LIGO SURF Proposal:
The impact of astrophysical population model choices on post-Newtonian deviation tests of general relativity

Ruby Knudsen, Mentor: Ethan Payne

May 2024

1 Background and Introduction

1.1 Gravitational Waves and General Relativity

Einstein put forward the theory of general relativity (GR) in the early 20th century, part of which theorized the idea of spacetime, and claimed that spacetime distortions occur where massive objects exist in its fabric. He summarized his theory mathematically through the Einstein field equations. Building off of this work, two years later, he posited the existence of gravitational waves (GW) — spatial distortions as transverse waves that originate from non-spherically symmetric quadrupolar disturbances before traveling away at the speed of light [2, 9]. Because GR theorizes the existence of GWs, we can use GWs as a method to test GR. There are several tests of GR, but GWs are the best probe for the theory.

Not all tests of GR were created equal, as summarized by Fig. (1). While all astrophysical objects have gravitational potential and cause spacetime curvature, the most extreme tests of GR exist in the strong-field regime. Potential is proportional to M/R^3, and curvature is proportional to M/R. The perihelion procession of Mercury probes GR, but not strenuously, since it has potential between 10^{-7} and 10^{-8}, and curvature between 10^{-32} and 10^{-33} cm^{-2} [14]. Due to the Schwarzchild radius, the event horizon of a black hole yields potential approximately 0.5, and is the most extreme possible potential. While Sagittarius A* and M87 both have potential near 0.5, their curvature is orders of magnitude lower than GWs' curvature [7]. Additionally, curvatures between 10^{-14} and 10^{-10} cm^{-2} are the most extreme for realistic astrophysical signals [7]. Probing GR in this region requires the detection of GWs from black hole and neutron star mergers. Thus, GW tests probe GR at the most extreme curvatures and potentials, and as such are the best tests of GR.

The Laser Interferometer Gravitational-wave Observatory (LIGO) — a joint project between the California Institute of Technology, the Massachusetts Institute of Technology, and the National Science Foundation — was built to detect these GWs and aims to further our understanding of GWs and test GR [1, 5]. LIGO has perpendicular arms that detect compact binary mergers using the interference of light due to the difference in length of the arms that GWs cause. This change in length is directly proportional to GW strength [1, 5]. The compact binary mergers, mostly binary black hole mergers, are then analyzed to check for consistency with GR. LIGO is sensitive to GW frequencies between 10 and 1000 Hz, so black holes of stellar mass are easiest to detect [10].

1.2 How do we model modifications to GR with individual observations?

There are many different ways of modeling GR deviations by modeling the GW signal. Here we will use one specific test: inferring deviations from the post-Newtonian (PN) approximation. Post-Newtonian approximation at the lowest order is the quadrupole formula, which estimates the emitted radiation from quadropolar mass distribution [13]. The PN expansion for gravitational-wave emission involves the dimensionless parameter v/c, where v is the orbital velocity of the binary, with the lowest order being the Newtonian-order v/c order, and higher orders being $(v/c)^2$, $(v/c)^3$, etc [15, 8]. PN tests are run by first constructing a post-Newtonian description of the GW inspiral in the frequency domain, before making modifications to

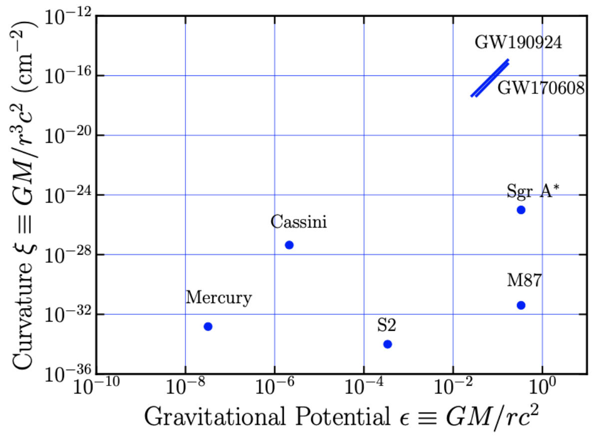

Figure 1: From [14]. This graphic shows a plot of gravitational potential vs. curvature for various tests of GR. The theoretical limit for the gravitational potential exists between 0.5 and 1.

individual parameters in the phase evolution [12]. This is done with the phase, not the amplitude, because LIGO is more sensitive to phase deviations than those of amplitudes.

$$\Phi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128} \times \sum_{k=0}^{7} \frac{1}{\eta^{\frac{k}{5}}} (\varphi_k + \varphi_{k,l} \ln \tilde{f}) \tilde{f}^{\frac{k-5}{3}} \tag{1}$$

In this equation, $\Phi(f)$ is the frequency domain GW phase under the stationary phase approximation $\tilde{f}$, the red-shifted chirp-mass multiplied by f (the frequency), divided by c^3. Additionally, t_c and ϕ_c are the coalescence time and phase respectively, η is the symmetric mass ratio, and φ_k, $\varphi_{k,l}$ are PN coefficients for the $k/2$ PN order [12]. After running these PN tests, a posterior is obtained by running Bayesian inference, a statistical framework.

Bayesian inference make statements about the Universe from data by creating a posterior distribution [16]. Creating this posterior distribution requires a likelihood to describe the measurement (noise, etc.) and a prior to encode prior beliefs about each event's parameters θ. Bayesian inference uses Bayes' Theorem,

$$p(\theta|d) = \frac{\mathcal{L}(d|\theta)\pi(\theta)}{\mathcal{Z}} \tag{2}$$

In this equation $\mathcal{L}(d|\theta)$ is the marginalized likelihood and $\pi(\theta)$ is the prior, and the evidence, which normalizes the distribution, is denoted as $\mathcal{Z}$ [16]. This same formalism is modified for use on multiple events.

Bayes theorem is used to make posterior distributions, which give probable parameters for each event analyzed, using Eq. (1). Posterior distributions can also be made for a population of events. This analysis of GWs and individual compact binary mergers introduces inferred deviations in the terms. These constraints are often in terms of posterior distributions on GR deviations of parametric post-Newtonian terms in a waveform expression [4].

1.3 How to we test GR with many observations?

Hierarchical inference is a method that first models the individual events from a source, and then the population of events to give the astrophysical population of the source given the events. The analysis returns parameter distributions for the population model, which is called the hyper-posterior [6].

In order to do this, events are compared with signal models that output many individual event posteriors, which are the probable parameters for each event. These posterior distributions from individual events are taken with a population model described by hyper-parameters, and a hyper-posterior — or an astrophysical population distribution — is inferred [16]. From this hyper-posterior, the astrophysical population has been inferred under the assumption of the choice of population model. A population model is important because otherwise an incorrect astrophysical population may be implicitly assumed, which in turn leads to bias in supposed measured deviations from GR [12].

To look at the population properties of a collection of events, the prior for each even θ is made conditional on hyper-parameters Λ, which parameterizes the astrophysical distribution from which the θs are drawn [16]. These hyper-parameters are very important because they parameterize the shape of the inferred astrophysical population, and one goal of population inference is estimating the posterior of these hyper-parameters. This will result in the likelihood of the data given the hyper-parameters, which is determined using Eq. (2).

A population likelihood is necessary for hierarchical inference. The population likelihood is a formula that allows simultaneous astrophysical population inference and GR testing, decreasing this bias [12, 11]. The equation for population likelihood is

$$p(\{d\}|\Lambda) = \frac{1}{\xi(\Lambda)^N} \prod_{i=1}^{N} \int d\theta_i p(d_i|\theta_i)\pi(\theta_i|\Lambda) \tag{3}$$

Here, $\{d\}$ is the collection of N observations, $\xi(\Lambda)$ accounts for selection biases and is the detectable fraction of observations given the population hyper-parameters, $p(d_i|\theta_i)$ is the likelihood for each individual event, and $\pi(\theta_i|\Lambda)$ are the hyper-priors for each event [12]. Thus, $\pi(\theta_i|\Lambda)$ is where the population distribution is encoded.

This equation shows how individual observations are put together for hierarchical inference. As stated earlier, this hierarchical approach requires a population model, so we therefore must choose one population model to use. Currently, we are not sure in what manner the different choices of population model impact GR tests. This leads to my project.

1.4 Project Proposal

Population distributions depend in part on the choice of population model, thus the choice of population model must be carefully considered. So far, this approach of joint inference has, at the population level, yielded GR deviations more consistent with GR by about 0.4σ, when using a PowerLaw+Peak population distribution for the mass of a black hole [12, 3]. However, it is possible that deviation from general relativity could be absorbed or hidden by an incorrect, assumed astrophysical population, so it is important to study the impact of different astrophysical population models on inferred GR deviation constraints. It is true that at some point that the incorrect choice will also lead to biases, the question is when, and how badly. Testing this using different population models is what I aim to do.

2 Objectives

- Generate simulated catalogs of events drawn from different astrophysical population models
- With injected signals, use both mis-specified and correct population models to determine the implications of astrophysical population mis-specification, including whether it would be possible to determine that mis-specification had occurred *post hoc*
- Determine how including different astrophysical population models influences GW tests of GR, and through this explore the significance of model choices for future tests

3 Approach

I will work with my mentor to expand on the work in reference [12] to study the impact or bias from mis-specified population models in testing general relativity. In order to do this I will pick two to three population models, and generate a simulated set of observations from each. I will then analyze these injected signals with the PN deviation gravitational waveform model, before inferring each set with each model. With this information, I will see when and where each population model breaks down. I will also compare the different population models (mis-specified and not) to each other, and to the work completed in reference [12].

I will simulate catalogs of events using JupyterNotebook, NumPyro, Jax, and Python code with various population models to constrain post-Newtonian phase coefficient deviations.

4 Work Plan

- Week 1-2: Get set up with NumPyro and Jax in JupyterNotebook and experimenting with running population inference calculations.
- Week 3: Identify and read up on population models to test; begin coding the population models into the inference code
- Week 4: Figure out how to create a simulated catalog of events to run through code
- Weeks 5-6: Run analyses using the targeted population models on the simulated catalog of events
- Weeks 7-8: Analyze hyper-posteriors generated by running analyses; compare them to each other; check for realistic population constraint assumptions
- Weeks 9-10: Wrap up any final analysis, create poster and presentation, work on write-up/paper, final presentation

References

[1] Junaid Aasi, BP Abbott, Richard Abbott, Thomas Abbott, MR Abernathy, Kendall Ackley, Carl Adams, Thomas Adams, Paolo Addesso, RX Adhikari, et al. Advanced ligo. *Classical and quantum gravity*, 32(7):074001, 2015.

[2] Benjamin P Abbott, Richard Abbott, TDe Abbott, MR Abernathy, Fausto Acernese, Kendall Ackley, Carl Adams, Thomas Adams, Paolo Addesso, Rana X Adhikari, et al. Observation of gravitational waves from a binary black hole merger. *Physical review letters*, 116(6):061102, 2016.

[3] R Abbott, TD Abbott, F Acernese, K Ackley, C Adams, N Adhikari, RX Adhikari, VB Adya, C Affeldt, D Agarwal, et al. Population of merging compact binaries inferred using gravitational waves through gwtc-3. *Physical Review X*, 13(1):011048, 2023.

[4] R Abbott, H Abe, F Acernese, K Ackley, N Adhikari, RX Adhikari, VK Adkins, VB Adya, C Affeldt, D Agarwal, et al. Tests of general relativity with gwtc-3. *arXiv preprint arXiv:2112.06861*, 2021.

[5] Fet al Acernese, M Agathos, K Agatsuma, Damiano Aisa, N Allemandou, Aea Allocca, J Amarni, Pia Astone, G Balestri, G Ballardin, et al. Advanced virgo: a second-generation interferometric gravitational wave detector. *Classical and Quantum Gravity*, 32(2):024001, 2014.

[6] Matthew R Adams, Neil J Cornish, and Tyson B Littenberg. Astrophysical model selection in gravitational wave astronomy. *Physical Review D*, 86(12):124032, 2012.

[7] Tessa Baker, Dimitrios Psaltis, and Constantinos Skordis. Linking tests of gravity on all scales: from the strong-field regime to cosmology. *The Astrophysical Journal*, 802(1):63, 2015.

[8] Luc Blanchet and Thibault Damour. Post-newtonian generation of gravitational waves. In *Annales de l'IHP Physique théorique*, volume 50, pages 377–408, 1989.

[9] Albert Einstein and Emil Warburg. *Die Relativitätstheorie*. Springer, 1911.

[10] Davide Gerosa, Geraint Pratten, and Alberto Vecchio. Gravitational-wave selection effects using neural-network classifiers. *Physical Review D*, 102(10):103020, 2020.

[11] Ryan Magee, Maximiliano Isi, Ethan Payne, Katerina Chatziioannou, Will M. Farr, Geraint Pratten, and Salvatore Vitale. Impact of selection biases on tests of general relativity with gravitational-wave inspirals. *Phys. Rev. D*, 109(2):023014, 2024.

[12] Ethan Payne, Maximiliano Isi, Katerina Chatziioannou, and Will M. Farr. Fortifying gravitational-wave tests of general relativity against astrophysical assumptions. *Phys. Rev. D*, 108(12):124060, 2023.

[13] William H Press and Kip S Thorne. Gravitational-wave astronomy. *Annual Review of Astronomy and Astrophysics*, 10(1):335–374, 1972.

[14] Dimitrios Psaltis, Colm Talbot, Ethan Payne, and Ilya Mandel. Probing the black hole metric: Black hole shadows and binary black-hole inspirals. *Physical Review D*, 103(10):104036, 2021.

[15] Hideyuki Tagoshi, Masaru Shibata, Takahiro Tanaka, and Misao Sasaki. Post-newtonian expansion of gravitational waves from a particle in circular orbit around a rotating black hole: Up to o (v 8) beyond the quadrupole formula. *Physical Review D*, 54(2):1439, 1996.

[16] Eric Thrane and Colm Talbot. An introduction to bayesian inference in gravitational-wave astronomy: parameter estimation, model selection, and hierarchical models. *Publications of the Astronomical Society of Australia*, 36:e010, 2019.