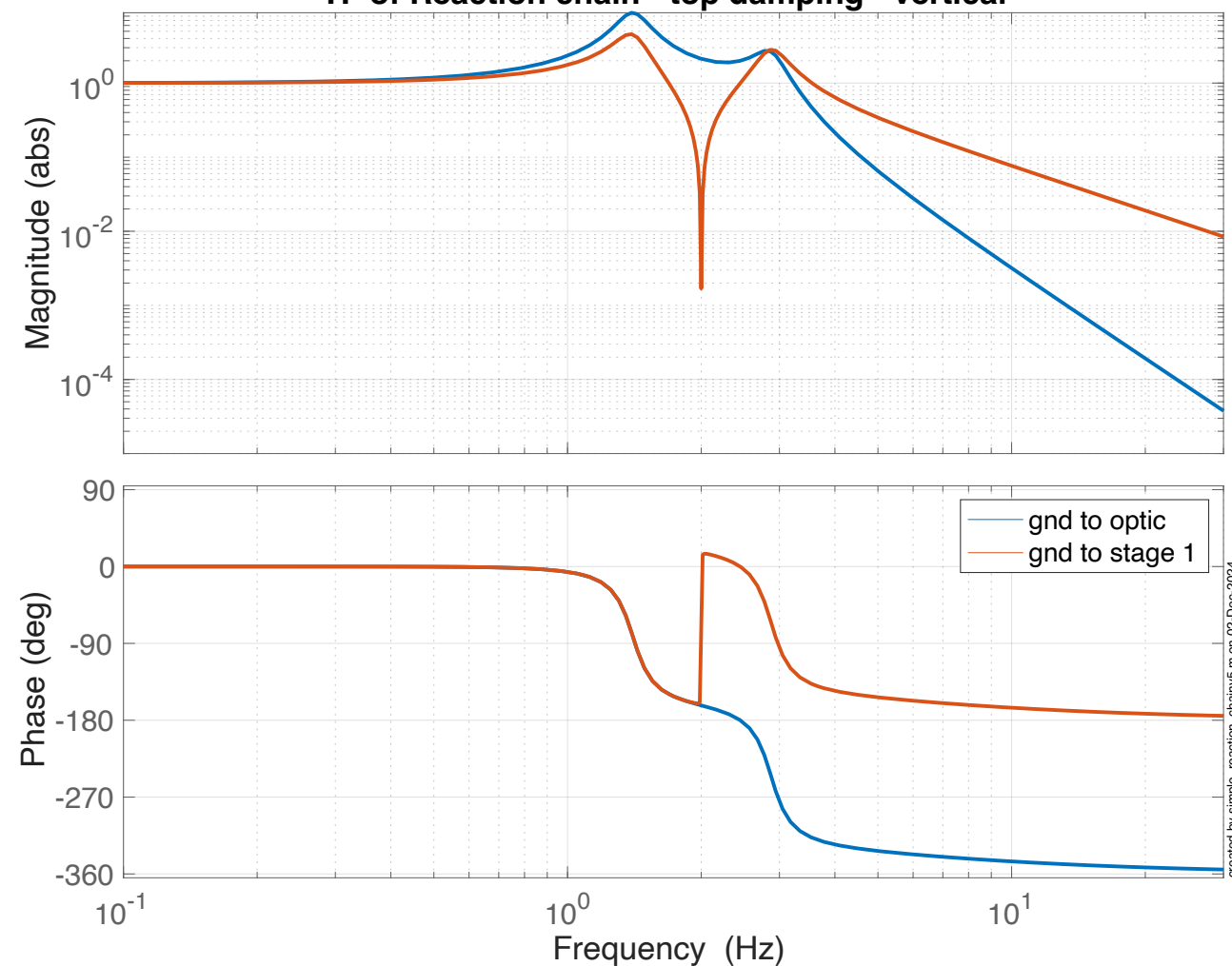
Fringing fields of the ESD will likely cause cross coupling between vertical motion and pitch force (transverse to yaw)



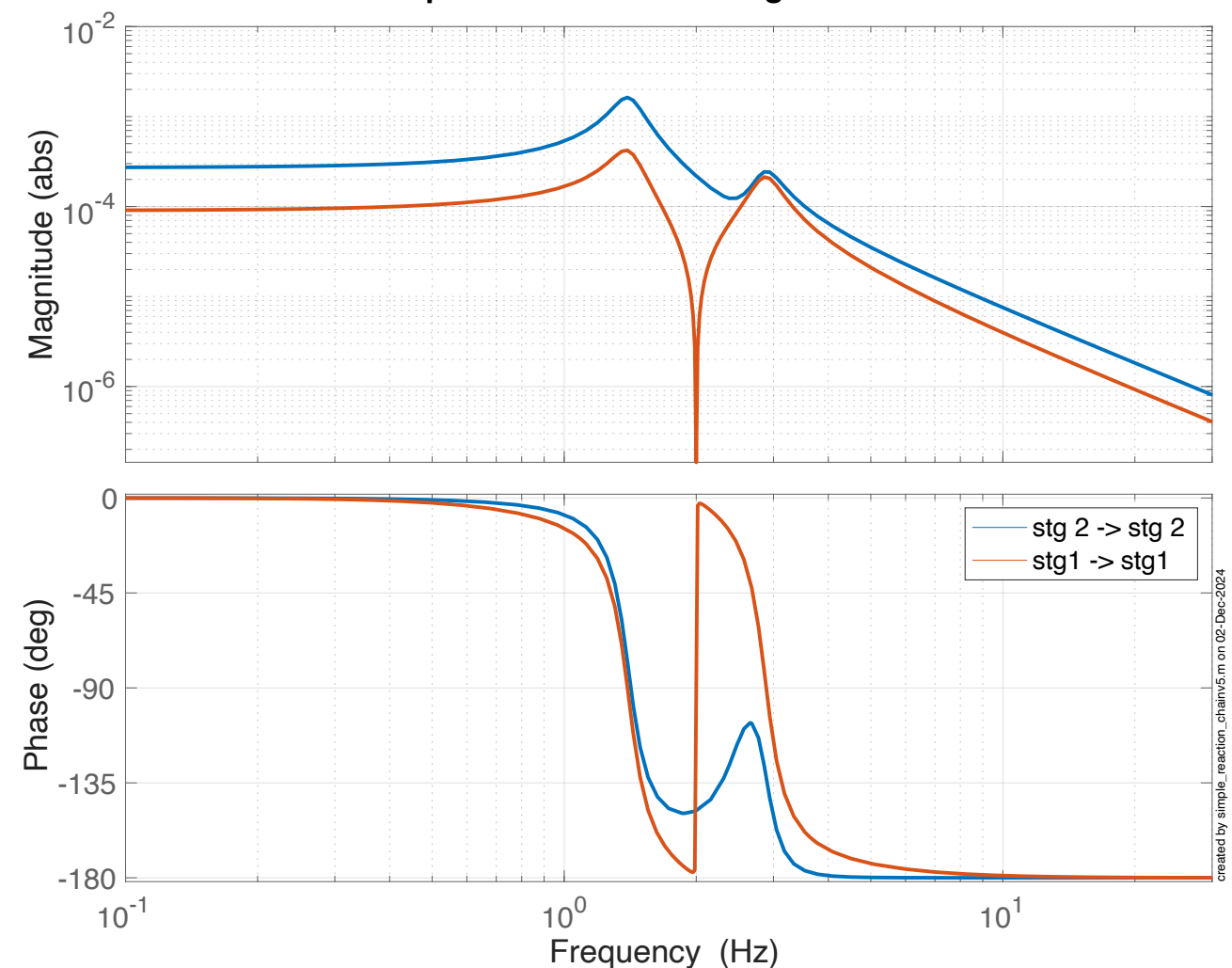
Vertical Plant

- set uncoupled vertical modes at 2 Hz, set to allow small springs.
- spring at 50% yield .The “active triangle” size is:
 - lower stage springs: 10.8 cm long, 1.0 mm thick
 - upper stage springs: 17.4 cm long, 1.8 mm thick
- real springs need base & tip attachment space
- coupled frequencies are 1.4 and 2.8 Hz (well below UGF of ISI)
- TFs and compliance are well behaved

TF of Reaction chain - top damping - vertical



Compliance of reaction stages - Vertical



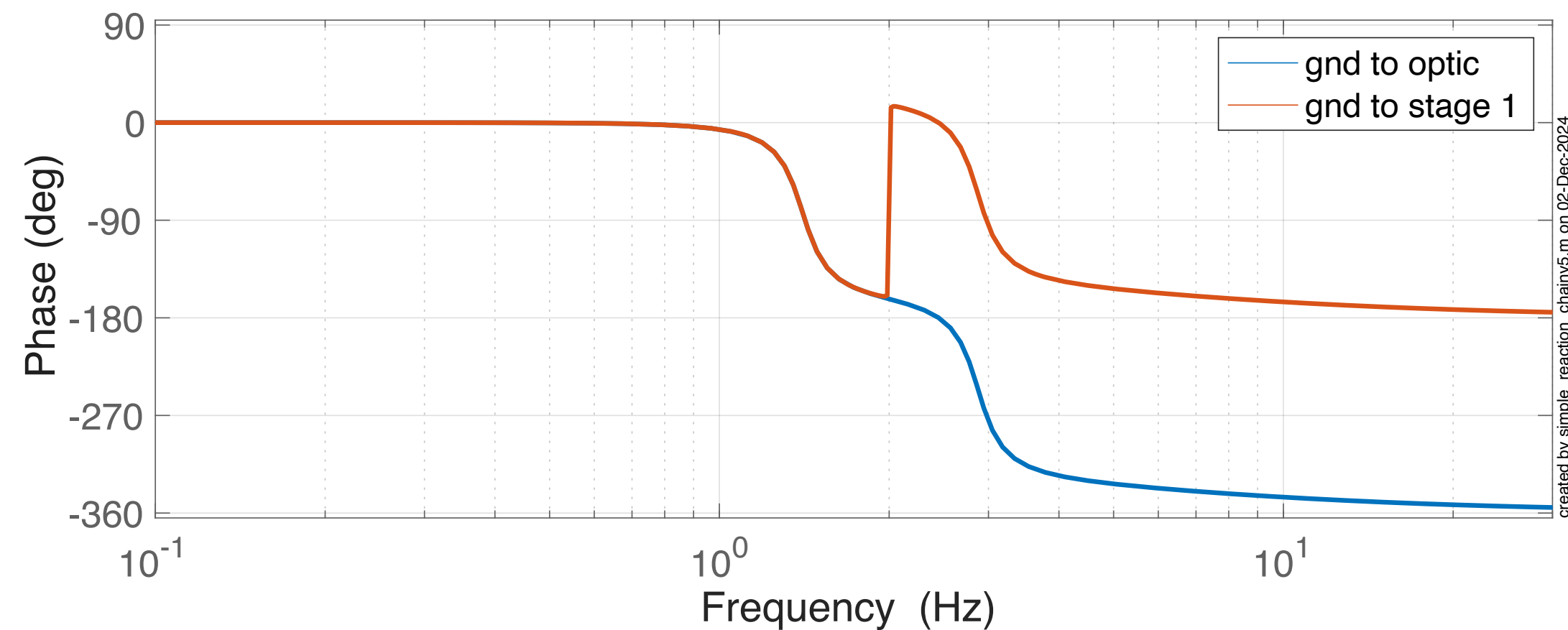
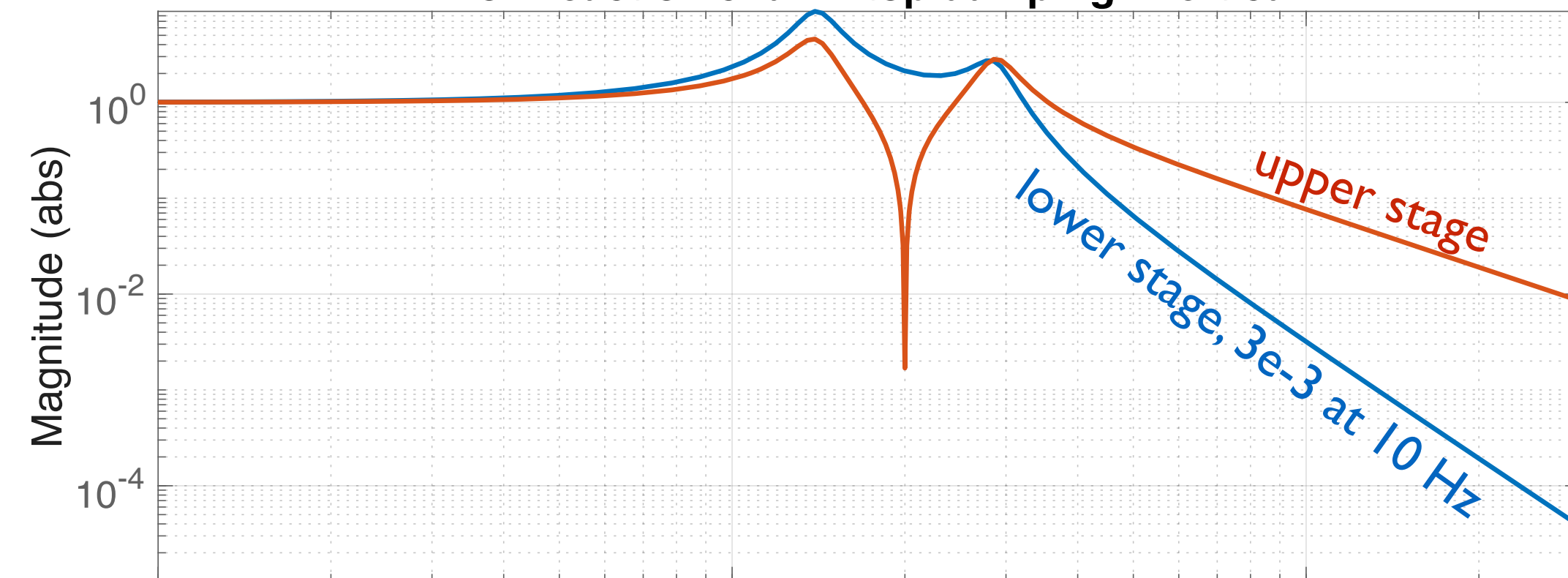


Vertical Plant

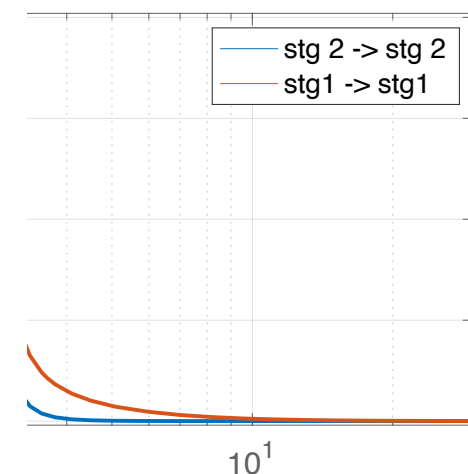
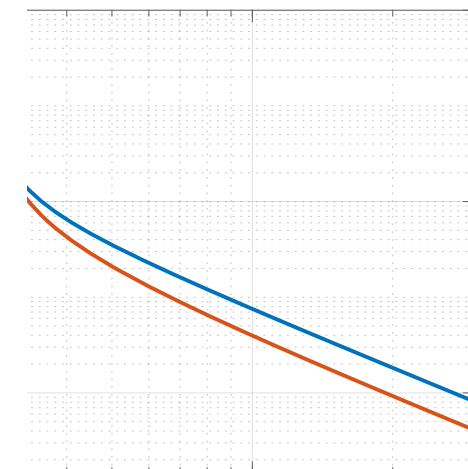


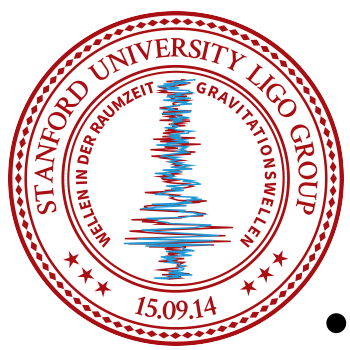
prings.

TF of Reaction chain - top damping - vertical



as - Vertical

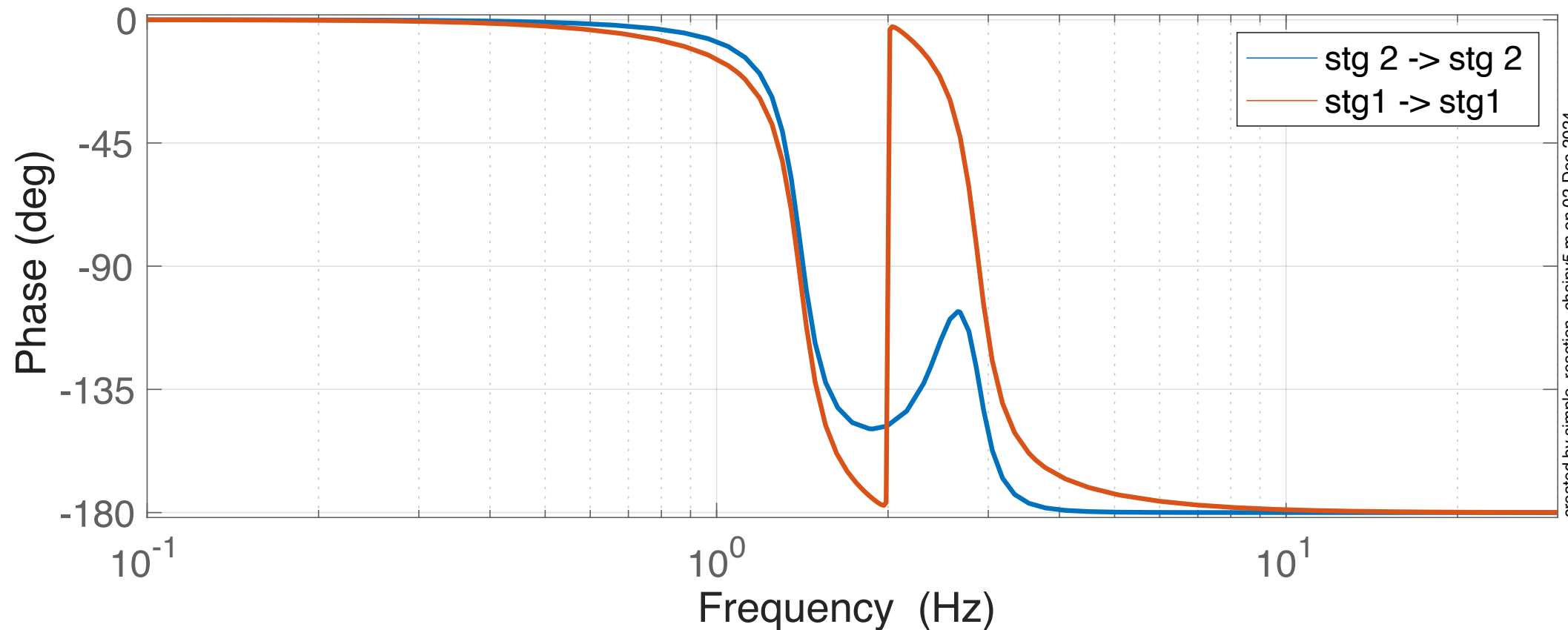
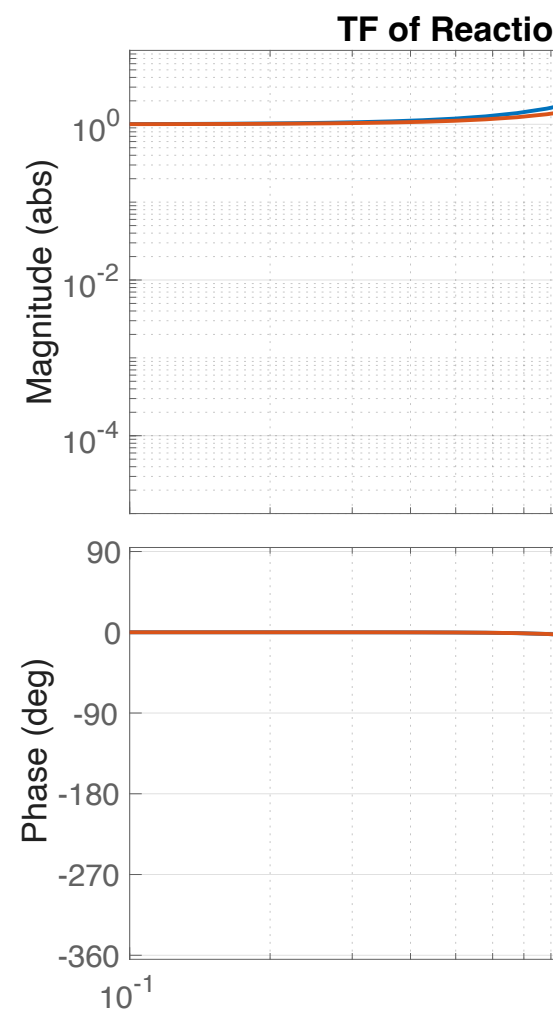
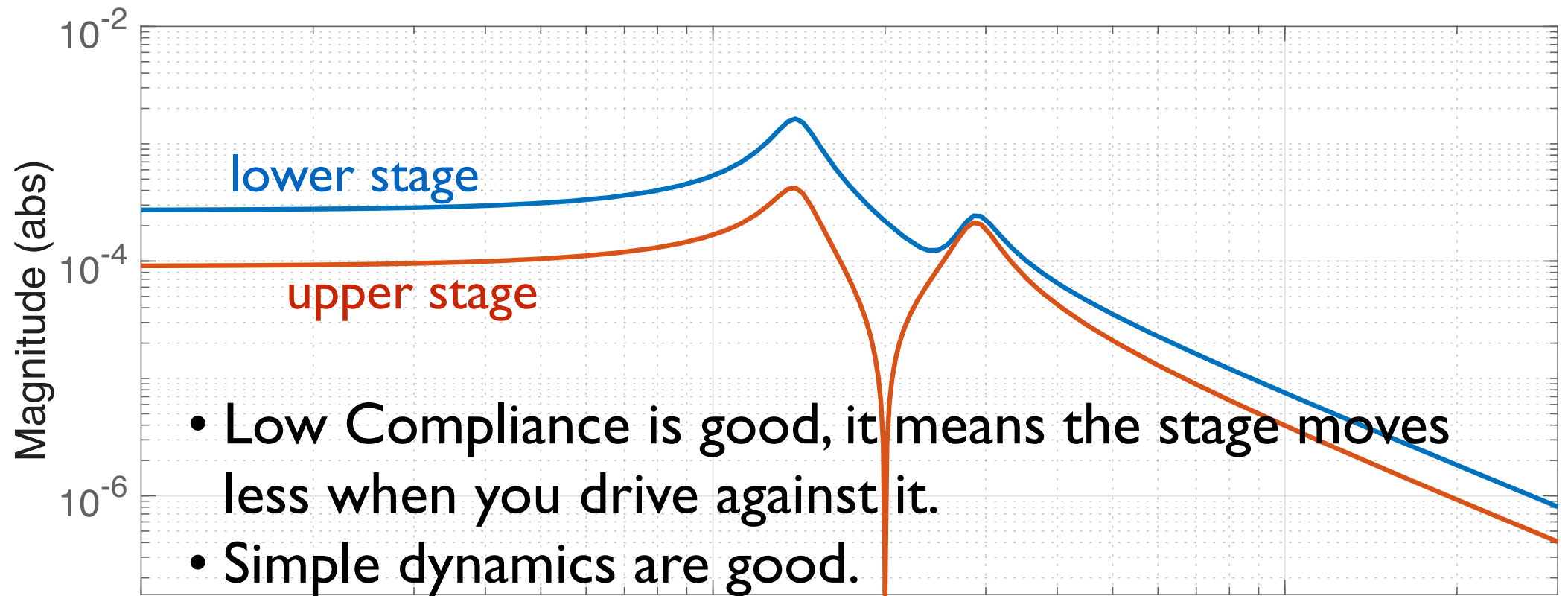


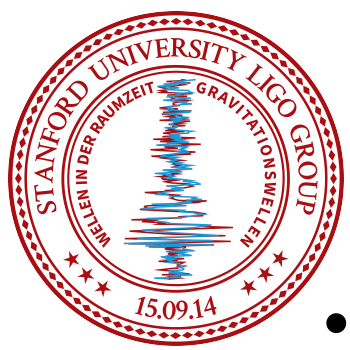


Vertical Plant

Compliance of reaction stages - Vertical

- set up
- spring
 - low
 - upper
- real system
- coupled
- TFs at

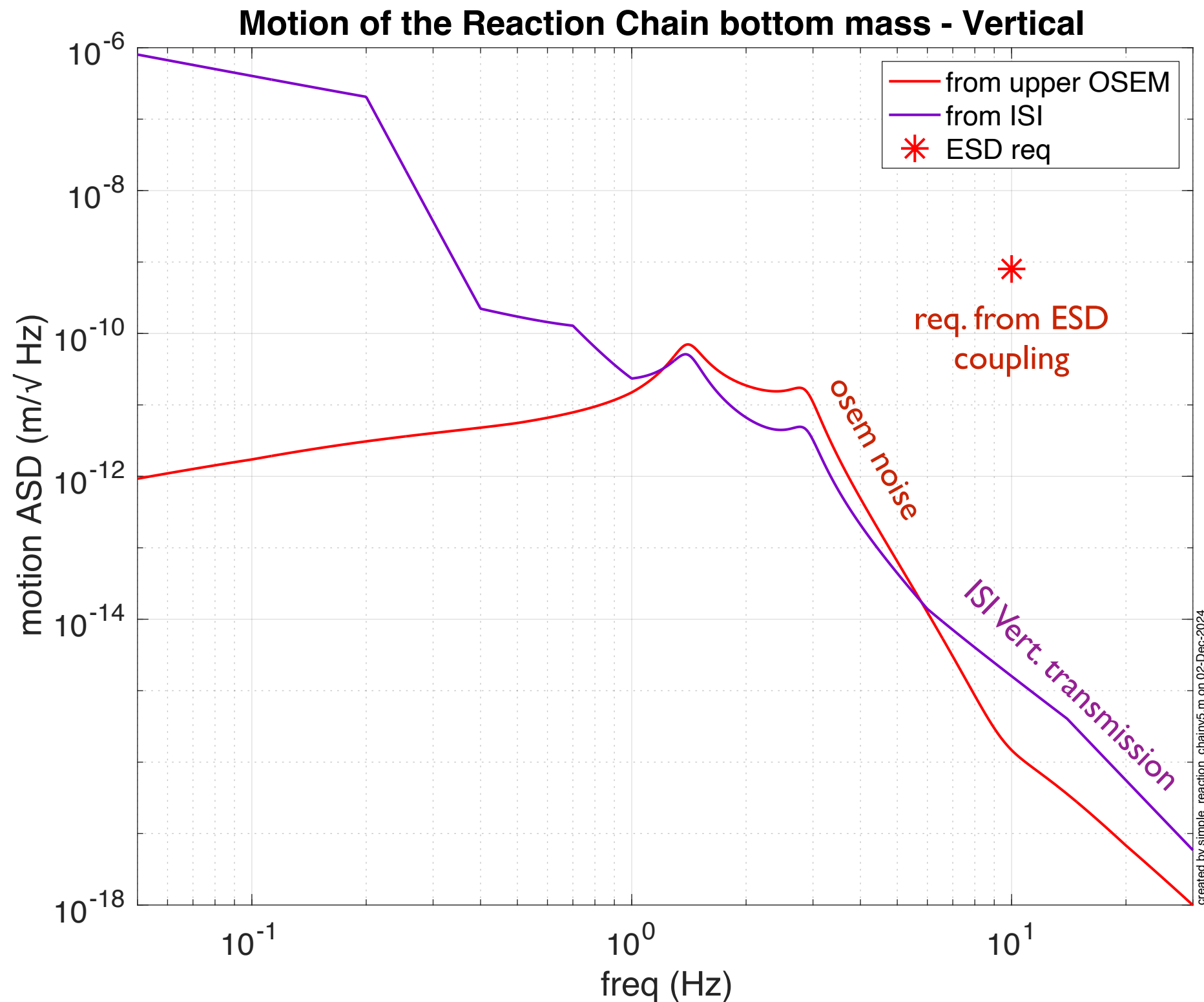


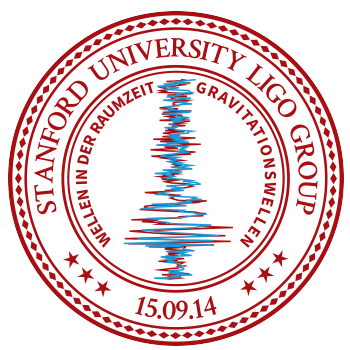


Vertical Isolation



- Isolation Performance is significantly better than you need

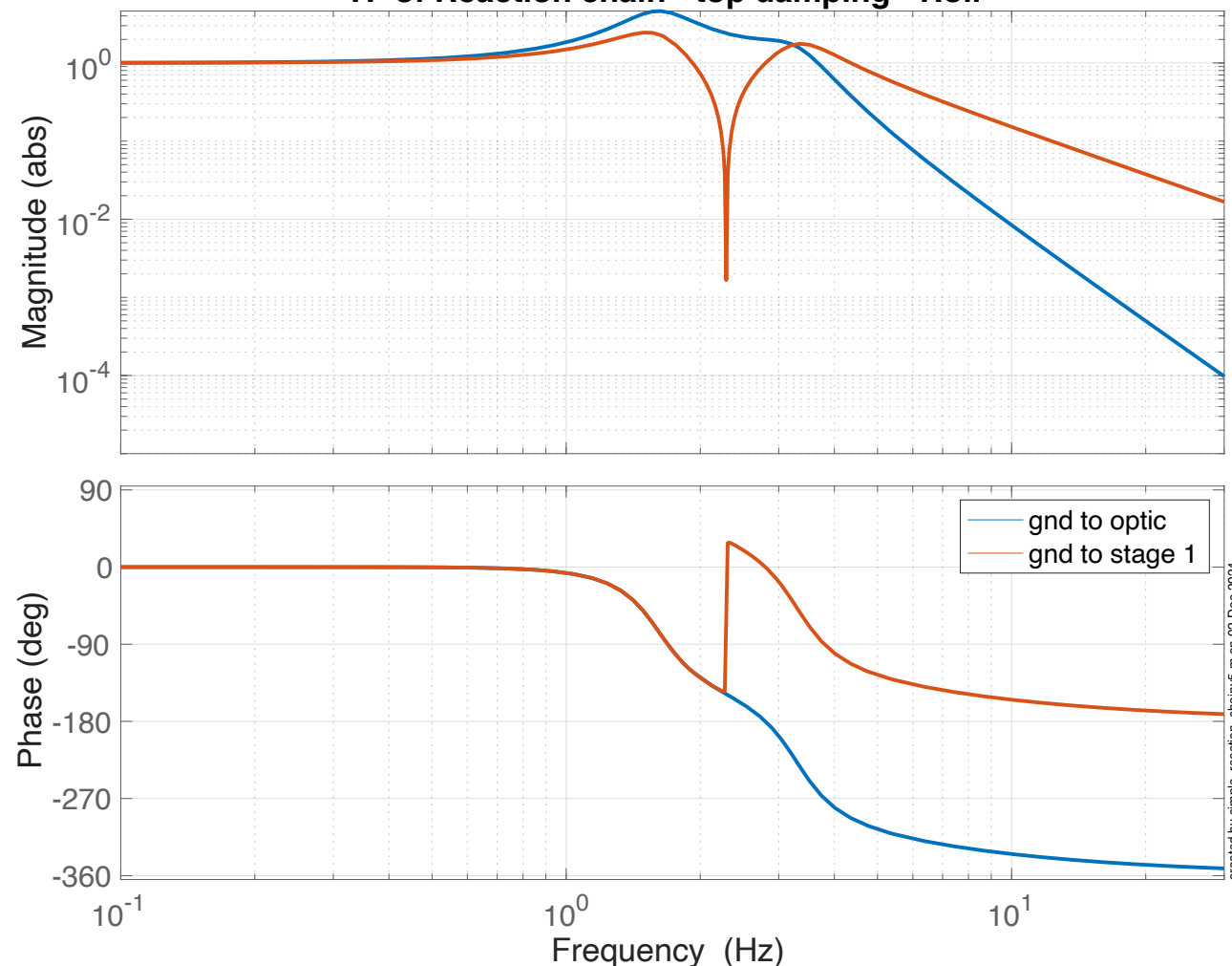




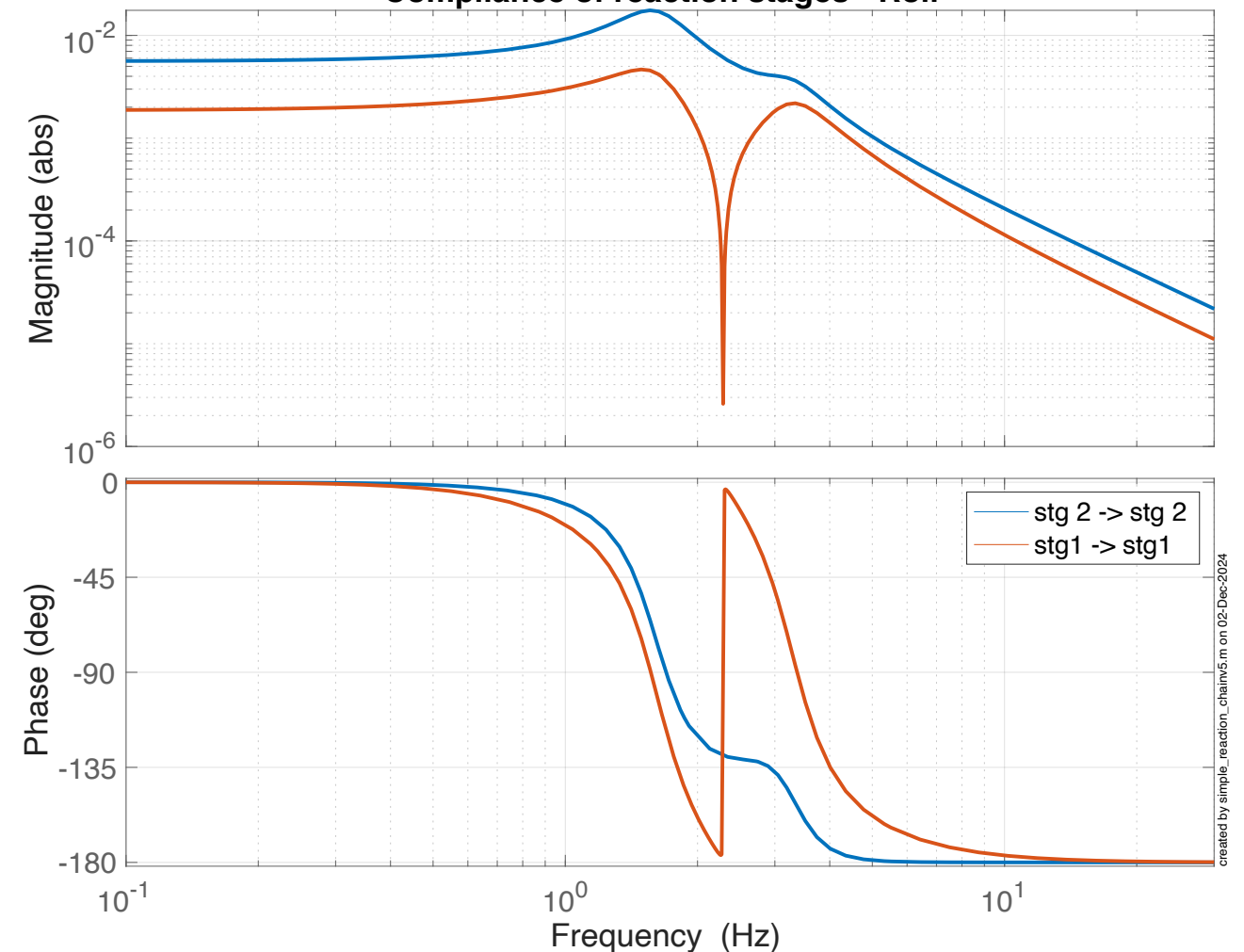
Roll Plant

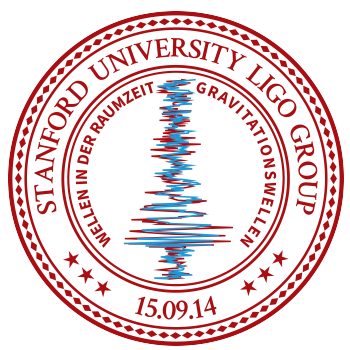
- Determined by vertical springs, wire spacing, roll moments
- Lower stage: wire spacing set by OD of lower mass
 - Moment set by geometry of lower mass
- Upper stage: wire spacing set to match lower mass (arbitrary)
 - Moment set to be 2x lower moment (arbitrary)
- coupled frequencies are 1.6 and **3.2 Hz**
(3.2 is highest mode - still below 10 Hz, and UGF of ISI)

TF of Reaction chain - top damping - Roll



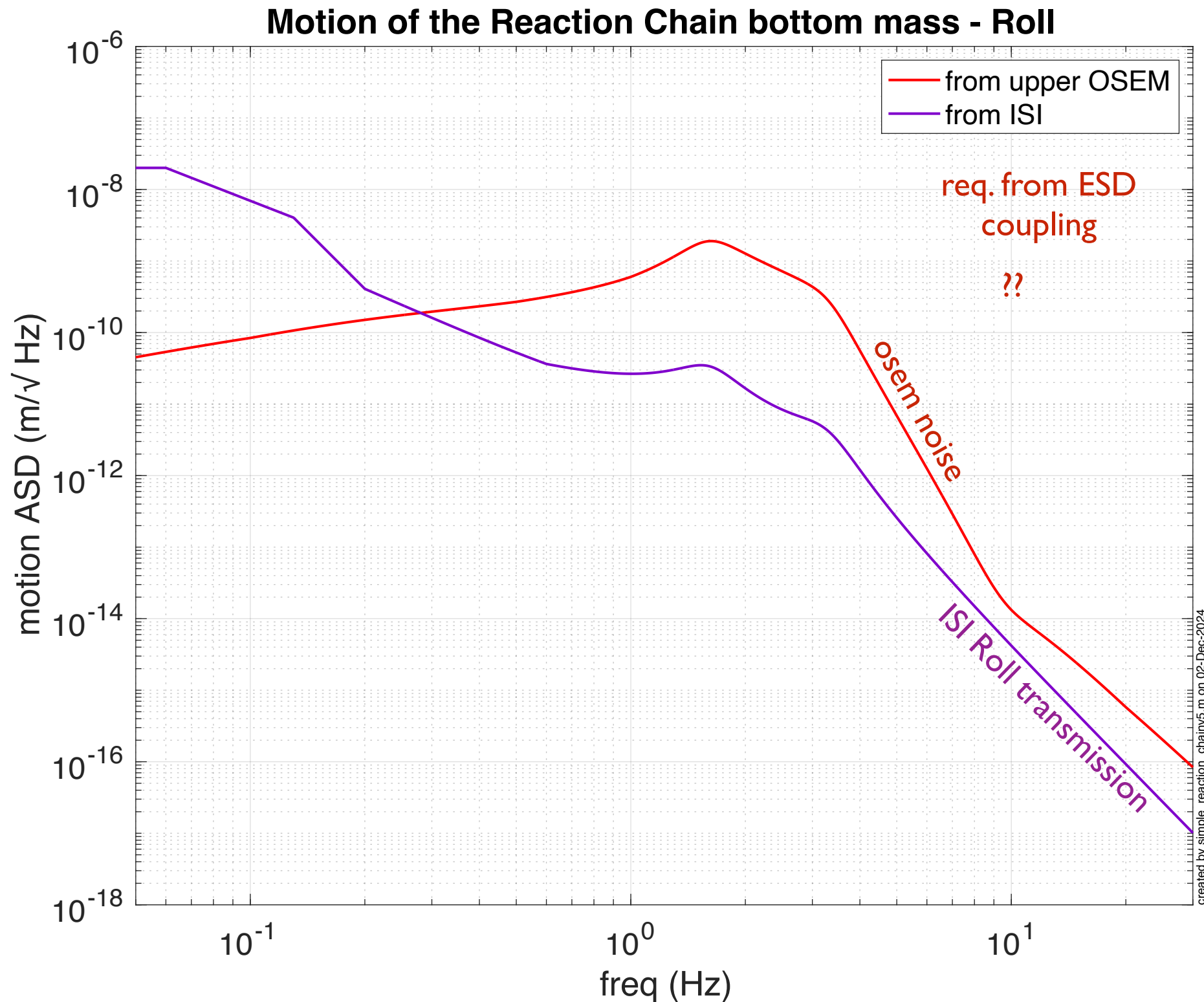
Compliance of reaction stages - Roll

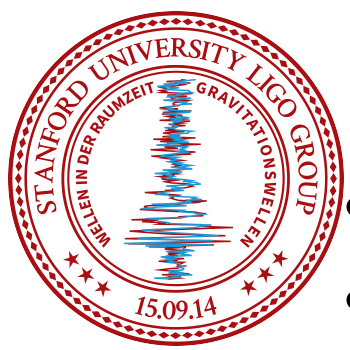




Roll Isolation

- Is there a requirement for roll isolation?



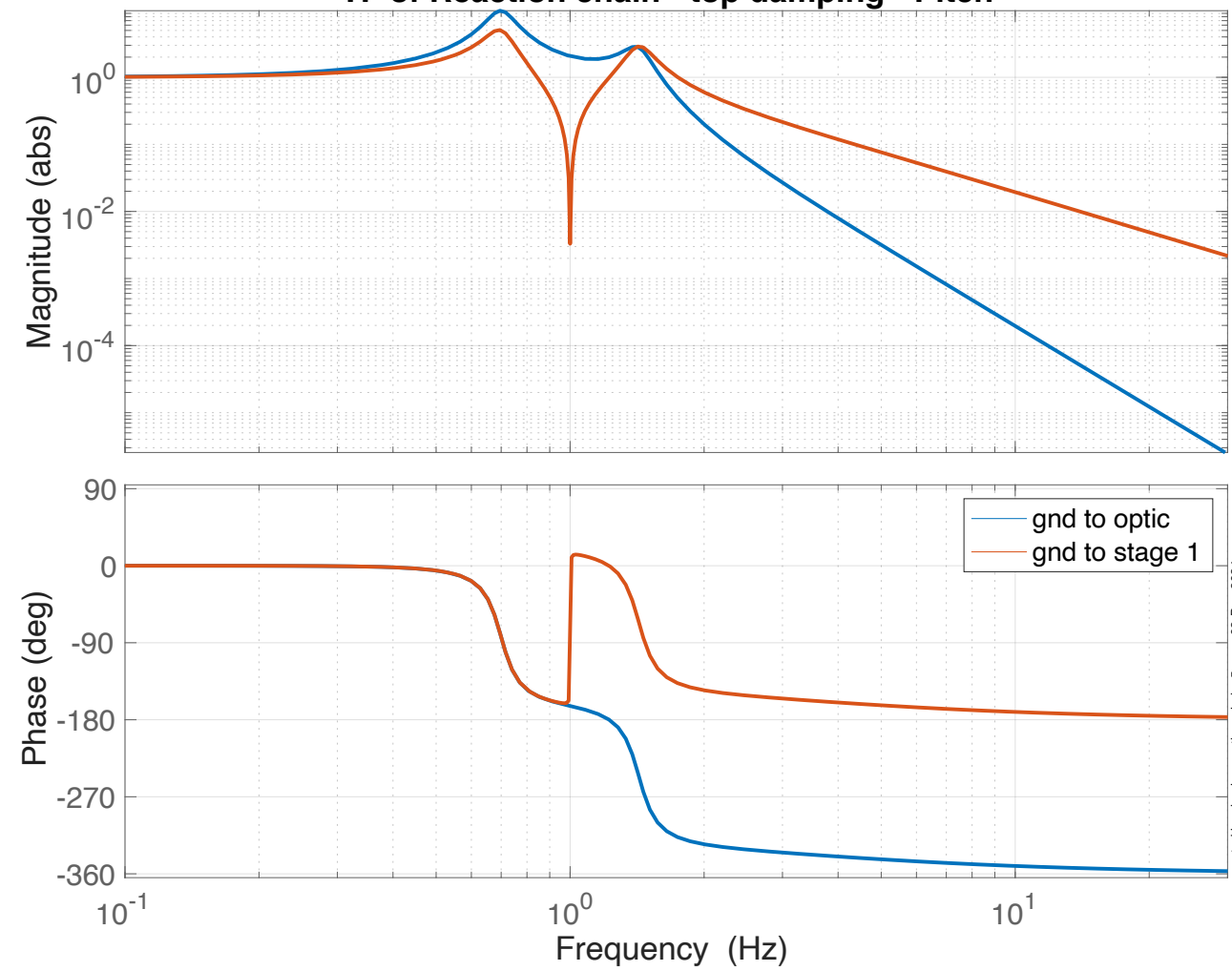


Pitch Plant

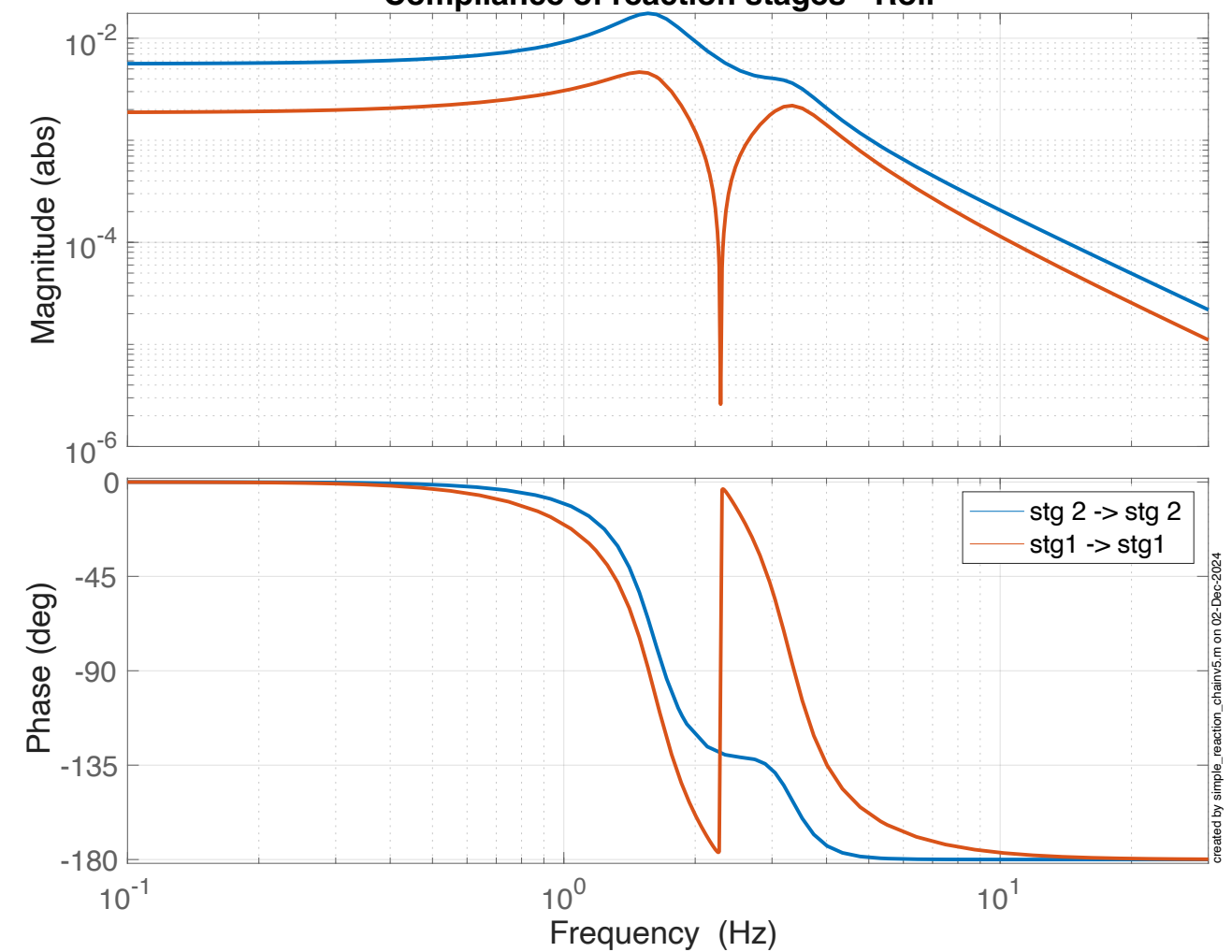


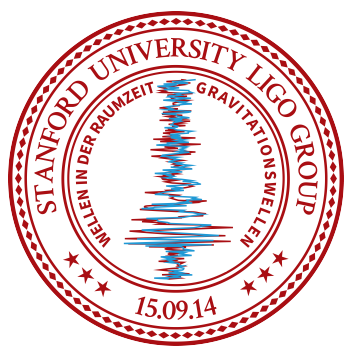
- Determined by vertical springs, wire spacing, pitch moments
- Lower stage: pick 1 Hz uncoupled freq's
 - wire spacing becomes 15 cm blade-to-blade
 - limited by width of blades (5.4 cm each) and upper stage
 - Moment set by geometry of lower mass
- Upper stage: pick 1 Hz uncoupled freq.
 - sets wire spacing (happens to be same as lower separation)
 - Moment set to be 2x lower moment (arbitrary)
- coupled frequencies are 0.7 and 1.4 Hz (good)

TF of Reaction chain - top damping - Pitch



Compliance of reaction stages - Roll

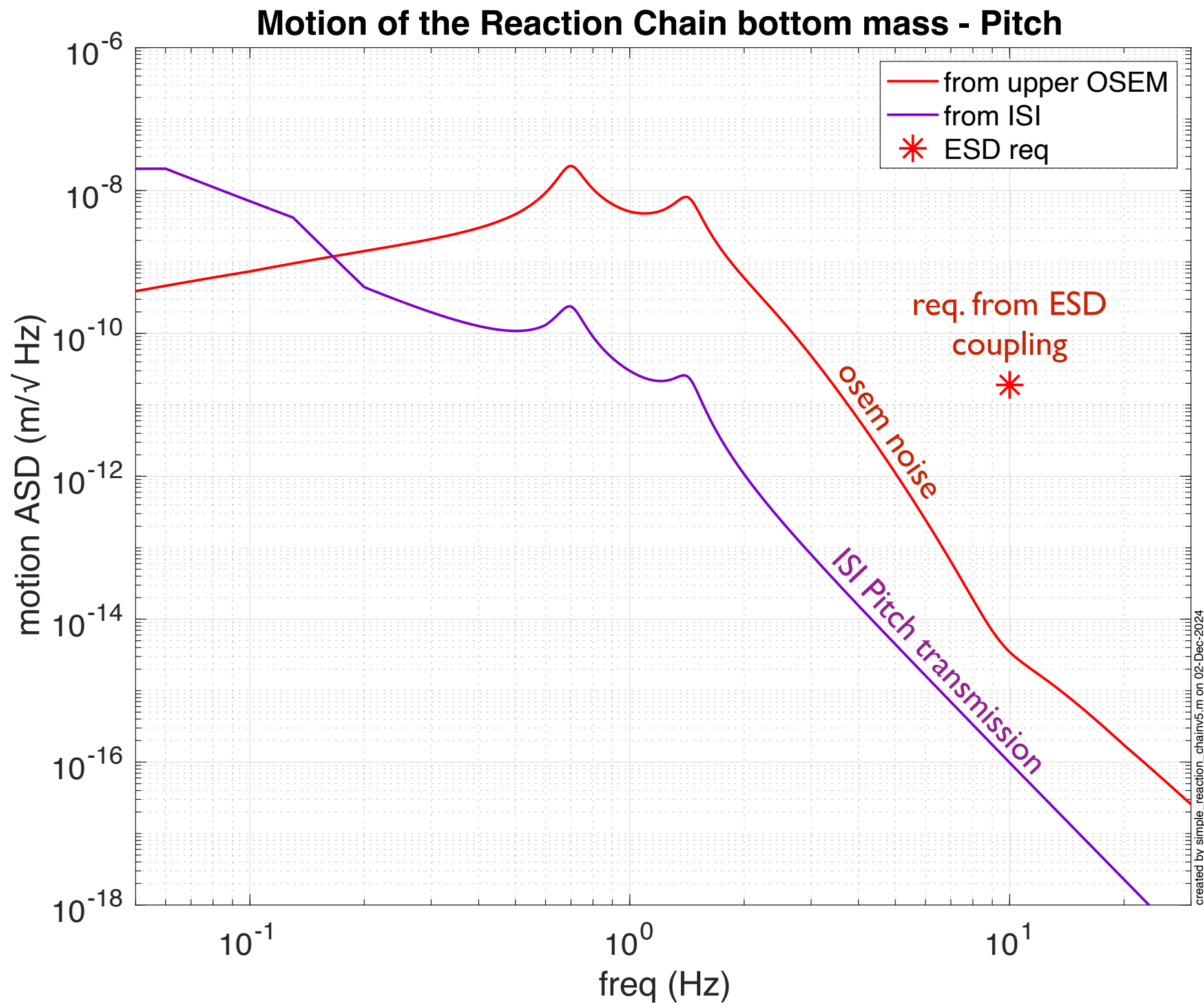


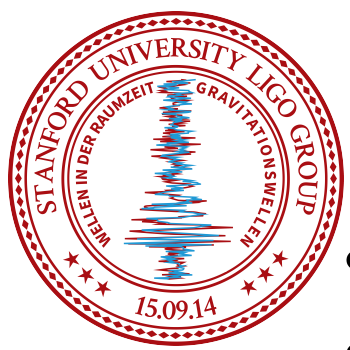


Pitch Isolation



- Requirement set by coupling of ESD springs in angle
- motion of test mass should be less than $1e-17$ rad/rtHz

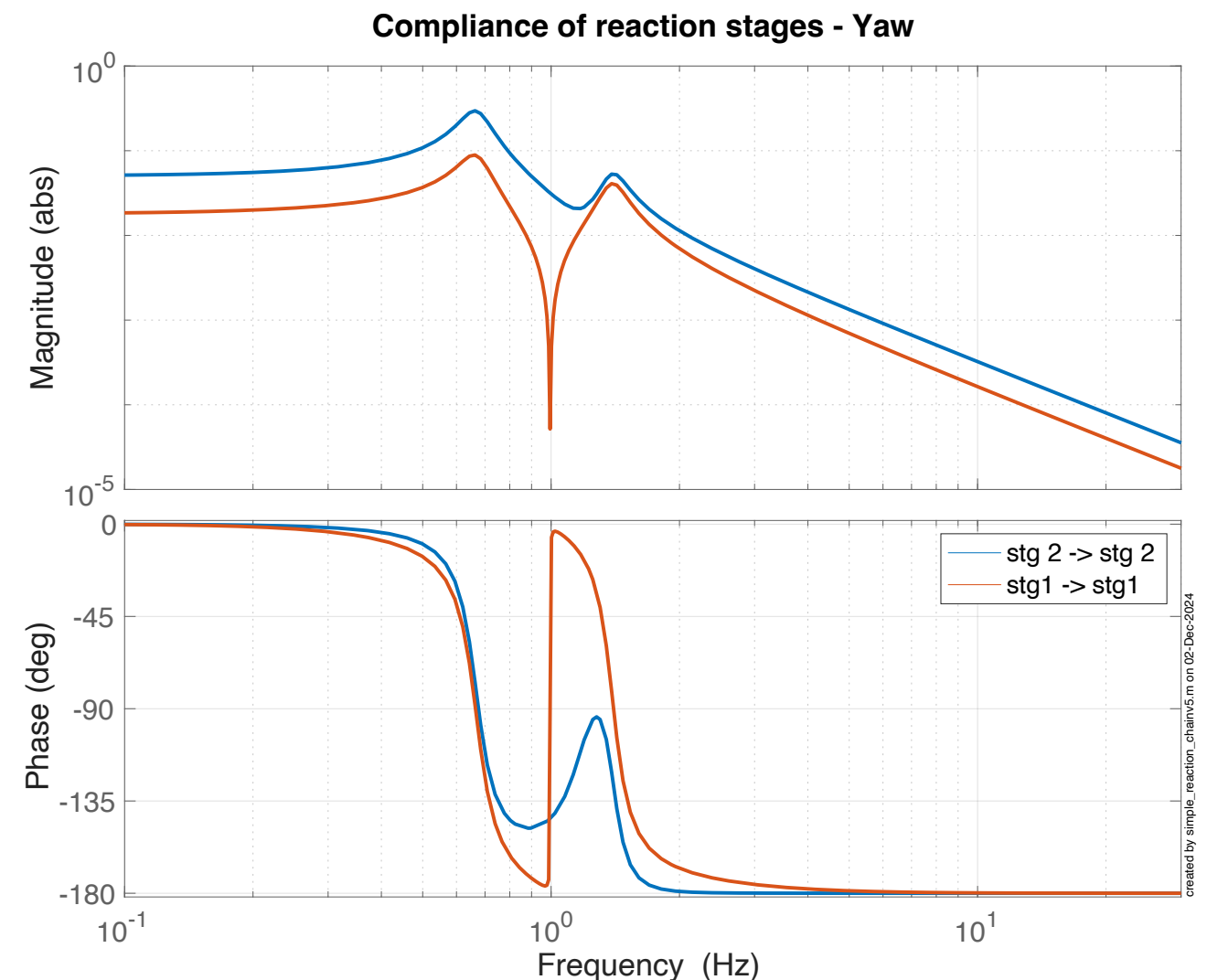
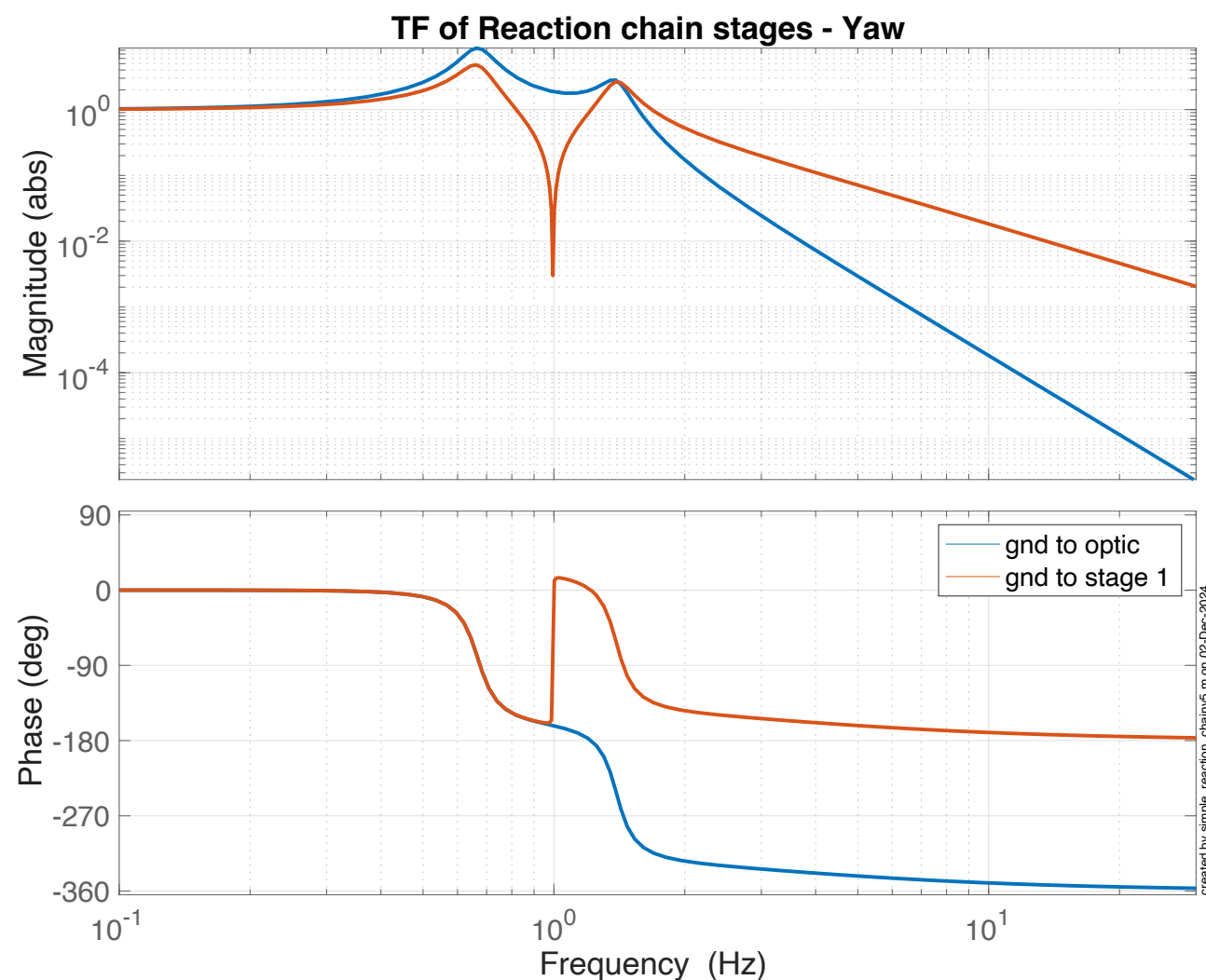


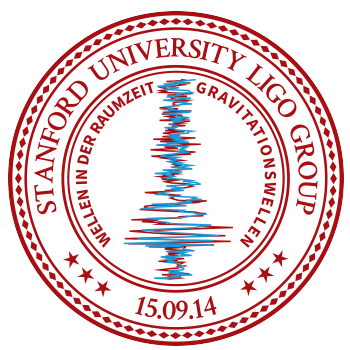


Yaw Plant



- Pre-determined by pitch/roll wire spacing, yaw moments
- Lower stage
 - Moment set by geometry of lower mass
- Upper stage
 - Moment set to be 2x lower moment (arbitrary)
- coupled frequencies are 0.68 and 1.38 Hz (good)

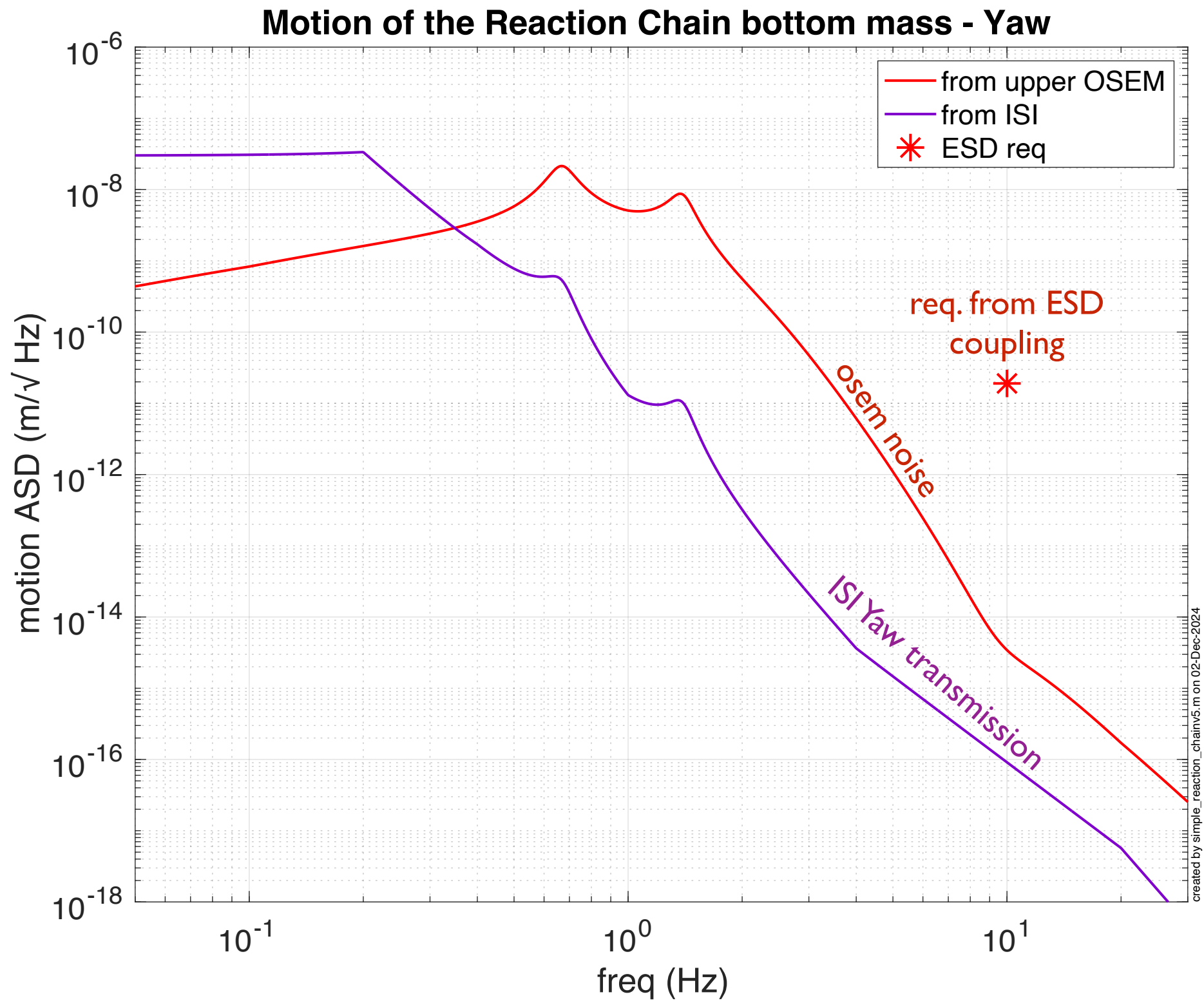


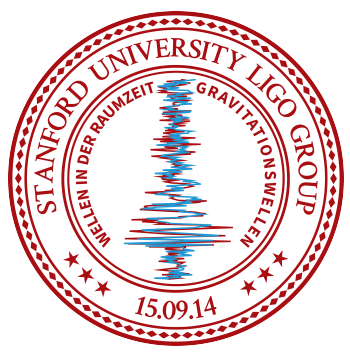


Yaw Isolation



- Same as Pitch requirement

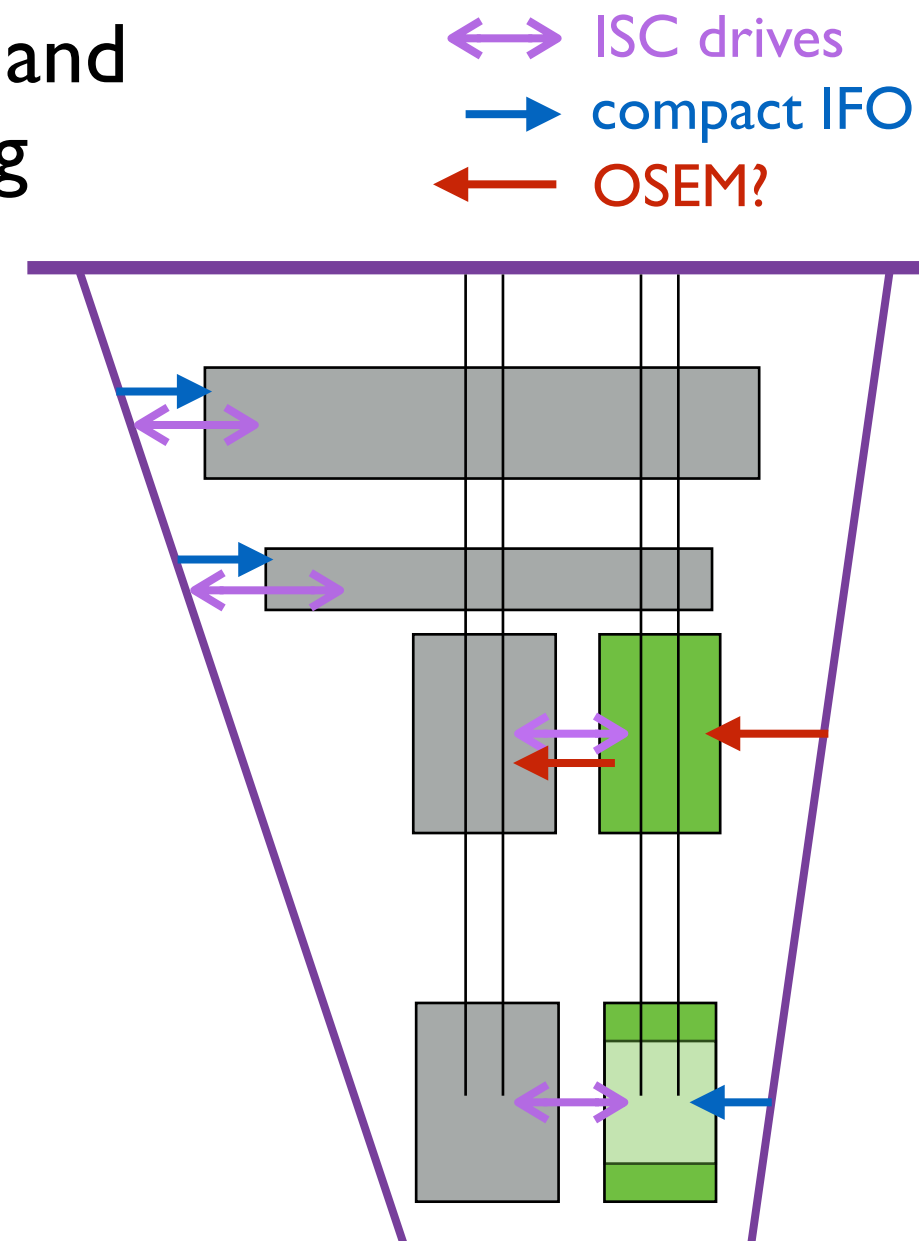


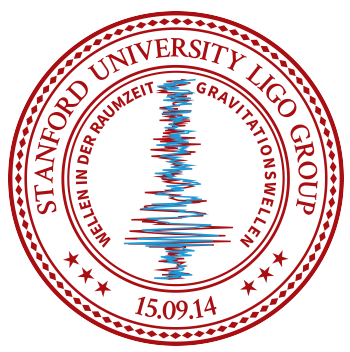


Design questions



- With a double pendulum, all the noise req's are met for ESD
- All 12 modes are below 4 Hz and well damped
 - Good for control of SUS, good for control of ISI
- OSEM is fine for noise, but
the compact IFO sensors are better calibrated and more linear, which may help with cross-coupling
- Is 32 cm the right size for the hole?
- Do we need some lock acquisition simulations to look for drives on the ISI & peak forces on the ESD?
- Where do we do R0 tracking?
main UIM vs. RC upper stage?
- I think we have enough info to start sketching
-

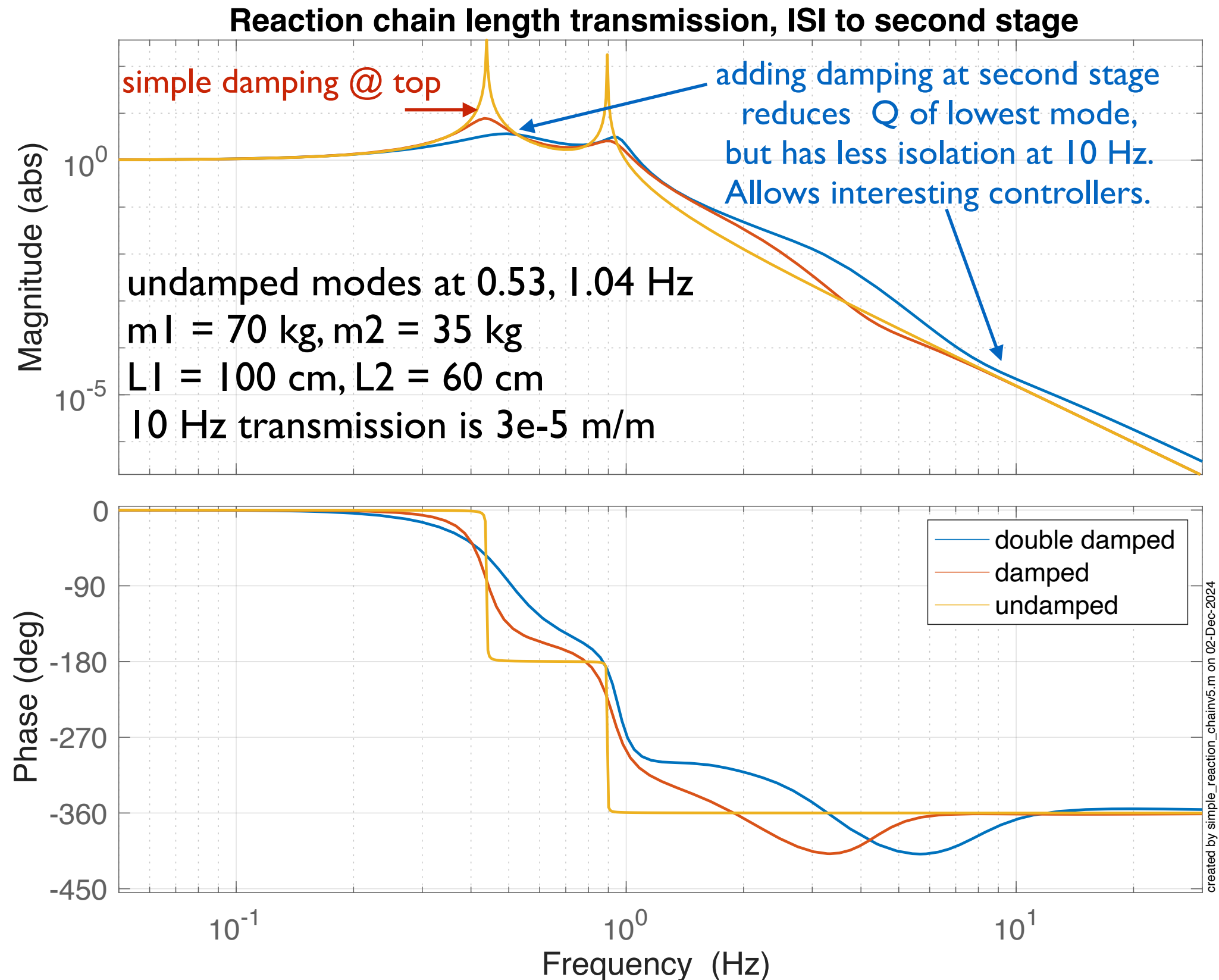


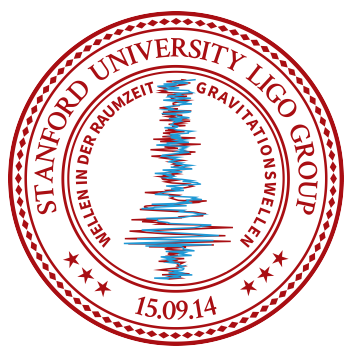


older slides...



Double RC transmission





Reaction Chain



Thinking about the reaction chain requirements,

Because - If the TOP and UIM are wide,

then the **aLIGO reaction chain** doesn't work.

- What are the requirements for an updated reaction chain?
- Can we use a double?

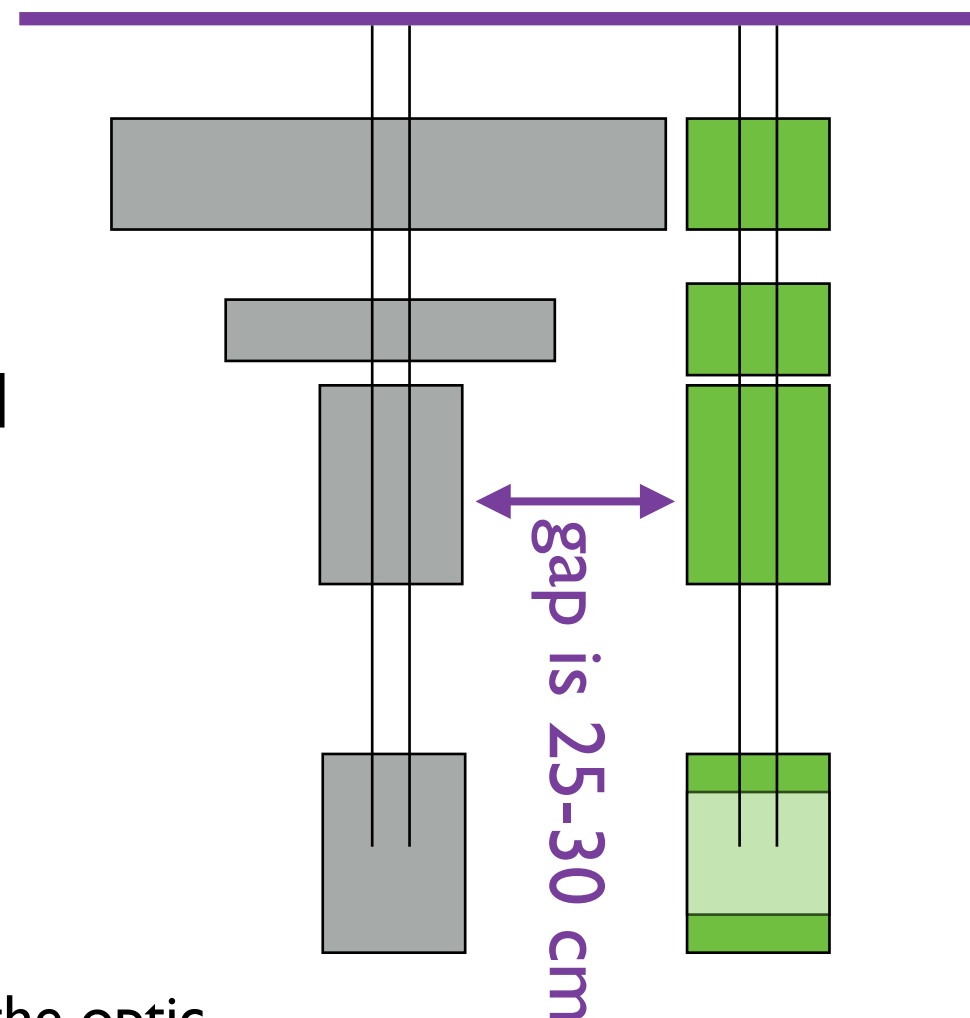
These are roughly drawn from

Stanford's A# calculations.

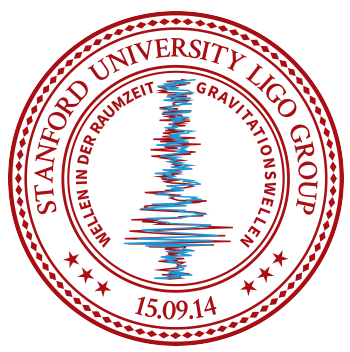
100 kg mirror is 27 cm thick, and

the TOP mass is 70-80 cm wide.

Vertical spacing is 34, 34, 34, and
60 cm.



$$(70 \text{ cm} - 27 \text{ cm}) / 2 + 5 \text{ cm top gap} = 26.5 \text{ cm at the optic}$$

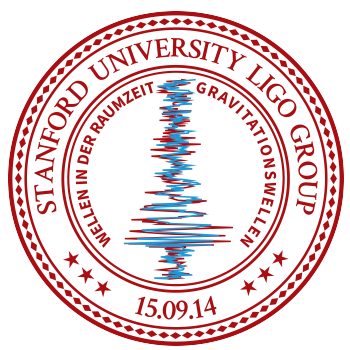


Requirements?



What are the requirements for the reaction chain for the Quad?

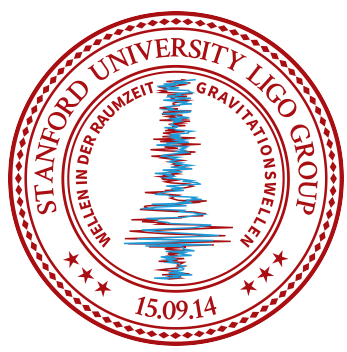
1. Quiet place for ISC actuators to push against
See this doc for length calc's. Angle is TBD.
2. Dynamics OK for ISC reaction (see 3.3 Hz issues)
3. Isolates the ISI from ISC kicks (see Beamsplitter)
4. Low relative velocity to manage scattered light (see R0 tracking)
5. Less than 200 kg and fits into space
(see the back sides of all the aLIGO Quad masses are in-plane)
6. What about suspending the compensation plate?
7. **Q: What else?**



How about a double?



1. Quiet place: Calc's for BOSEMs on triple indicate ISC noise OK even for HAM isolation and no reaction chain
2. Dynamics: dynamics of damped double are much simpler than quad damped only from the top.
3. Kicks to ISI: need to check, but should be fine if high freq. drives sent to bottom 2 stages of quad.
4. Tracking for scatter: should be fine, R0 tracking for double should be simpler than for a quad because dynamics are simpler.
5. Size and Weight: Fits. Modeled with 40 and 80 kg mass (easily fits 200 kg budget). Wires interfere with UIM (requires a hole).
6. What about suspending the compensation plate?
7. **Q: What else?**



Coupling calc



**T2300112, based on
pg 233 of P1000032**

What is the spring rate for the Advanced LIGO ESD, and what might we expect for A#?

For aLIGO From John Miller's thesis P1000032, pg 233, we get

$$\alpha = 9.54 * 10^{-16} * x^{-2.38} N/V^2 \quad (1)$$

so the slope is

$$\frac{d\alpha}{dx} = -2.38 * 9.54 * 10^{-16} * x^{-3.38} N/(m * V^2) \quad (2)$$

at $x = 5 * 10^{-3}$ meters, this becomes

$$\frac{d\alpha}{dx} = -1.36 * 10^{-7} N/(m * V^2) \quad (3)$$

The Advanced LIGO bias can go as high as 400 V, although for O4 it is expected to be 100 V at LLO, and LHO is also considering working at 100 V bias. We make the conservative choice of 400 V for this calculation. This results in a spring rate, k_{aLIGO} , of

$$k_{aLIGO} = \frac{dF}{dx} = -1.36 * 10^{-7} * V^2 N/m = -0.0218 N/m \text{ at } 400 \text{ V for aLIGO} \quad (4)$$

At 100 V bias, this drops to $k = -1.36 * 10^{-3} N/m$.

No real work has been done on the ESD for A#, so we assume the same basic electrode geometry, bias, and 5 mm gap. Because the mass is 2.5 times larger, we scale the electrode size up by 2.5, and estimate the spring rate $k_{A\#}$ for A# ESD to be

$$k_{A\#} = -0.0544 N/m \text{ at } 400 \text{ V for A\#} \quad (5)$$

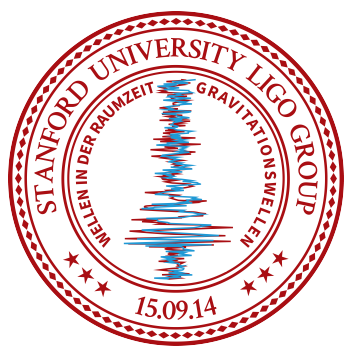
Now, we can estimate the allowed motion of the reaction mass at 10 Hz. When the reaction mass moves, this spring produces a force on the optic.

$$m * \omega^2 * x_{optic} = k * x_{RM} \quad (6)$$

If we set the motion requirement of the optic to be less than 10^{-19} m/rtHz at 10 Hz, then the allowed motion of the reaction mass is

$$100kg * (2\pi * 10)^2 * 10^{-19} \text{m/rtHz} / (-0.0545 N/m) > x_{RM} \quad (7)$$

$$x_{RM} < 7.26 * 10^{-13} \text{m/rtHz at } 10 \text{ Hz} \quad (8)$$



Triples - no reaction chain



In 2002, Phil Willems showed the triples don't need a reaction chain ([T020059](#))

Updated in 2008 by J. Kissel, N. Roberston, M. Barton, P. Willems

Doupling via ISC drive at least 100x below coupling via SUS transmission

Now - ISC drives react directly against the SUS cage (and ISI)

Q: Do HAM triples have any issues with no reaction chain?

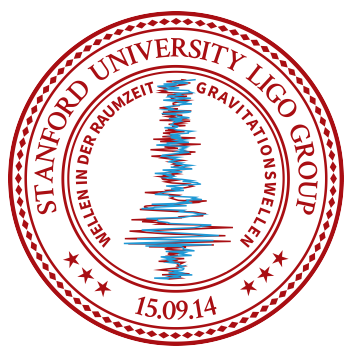
Triples includes the Beamsplitter.

Beamsplitter **does** have an issue - there are no drives for the bottom stage (the beamsplitter optic) so high frequency lock acquisition is driven from the middle mass. (no reaction chain, no direct drive. HF drives are very large)

The back reaction from this drive creates so much disturbance on the ISI that the loops saturate. Thus, ISIs engage full isolation after MICH is locked.

Beamsplitter Drive issue would be resolved with OSEMs at bottom stage (True/ False?)

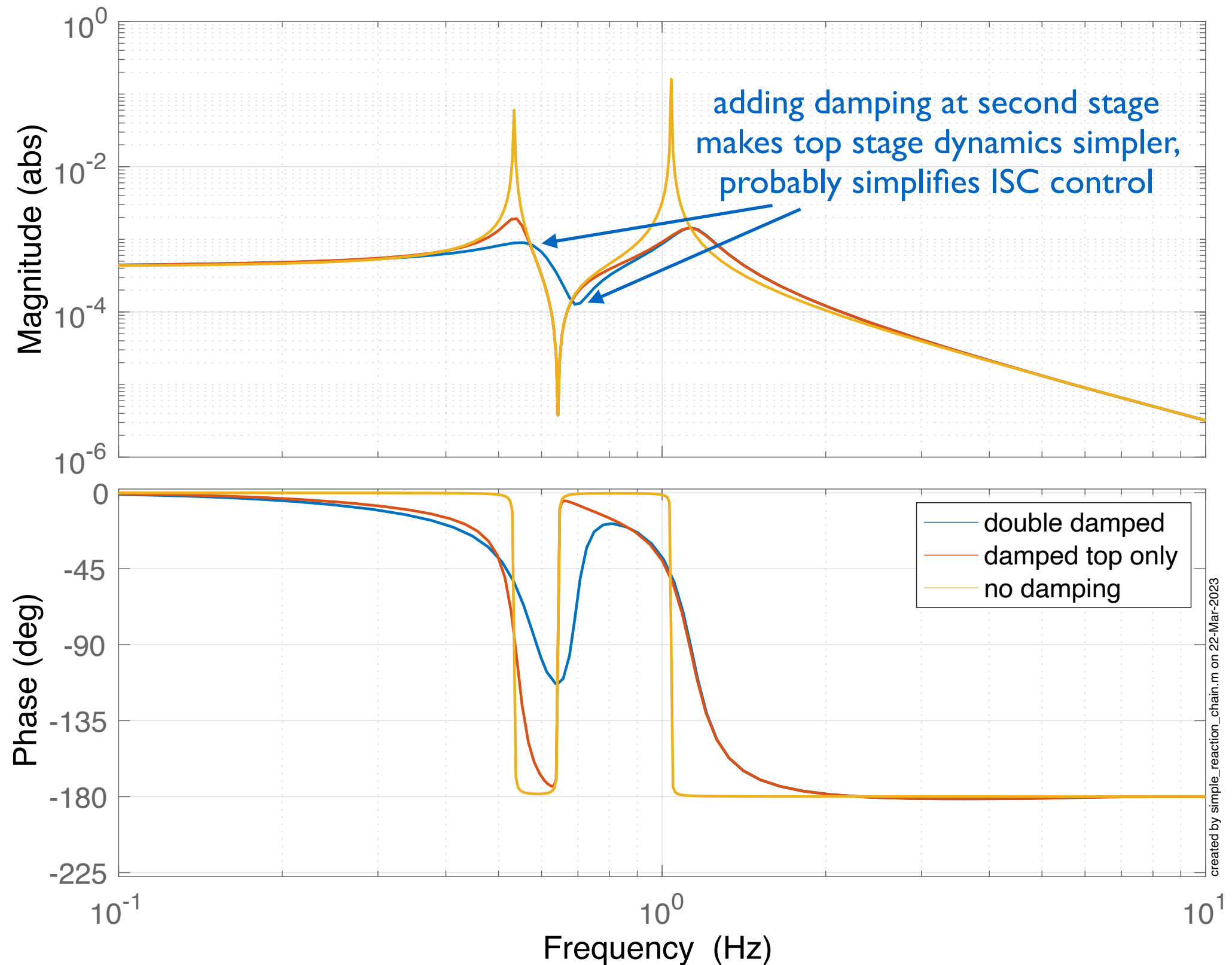
Quad reaction chain isolates the ISI from lower stage ISC drives.

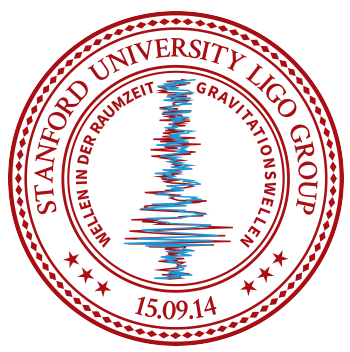


Dynamics of top stage



Compliance of the top stage

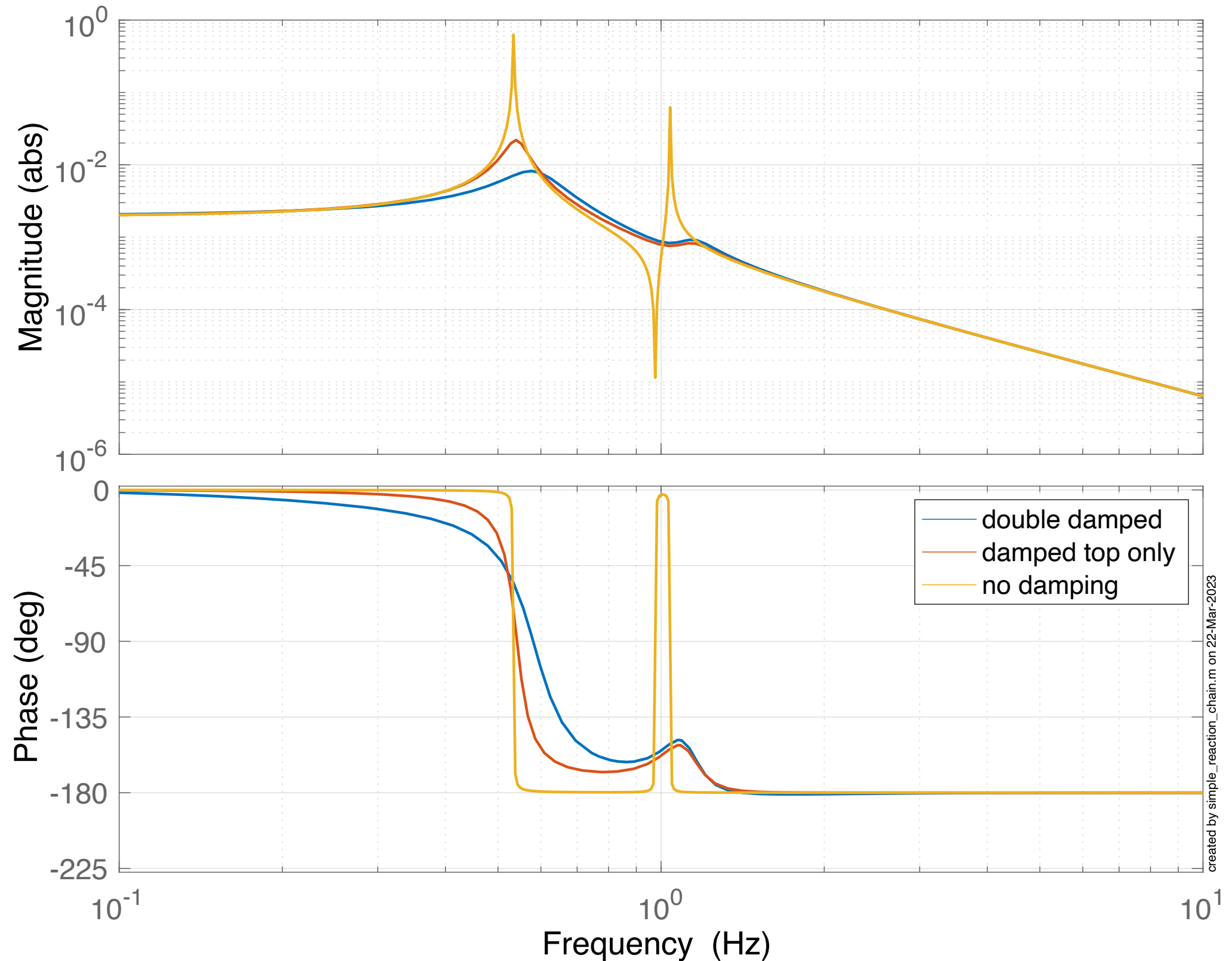


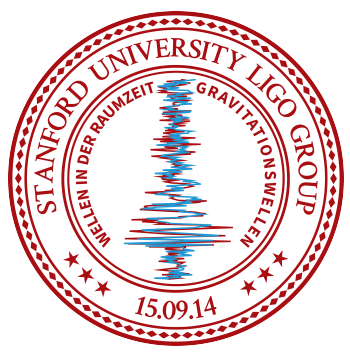


control on lower mass



Compliance of the lower stage

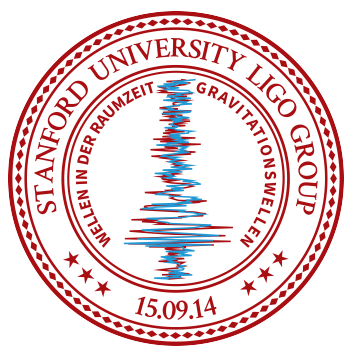




Conclusions



1. We need a reaction chain (ISC drive noise, scatter, ISI kicks)
2. A double seems to give large margin for isolation performance
3. but we haven't calculated angles,
4. or the vertical isolation requirements,
5. or the compensation plate.
6. The dynamics look really nice (no funny 3.3 Hz modes lurking)
7. It fits with the space and weight,
8. but probably requires a hole in the UIM structure.
9. Overall, it's probably easier to install than a quad
10. Please tell me what else we need to consider.



compensation plate?



Advanced LIGO System Design, Fritschel, Coyne, [T010075-v3](#), pg 14 (2015)

<i>Component</i>	<i>Number of suspension stages</i>	<i>Fiber type</i>	<i>Noise req. @10 Hz $m/\sqrt{Hz}$</i>
Test masses	4	fused silica	1×10^{-19}
Reaction masses (CP, ERM)	4	steel wire	???
Beamsplitter	3	steel wire	6.4×10^{-18}
Recycling cavity optics	3	steel wire	1×10^{-17}
Input mode cleaner mirrors	3	steel wire	3×10^{-15}
Output mode cleaner	2	steel wire	1×10^{-13}
Output Faraday isolator	1	steel wire	
ETM transmission monitor	2	steel wire	
IO SM/MM optics	1	steel wire	

Table 4: Summary of suspension types in the interferometer. The test mass suspensions

Other refs which I've looked at:

Cavity Optics Suspension Subsystem Design Requirements - [T010007](#)

Displacement Noise in Advanced LIGO Triple Suspensions - [T080192](#)