Investigating Eccentric & Precessing Waveform Models

Isaiah Tyler
Loyola Marymount University
Lucy M Thomas & Taylor Knapp

Waveform Templates

- Data Analysis algorithms that search for GW events utilize the method of matched filtering
  - correlates theoretical template signals to the detector output data

- Template banks → increased physics output of data analysis at LIGO
  - Parameter estimation relies on waveform templates

- Currently no models that include both eccentricity and precession

GstLAL all-sky (O4) template bank:

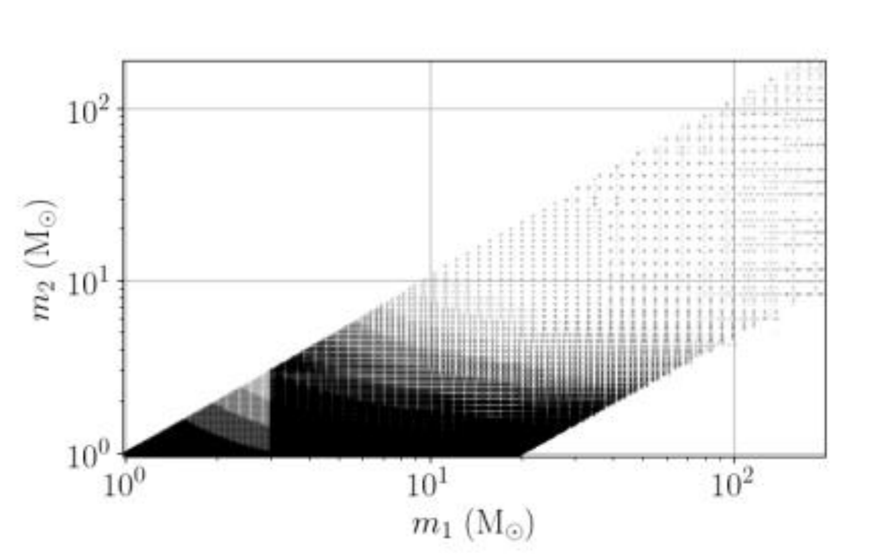
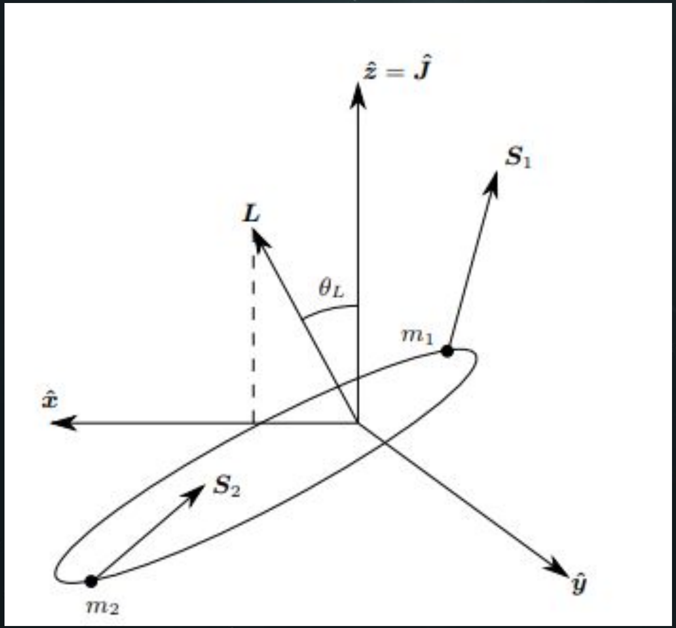


Image Credit: Sakon, S et al. (2024). Template bank for compact binary mergers in the fourth observing run of Advanced LIGO, Advanced Virgo, and KAGRA. *Physical Review D*, *109*(4), 044066.

Precession & Eccentricity



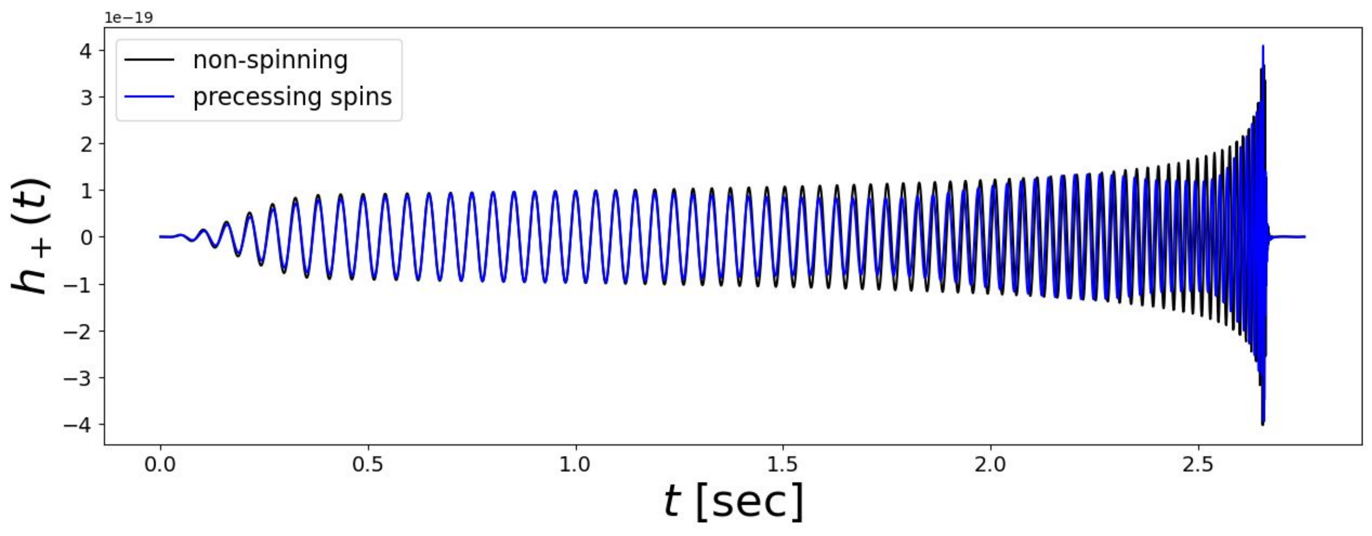
- Misalignment of spins → changes in orientation of orbital plane (precession)
- Eccentricity is a dimensionless parameter that describes how much an orbit deviates from a circular orbit

Image credit: Klein, A. (2021). EFPE: Efficient fully precessing eccentric gravitational waveforms for binaries with long inspirals

Precession in the Time Domain

- Spin precession → modulations in the amplitude and phase of the strain.
  - Increasing the magnitude of the spins → stronger modulations

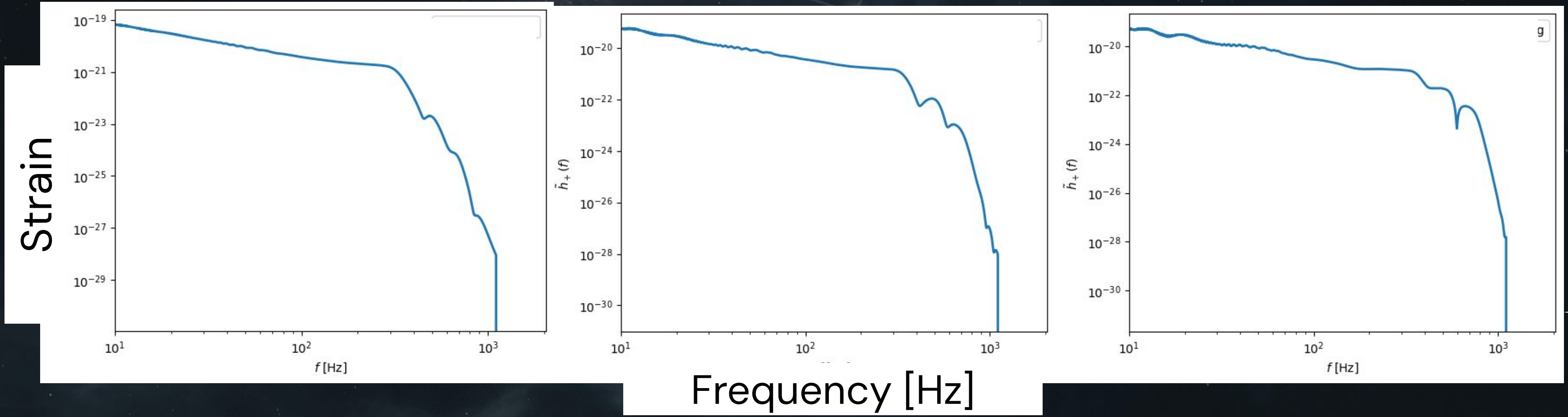
Approximant: IMRPhenomTPHM



Precession in the Frequency Domain

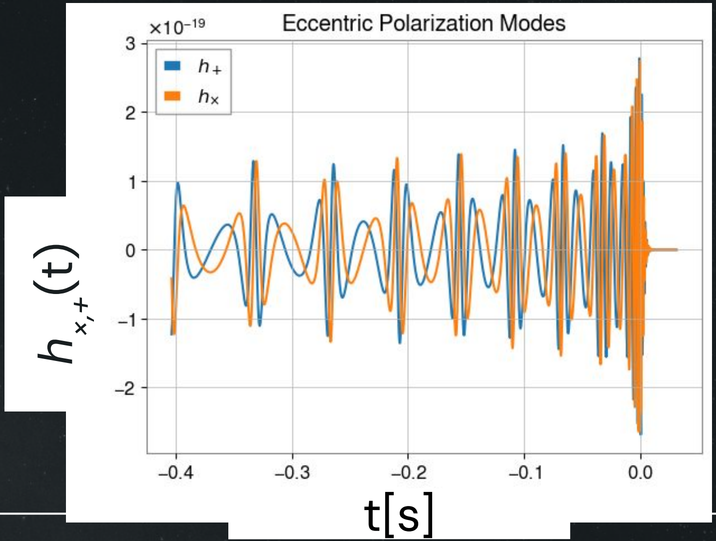
- Systems with more spin exhibit more modulations during the inspiral at low frequencies
- Exhibits different modulation behavior in the merger-ringdown at high frequencies

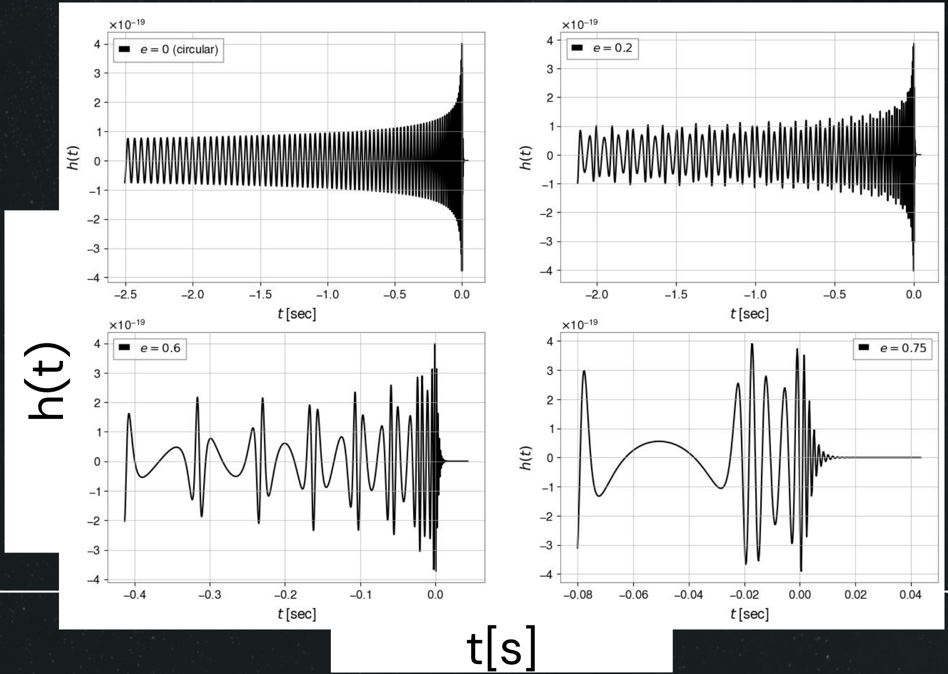
Approximant: IMRPhenomXPHM



Eccentricity in the Time Domain

- The duration of the coalescence decreases as the eccentricity increases since the radiation reaction occurs at a higher rate
- The amplitude of the peaks during inspiral increases as eccentricity increases because the periastron of each orbit is closer (more energy is radiated)





Model: TEOBResumS-DALI

Orbital Evolution

- Goal: Obtain $h_+(t)$ and $h_\times(t)$ polarizations
  - Calculate the dynamics and use those to calculate the strain
- Investigate the changes in the orbital evolution under varying initial conditions

Orbital Evolution Equations:

$$
\begin{aligned}
\dot{\omega} &= \eta \bar{x}^{11/2}(1-e_t^2)^2\left[\mathcal{O}_{\rm N} + \mathcal{O}_{\rm 1PN}\bar{x} + \mathcal{O}_{\rm 1.5PN}^{\text{hered, SO}}\bar{x}^{3/2} + \mathcal{O}_{\rm 2PN}^{\text{incl. SS}}\bar{x}^2 + \mathcal{O}_{\rm 2.5PN}^{\text{hered}}\bar{x}^{5/2} + \mathcal{O}_{\rm 3PN}^{\text{incl. hered}}\bar{x}^3\right],\\
\dot{e_t^2} &= -2\eta \bar{x}^4(1-e_t^2)^{3/2}\left[\mathcal{E}_{\rm N} + \mathcal{E}_{\rm 1PN}\bar{x} + \mathcal{E}_{\rm 1.5PN}^{\text{hered, SO}}\bar{x}^{3/2} + \mathcal{E}_{\rm 2PN}^{\text{incl. SS}}\bar{x}^2 + \mathcal{E}_{\rm 2.5PN}^{\text{hered}}\bar{x}^{5/2} + \mathcal{E}_{\rm 3PN}^{\text{incl. hered}}\bar{x}^3\right],\\
\dot{k} &= \eta \bar{x}^4(1-e_t^2)^{3/2}\left[\mathcal{K}_{\rm 1PN}\bar{x} + \mathcal{K}_{\rm 1.5PN}^{\text{SO}}\bar{x}^{3/2} + \mathcal{K}_{\rm 2PN}^{\text{incl. SS}}\bar{x}^2 + \mathcal{K}_{\rm 2.5PN}^{\text{hered}}\bar{x}^{5/2} + \mathcal{K}_{\rm 3PN}\bar{x}^3\right],\\
\dot{l} &= \frac{\omega}{1+k},\\
\dot{\lambda} &= \omega,
\end{aligned}
$$

Precession Evolution Equations:

$$
\begin{aligned}
\dot{\mathbf{S}}_1 &= \frac{1}{2a^3(1-e^2)^{3/2}}\left[\left(4+3q-\frac{3(\mathbf{S}_2+q\mathbf{S}_1)\cdot\mathbf{L}}{L^2}\right)\mathbf{L}+\mathbf{S}_2\right]\times\mathbf{S}_1,\\
\dot{\mathbf{S}}_2 &= \frac{1}{2a^3(1-e^2)^{3/2}}\left[\left(4+3q^{-1}-\frac{3(\mathbf{S}_1+q^{-1}\mathbf{S}_2)\cdot\mathbf{L}}{L^2}\right)\mathbf{L}+\mathbf{S}_1\right]\times\mathbf{S}_2,\\
\dot{\mathbf{L}} &= \boldsymbol{\omega}_p\times\mathbf{L}.
\end{aligned}
$$

$$\boldsymbol{\omega}_p := \delta_1\mathbf{S}_1 + \delta_2\mathbf{S}_2, \tag{9a}$$

$$\delta_1 := \frac{1}{2a^3(1-e^2)^{3/2}}\left(4+3q-\frac{3(\mathbf{S}_2+q\mathbf{S}_1)\cdot\mathbf{L}}{L^2}\right), \tag{9b}$$

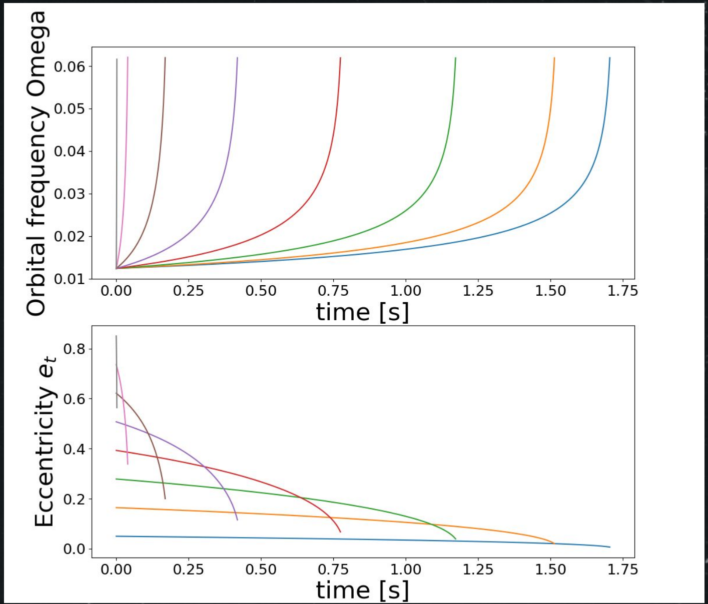
$$\delta_2 := \frac{1}{2a^3(1-e^2)^{3/2}}\left(4+3q^{-1}-\frac{3(\mathbf{S}_1+q^{-1}\mathbf{S}_2)\cdot\mathbf{L}}{L^2}\right), \tag{9c}$$

Equations: LIGO Document T2400230-v1

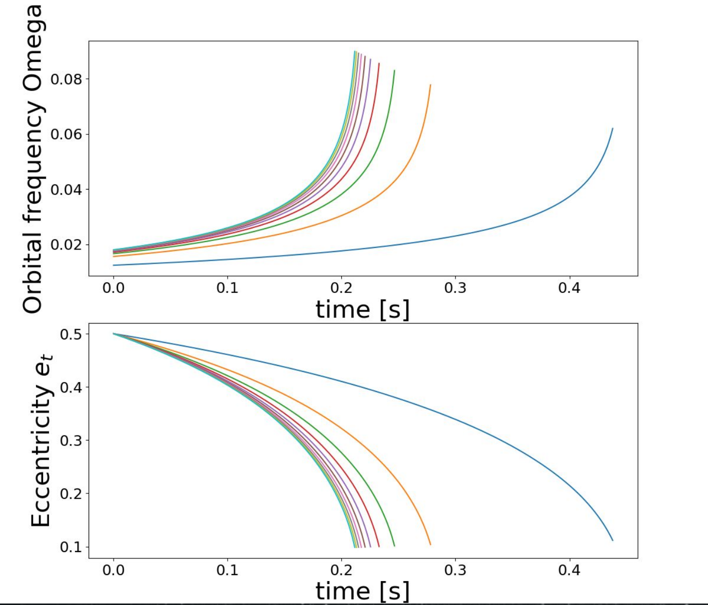
Orbital Evolution

- Increasing mass ratio & initial eccentricity
  - The orbit circularizes quicker
  - The orbital frequency reaches a high maximum

Range of initial eccentricities: (0.05<e<0.85)



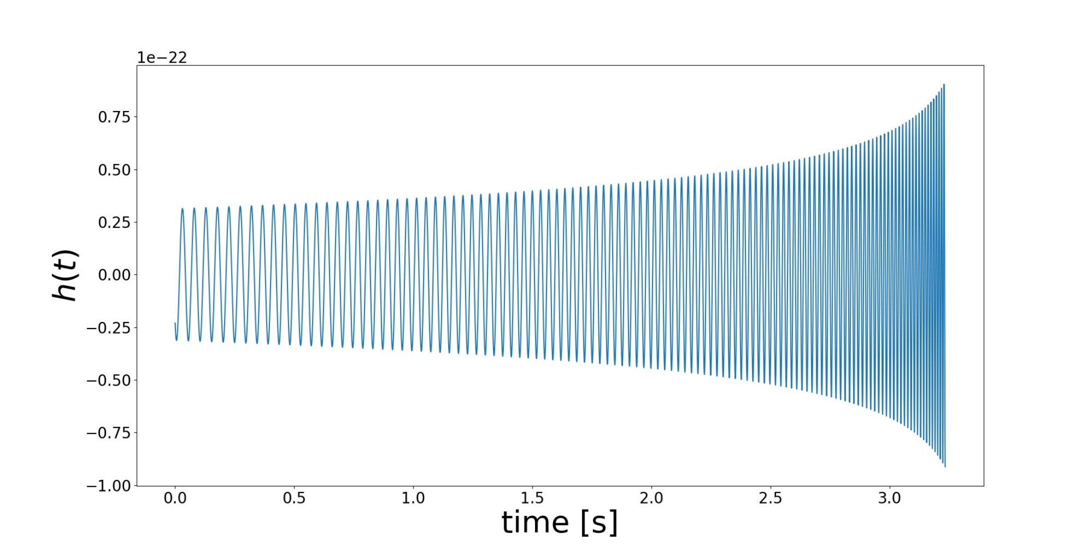
Range of mass ratios: (1<q<20)



OPN Equations

$$h_+(t) = \frac{4}{r}(\frac{GM_c}{c^2})^{\frac{5}{3}}(\frac{\pi f_{gw}}{c})^{\frac{2}{3}}\frac{1+cos^2(\theta)}{2}cos(2\pi f_{gw}t_{ret}+2\phi)$$

$$h_\times(t) = \frac{4}{r}(\frac{GM_c}{c^2})^{\frac{5}{3}}(\frac{\pi f_{gw}}{c})^{\frac{2}{3}}cos(\theta)sin(2\pi f_{gw}t_{ret}+2\phi)$$



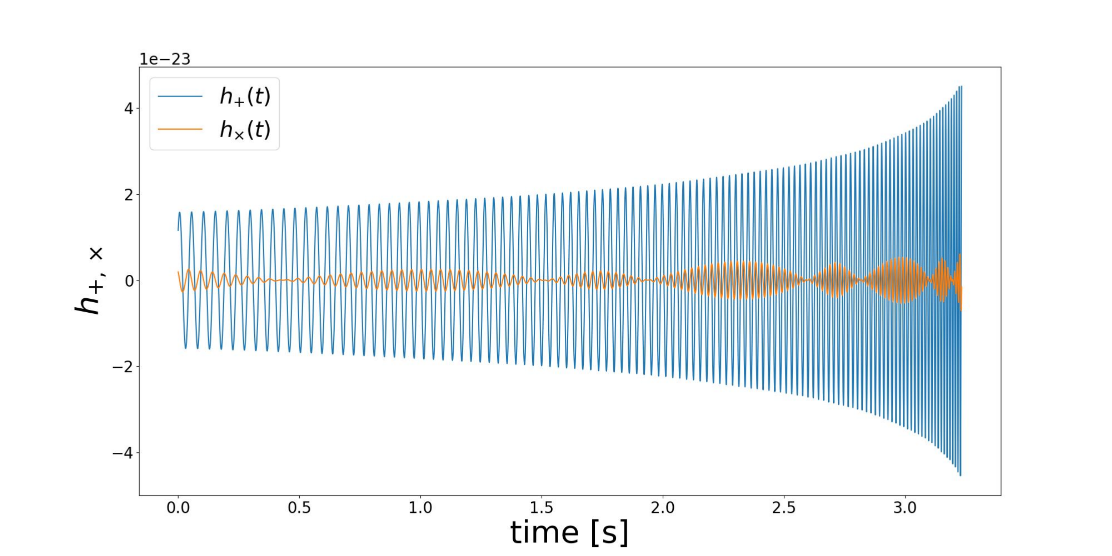
- Equations assume circular obit
- Does not exhibit modulation behavior as expected from eccentric or precessing waveforms.

Maggiore, M. (2007). *Gravitational waves* (Vol. 1). Oxford university press.

1.5PN Equations

$$h_{+,\times} = \frac{2\eta G m}{c^4 R}\left(\frac{G m \Omega}{c^3}\right)^{\frac{2}{3}} H_{+,\times}$$

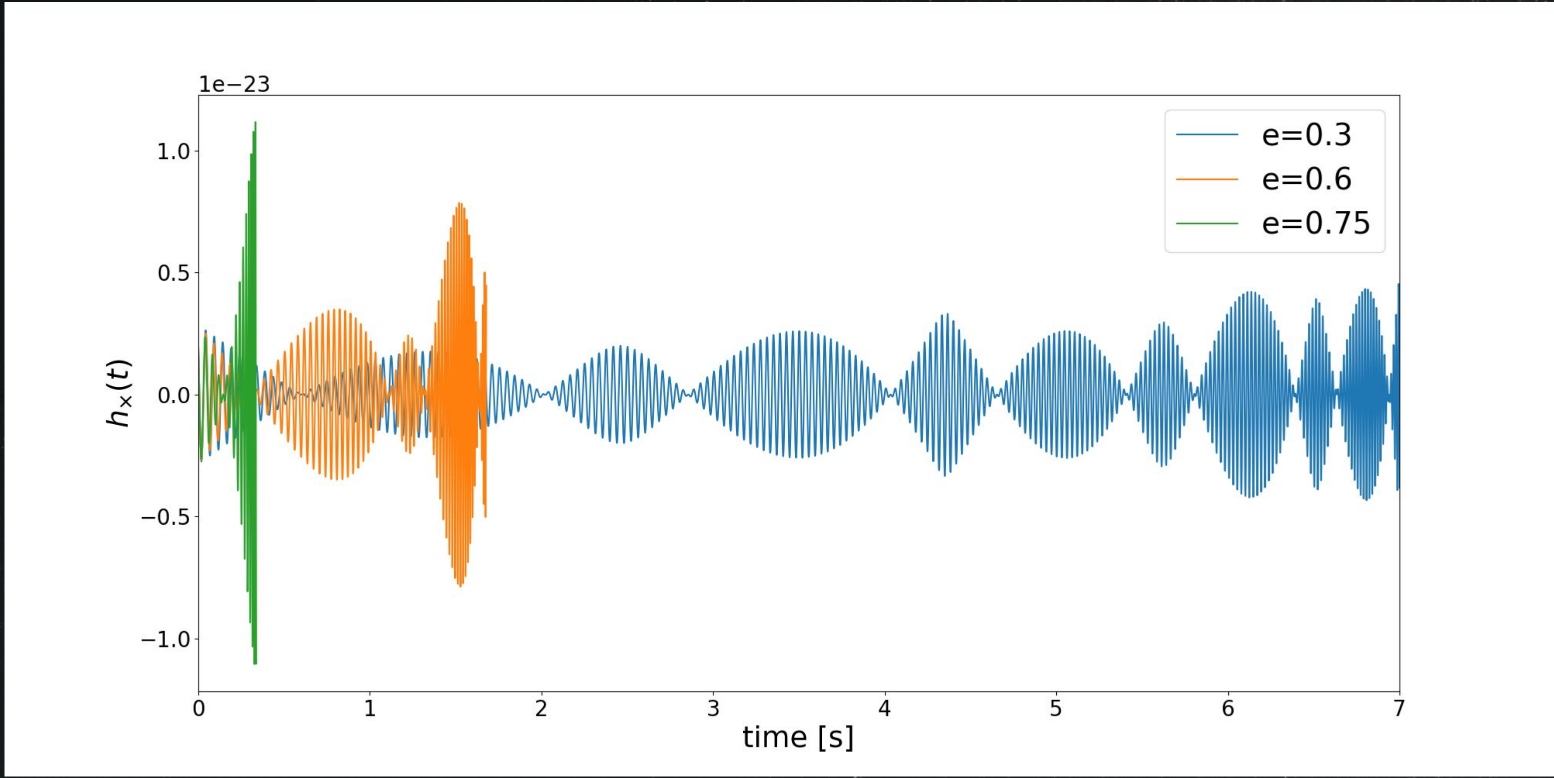
$$H_{+,\times} := H^{[0]}_{+,\times} + \Delta\beta H^{[1/2]}_{+,\times} + \beta^2 H^{[1]}_{+,\times} + \Delta\beta^3 H^{[3/2]}_{+,\times} + \beta^3 H_{\mathrm{tail},+,\times} + \mathcal{O}(\beta^4)$$



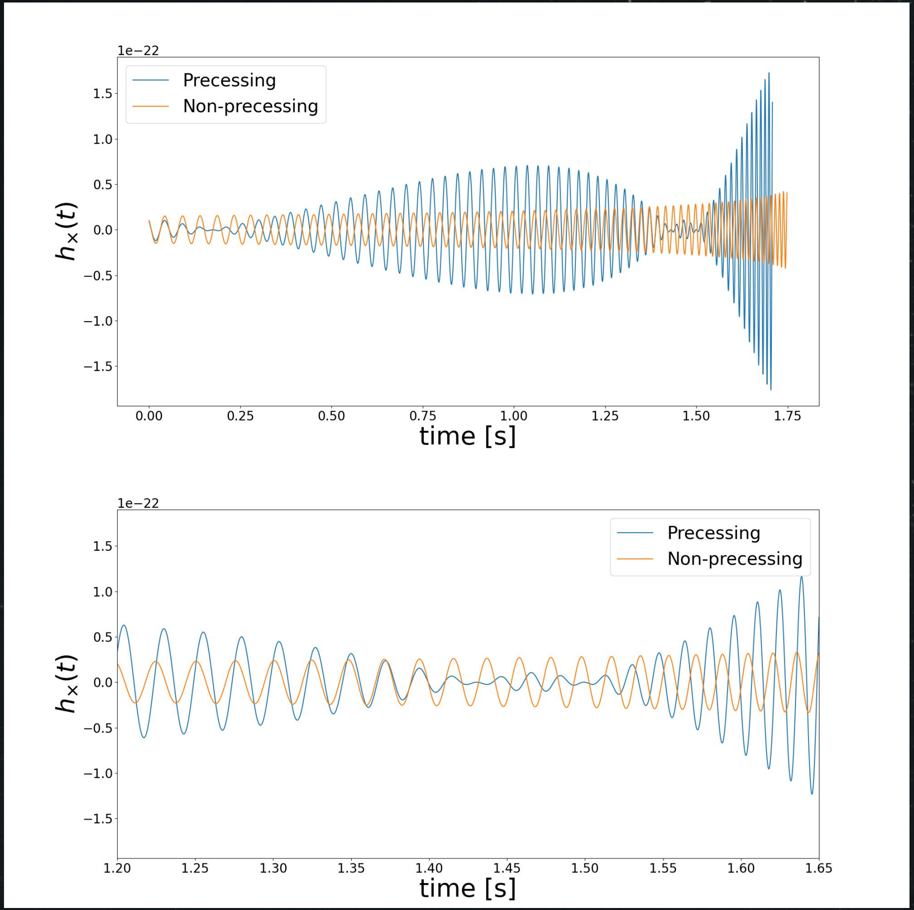
- Equations assume a circular orbit
- Modulations appear in the $h_\times$ mode

Poisson, E., & Will, C. M. (2014). *Gravity: Newtonian, post-newtonian, relativistic*. Cambridge University Press.

Isolating Eccentricity Effects



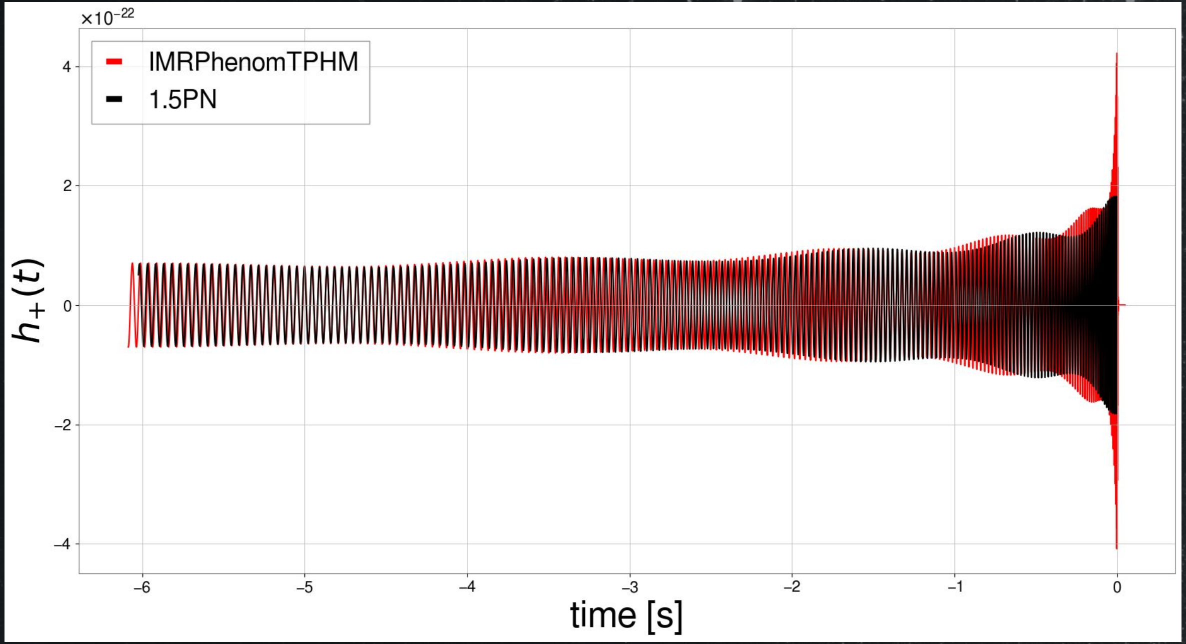
Isolating Eccentricity Effects



- No modulations appear in the non-precessing, eccentric waveform

Comparison of Precessing Waveforms

- Comparing new code to IMRPhenomTPHM for same set of parameters
  - The two waveforms are identical in the early inspiral
  - Similar modulation behavior



Removing the Circular Assumption

$$h^{jk}(t, \boldsymbol{x}) = \frac{2\eta G m}{c^4 R}\Big[A^{jk}[0\text{PN}] + A^{jk}[0.5\text{PN}] + A^{jk}[1\text{PN}] + A^{jk}[1.5\text{PN}] + A^{jk}[\text{tail}] + O(c^{-4})\Big],$$

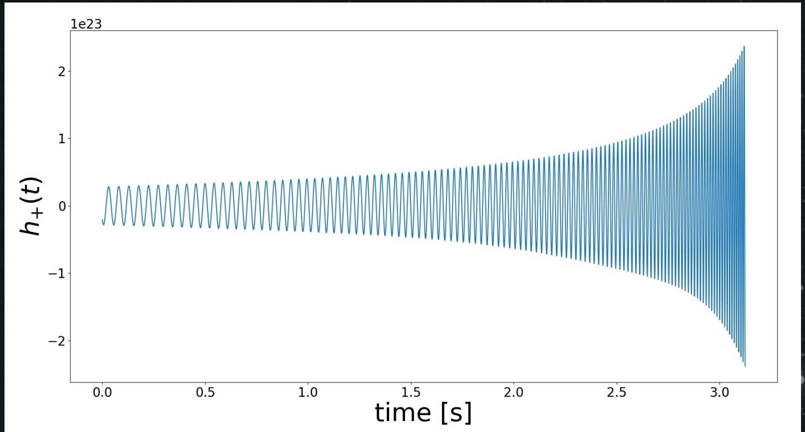
- A_{jk} is a function of orbital dynamics quantities
    - Mean orbital phase , separation , orientation of orbital plane, velocities

$$\boldsymbol{e}_X = [\cos\omega, -\sin\omega, 0],$$
$$\boldsymbol{e}_Y = [\cos\iota\sin\omega, \cos\iota\cos\omega, -\sin\iota],$$
$$\boldsymbol{e}_Z = [\sin\iota\sin\omega, \sin\iota\cos\omega, \cos\iota] = \boldsymbol{N},$$

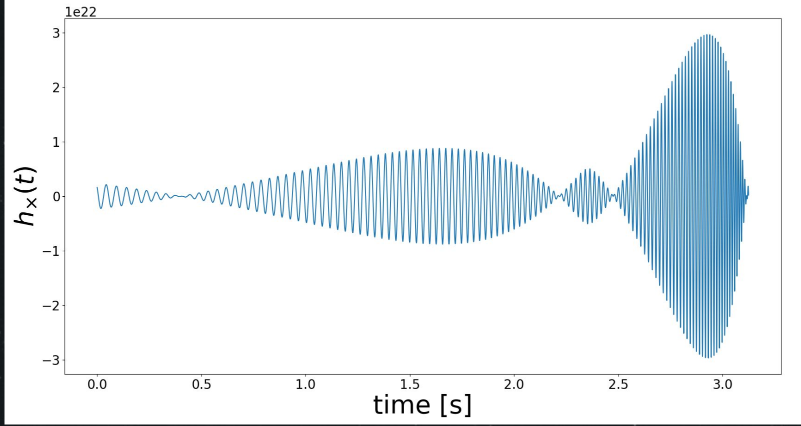
$$h_+ = \frac{1}{2}(e_X^j e_X^k - e_Y^j e_Y^k) h_{jk},$$
$$h_\times = \frac{1}{2}(e_X^j e_Y^k + e_Y^j e_X^k) h_{jk},$$

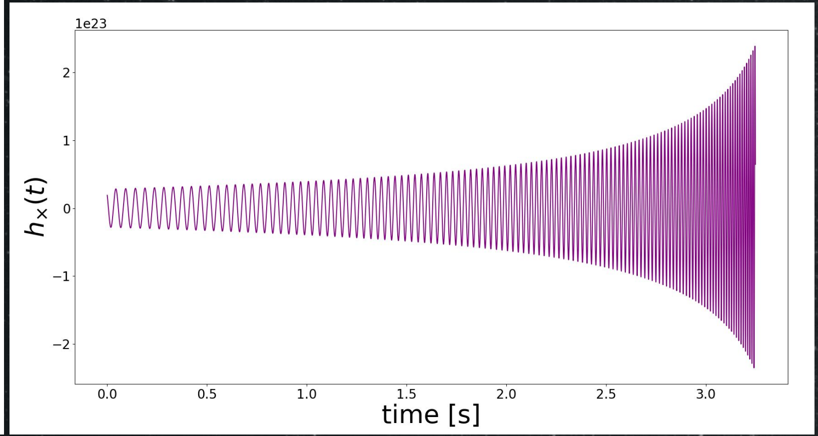
Poisson, E., & Will, C. M. (2014). *Gravity: Newtonian, post-newtonian, relativistic*. Cambridge University Press.

Removing the Circular Assumption



- Modular behavior appears in the $h_\times$ polarization but not h_+ similar to the previous set of waveforms
- No modulations appear when spin-precession is removed as seen in the purple waveform





Conclusion

- Waveforms generated from the new orbital and spin evolution equations could potentially replicate precessing waveforms from IMRPhenom approximants

- Orbit-averaging quantities such as the binary separation when deriving the orbital evolution equations could explain why the effects of eccentricity are not seen in the strain.

Acknowledgements

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