
The impact of
astrophysical population
model choices on
post-Newtonian deviation
tests of general relativity

Ruby Knudsen, UC Berkeley

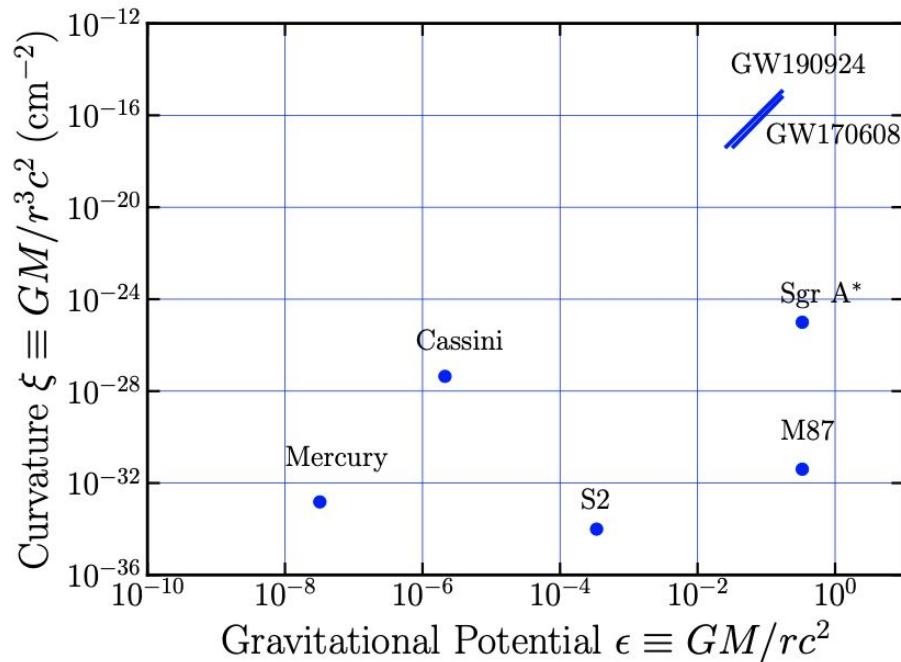
Mentor: Ethan Payne

Why we test GR with GWs

Einstein's theory of general relativity (GR)

Gravitational waves (GWs) as a consequence

Can use GWs to test GR → **the best test**



From Psaltis et al. (2021)

Testing GR (TGR) with individual events

Post-Newtonian tests

Lowest order: quadrupole formula

1. convert GW inspiral data into frequency domain
2. create PN description of inspiral
3. make modifications to PN phase evolution parameters

Bayes' Theorem

Learn about Universe from data by inferring the distribution probable for parameters of events analyzed

$$p(\theta|d) = \frac{p(d|\theta)\pi(\theta)}{\mathcal{Z}}$$

Diagram illustrating Bayes' Theorem with annotations:

- $p(\theta|d)$ is labeled **posterior** with a blue arrow pointing to it.
- $p(d|\theta)$ is labeled **likelihood** with a blue arrow pointing to it.
- $\pi(\theta)$ is labeled **prior** with a blue arrow pointing to it.
- \mathcal{Z} is labeled **evidence** with a blue arrow pointing to it.

Hierarchically testing GR

Hierarchical (inferring the population parameters from individual events) **TGR relies on inference of the astrophysical population**

The diagram shows the equation for the population likelihood $p(\{d\}|\Lambda)$ with several annotations and arrows:

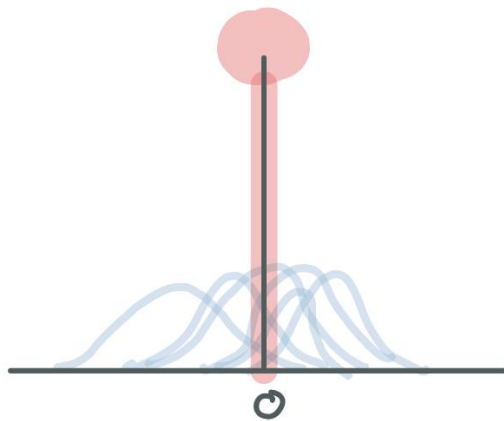
- population likelihood**: points to $p(\{d\}|\Lambda)$
- collection of N observations**: points to $\{d\}$
- hyper-parameters**: points to Λ
- detection fraction**: points to $\xi(\Lambda)$
- likelihood for each individual event**: points to $p(d_i|\theta_i)$
- prior for each event**: points to $\pi(\theta_i|\Lambda)$

$$p(\{d\}|\Lambda) = \frac{1}{\xi(\Lambda)^N} \prod_{i=1}^N \int d\theta_i p(d_i|\theta_i) \pi(\theta_i|\Lambda)$$

GR deviation model

Gaussian model for GR deviations \Rightarrow
(mean, SD) = (0,0)
implies **consistency with GR**

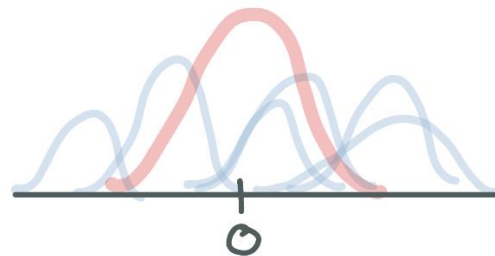
GR is TRUE



$$\delta\varphi = 0$$

GR is FALSE

$$\sigma \neq 0$$



$$\delta\varphi \neq 0$$

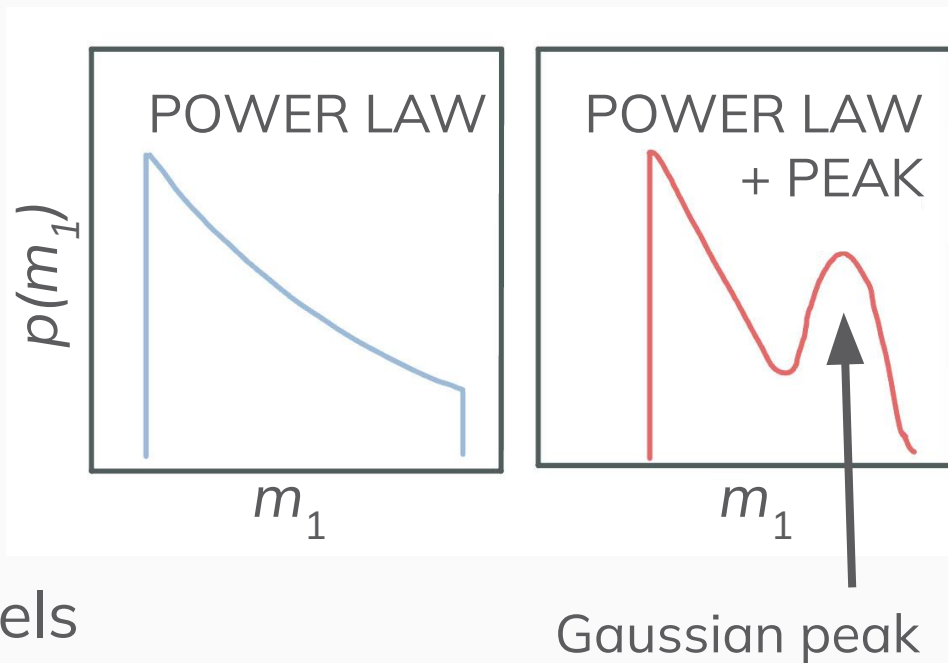
Primary mass population models

POWER LAW and
POWER LAW + PEAK

Inject a certain model

Recover with a certain
model

Some models can be
described by other models





Incorrect astrophysical
population model \Rightarrow biases in
supposed deviations from GR

Why is this important?

Assuming no astrophysical
population model is an
incorrect population model

Can implicitly assume an
astrophysically impossible
population model

I injected...

POWER LAW, POWER LAW +
PEAK

$\alpha = 2$

$\beta = 0$

$\mu = 0$

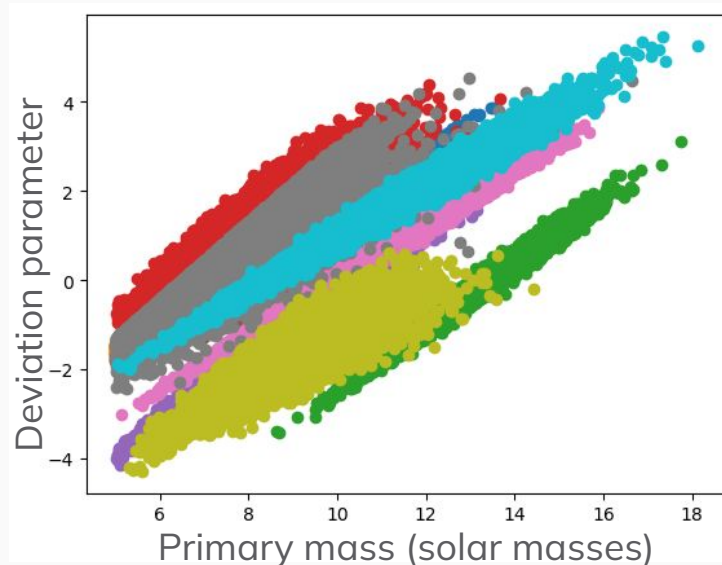
$\sigma = 0, 0.25, 0.5$

for only POWER LAW + PEAK:

Peak_frac = 0.1

Peak_mu = 35

Peak_sigma = 7



$N = 100$ events

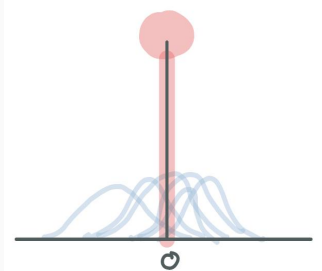
GWFish to simulate posteriors from
3rd generation GW detectors

Deviation parameter correlated to
mass

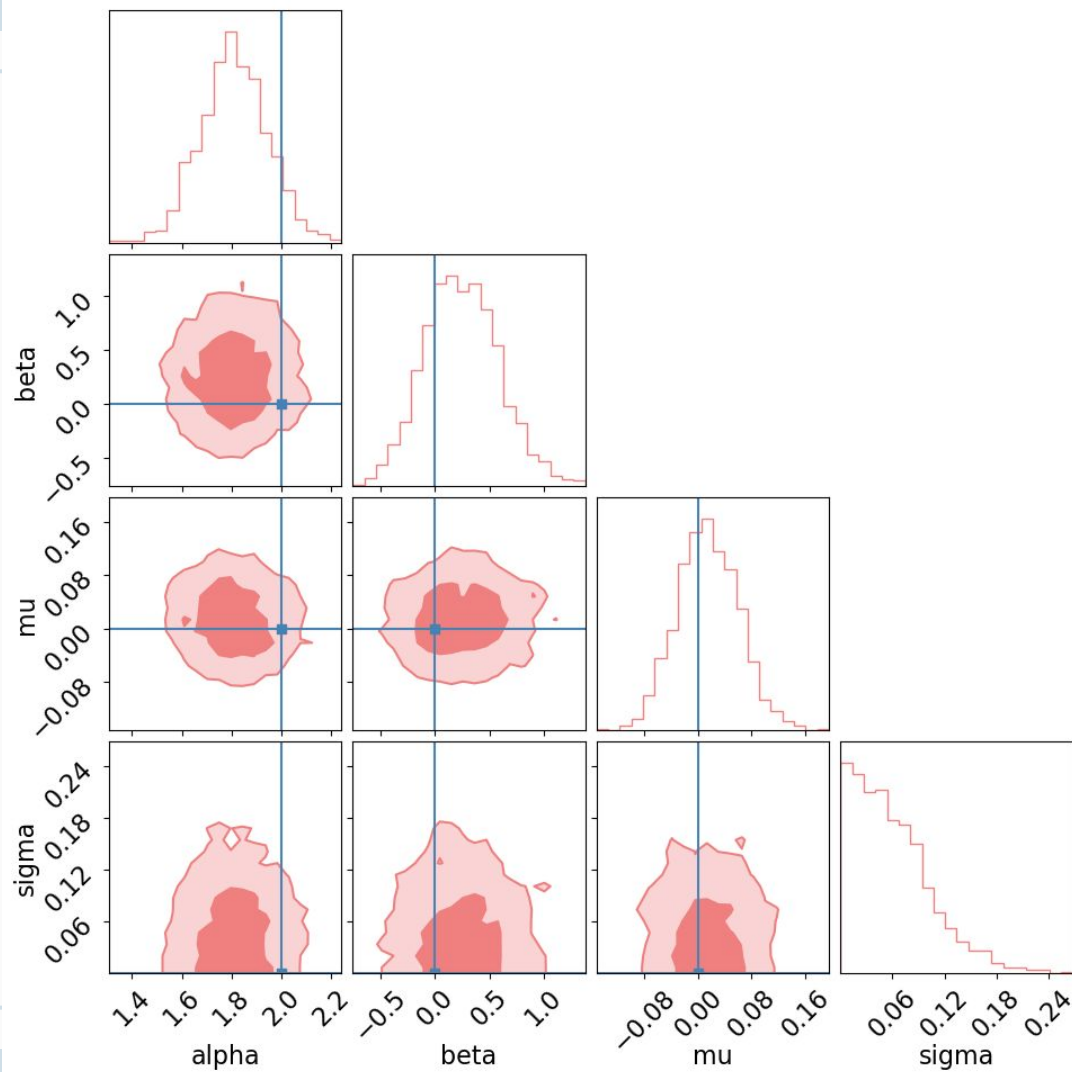
Injected:
POWER LAW
Recovered:
POWER LAW

GR is injected to
be **True**

GR is **TRUE**

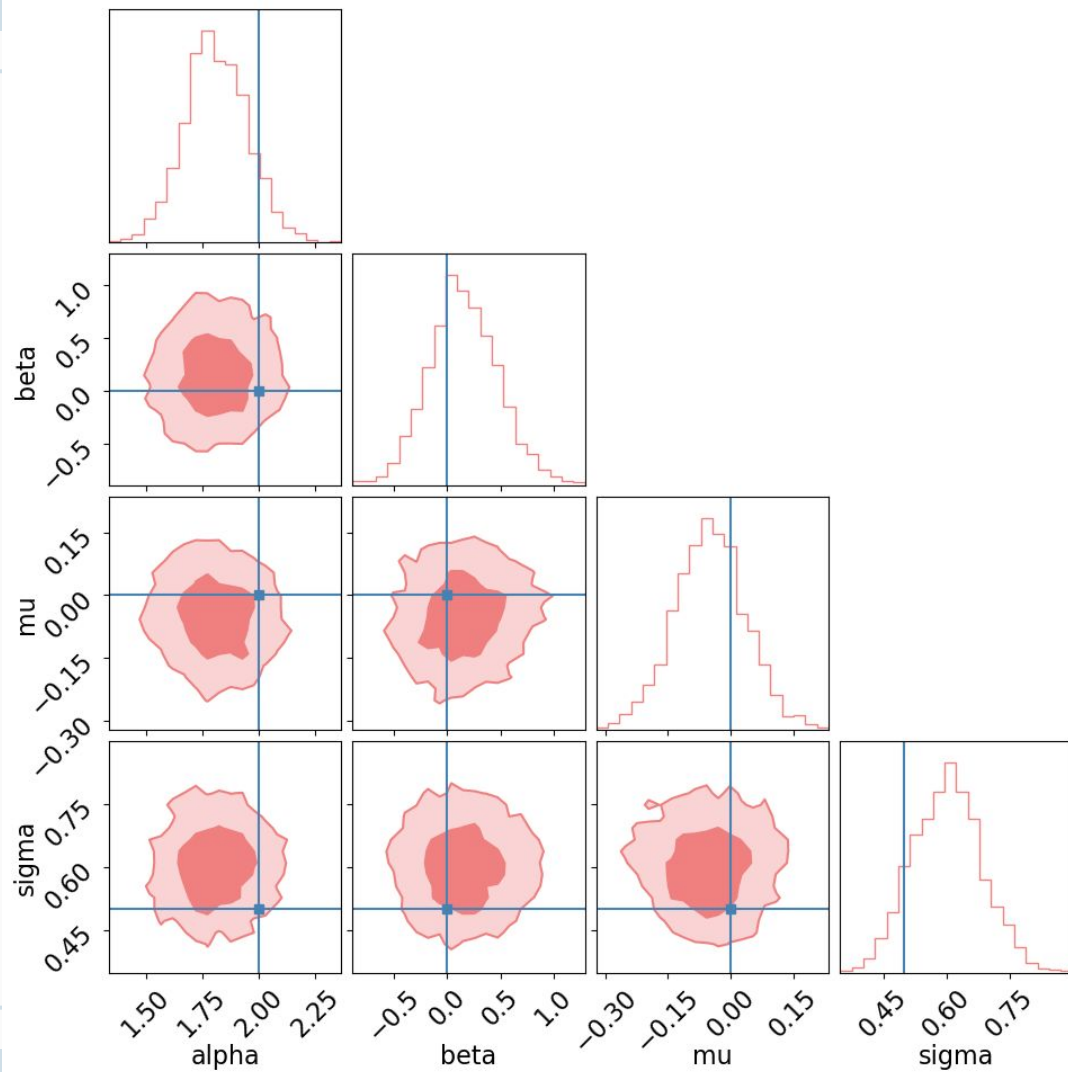
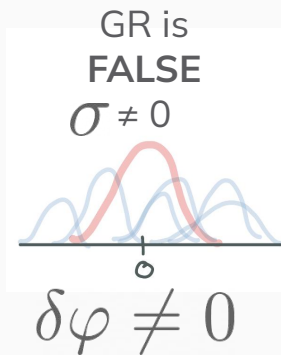


$$\delta\varphi = 0$$



Injected:
POWER LAW
Recovered:
POWER LAW

GR is injected to
be **False** with a
sigma of **0.5**

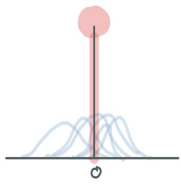


Injected:
POWER LAW +
PEAK

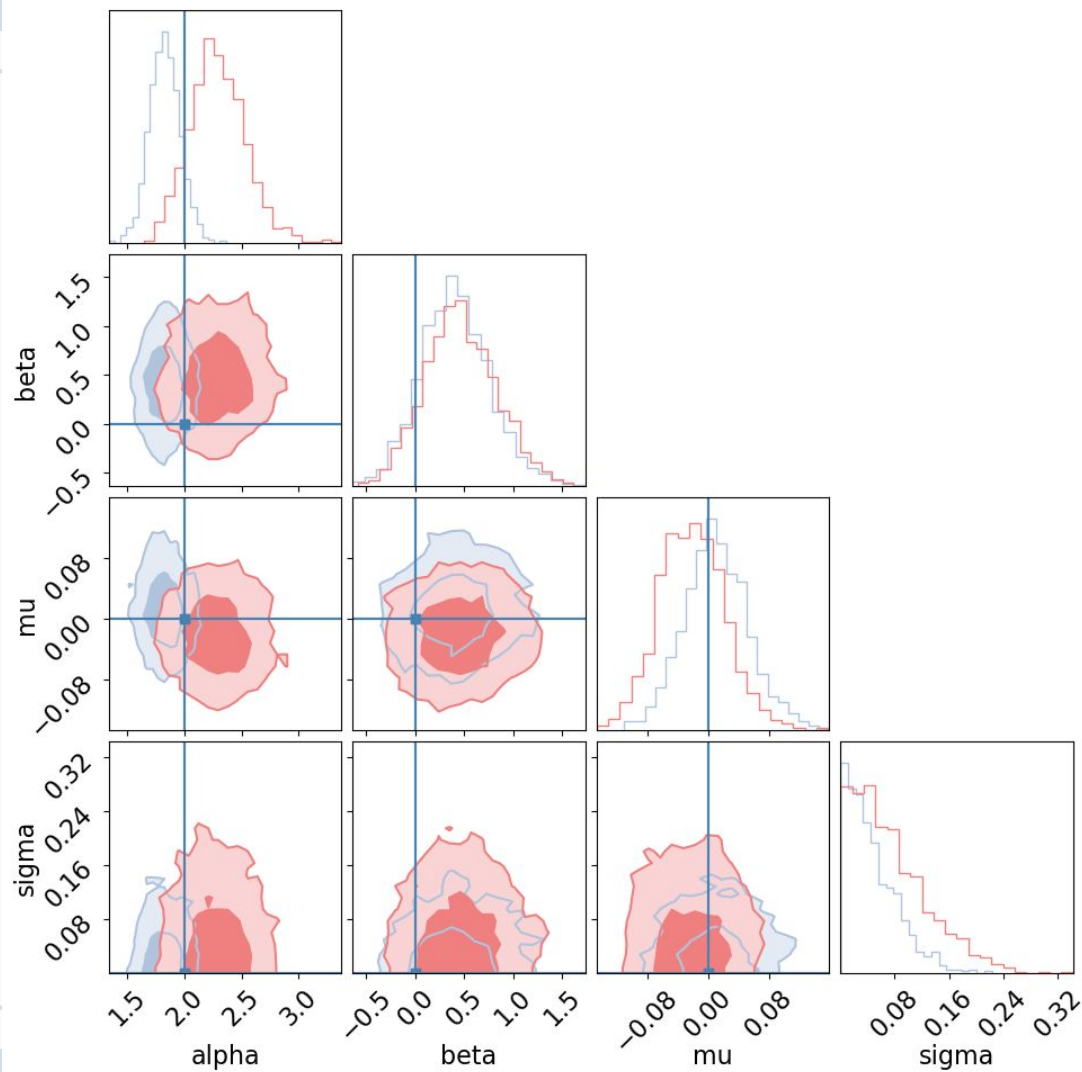
Recovered:
POWER LAW
POWER LAW +
PEAK

GR is injected to
be **True**

GR is **TRUE**



$$\delta\varphi = 0$$

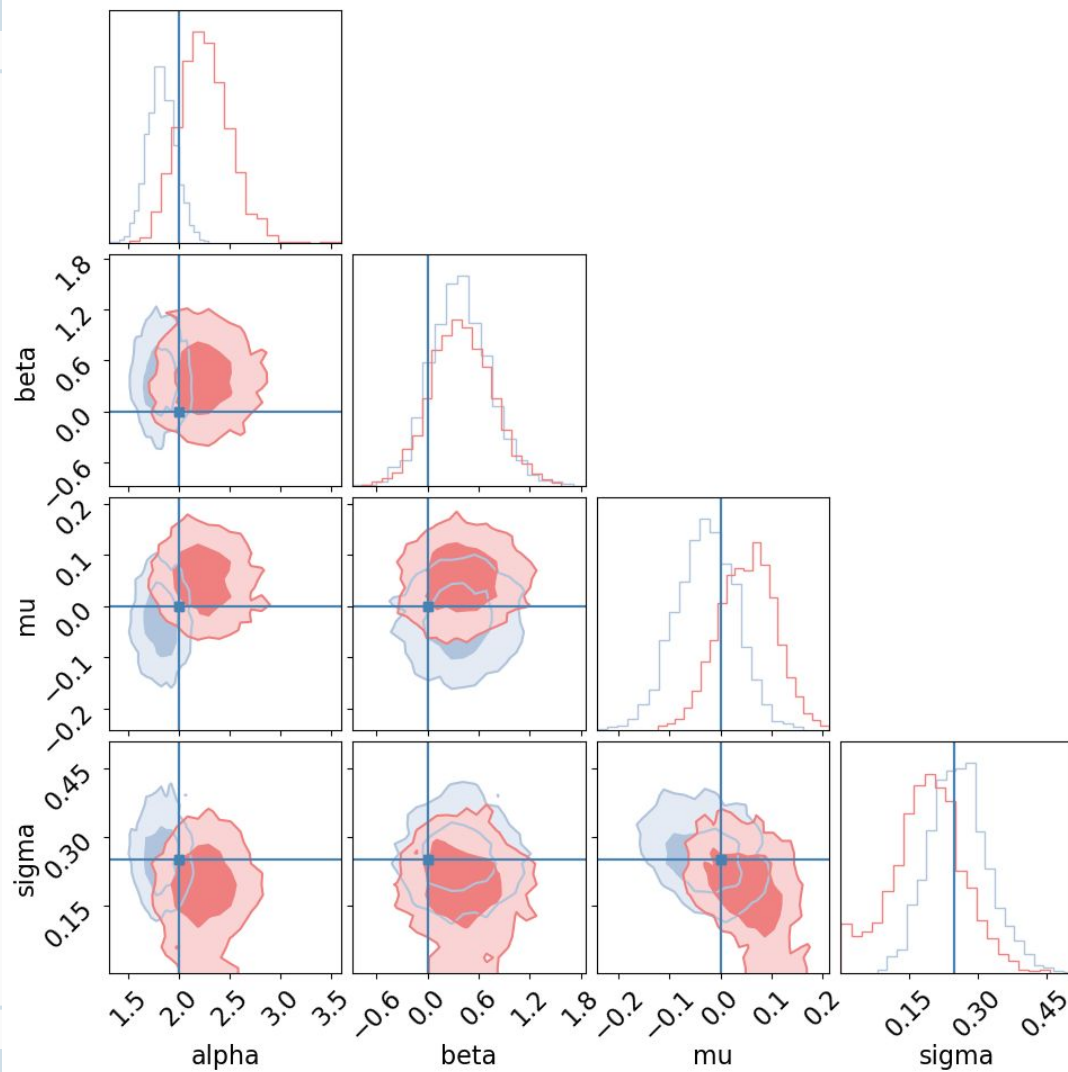
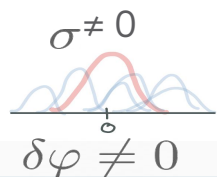


Injected:
POWER LAW +
PEAK

Recovered:
POWER LAW
POWER LAW +
PEAK

GR is injected to
be **False** with a
sigma of **0.25**

GR is **FALSE**



Injected:

POWER LAW +
PEAK

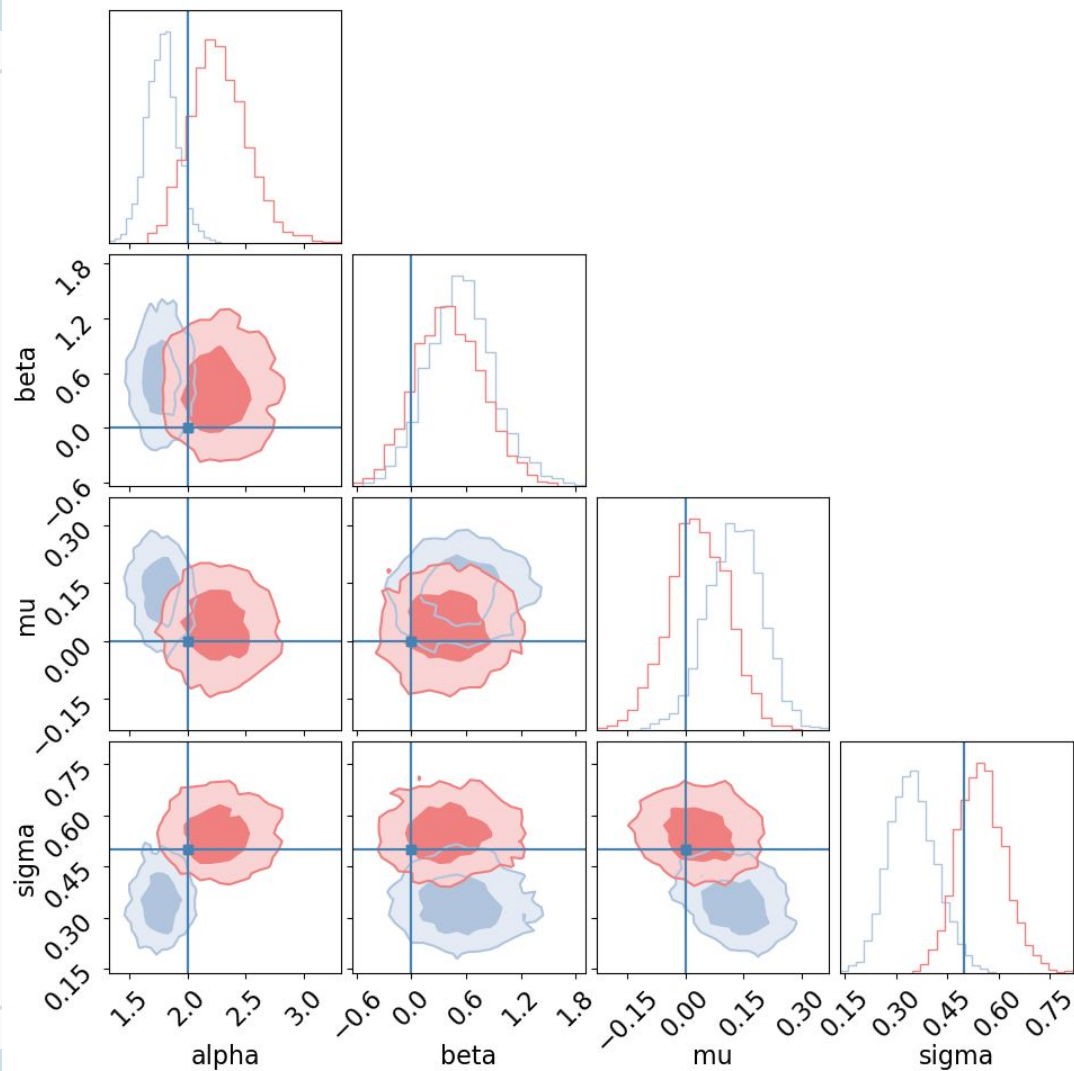
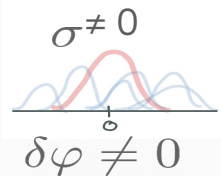
Recovered:

POWER LAW

POWER LAW +
PEAK

GR is injected to
be **False** with a
sigma of **0.5**

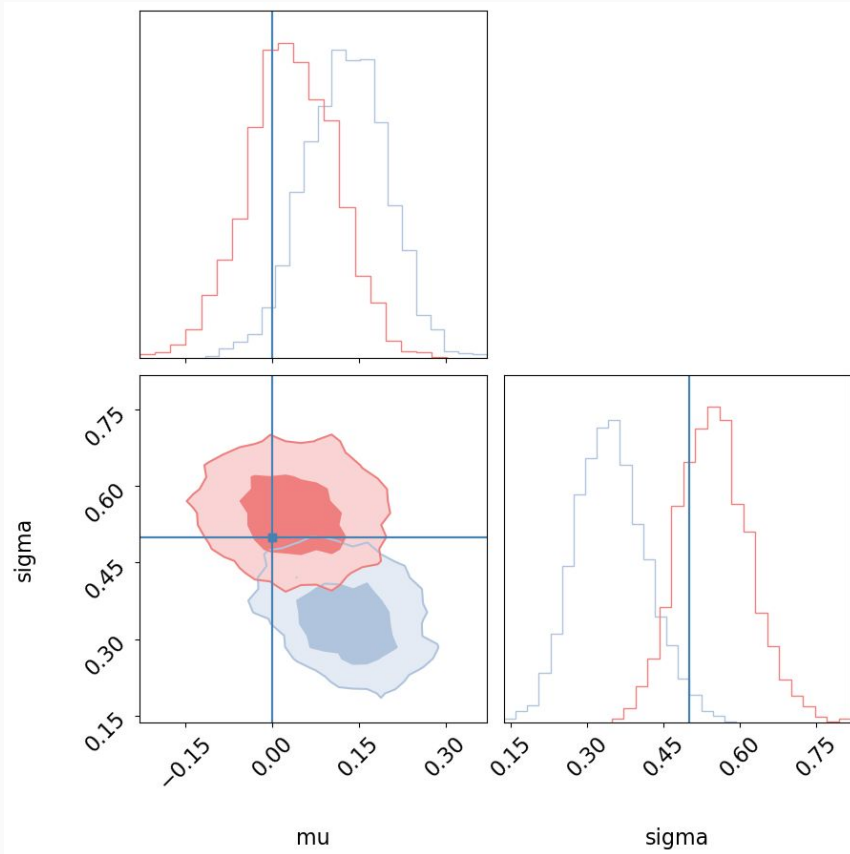
GR is **FALSE**



Implications

Using a misspecified astrophysical population model leads to **inaccurate recoveries of GR deviations**

⇒ it could be plausible that **inconsistencies with GR are masked** by using a misspecified astrophysical population model





Conclusion

Hierarchical TGR (Gaussian model of GR deviations) relies on an astrophysical population model

Generated simulated catalog of 100 events using POWER LAW and POWER LAW + PEAK models, sampled with GWFish

Correlated primary mass and GR deviation parameter to simulate PN deviation test

Markov chain Monte Carlo to recover GR deviation parameters using each model

Acknowledgements



This work was supported by the National Science Foundation Research Experience for Undergraduates (NSF REU) program, the LIGO Laboratory Summer Undergraduate Research Fellowship program (NSF LIGO), and the California Institute of Technology Student-Faculty Programs. I would like to thank my mentor, Ethan Payne, my fellow LIGO SURF-ers, my family and friends, and all of the faculty and mentors at the LIGO SURF program, especially Alan Weinstein and Jonah Kanner.

Caltech

CREDITS: This presentation template was created by [Slidesgo](#), and includes icons by [Flaticon](#), and infographics & images by [Freepik](#)

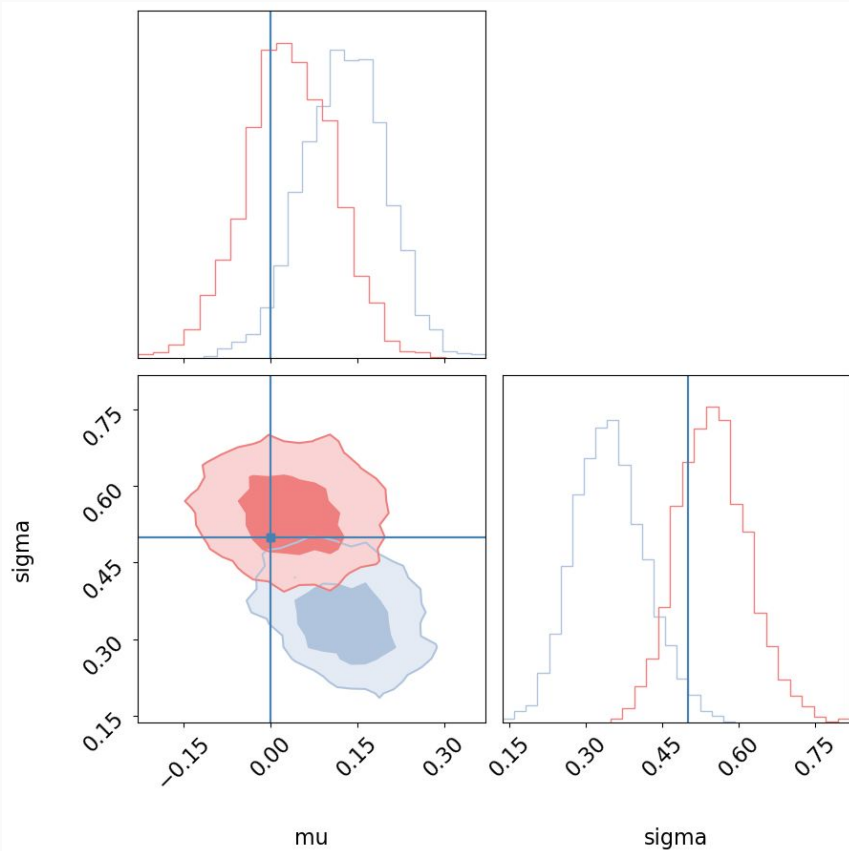




Important conclusions

Using a misspecified astrophysical population model leads to **inaccurate recoveries of GR deviations**

It is possible that **inconsistencies with GR are masked** by using a misspecified astrophysical population model





References

- [1] Junaid Aasi, BP Abbott, Richard Abbott, Thomas Abbott, MR Abernathy, Kendall Ackley, Carl Adams, Thomas Adams, Paolo Addesso, RX Adhikari, et al. Advanced ligo. *Classical and quantum gravity*, 32(7):074001, 2015.
- [2] Benjamin P Abbott, Richard Abbott, TDe Abbott, MR Abernathy, Fausto Acernese, Kendall Ackley, Carl Adams, Thomas Adams, Paolo Addesso, Rana X Adhikari, et al. Observation of gravitational waves from a binary black hole merger. *Physical review letters*, 116(6):061102, 2016.
- [3] R Abbott, TD Abbott, F Acernese, K Ackley, C Adams, N Adhikari, RX Adhikari, VB Adya, C Affeldt, D Agarwal, et al. Population of merging compact binaries inferred using gravitational waves through gwtc-3. *Physical Review X*, 13(1):011048, 2023.
- [4] Rich Abbott, TD Abbott, S Abraham, Fausto Acernese, K Ackley, A Adams, C Adams, RX Adhikari, VB Adya, Christoph Affeldt, et al. Population properties of compact objects from the second ligo–virgo gravitational-wave transient catalog. *The Astrophysical journal letters*, 913(1):L7, 2021.
- [5] Fet al Acernese, M Agathos, K Agatsuma, Damiano Aisa, N Allemandou, Aea Allocca, J Amarni, Pia Astone, G Balestri, G Ballardin, et al. Advanced virgo: a second-generation interferometric gravitational wave detector. *Classical and Quantum Gravity*, 32(2):024001, 2014.
- [6] Matthew R Adams, Neil J Cornish, and Tyson B Littenberg. Astrophysical model selection in gravitational wave astronomy. *Physical Review D*, 86(12):124032, 2012.



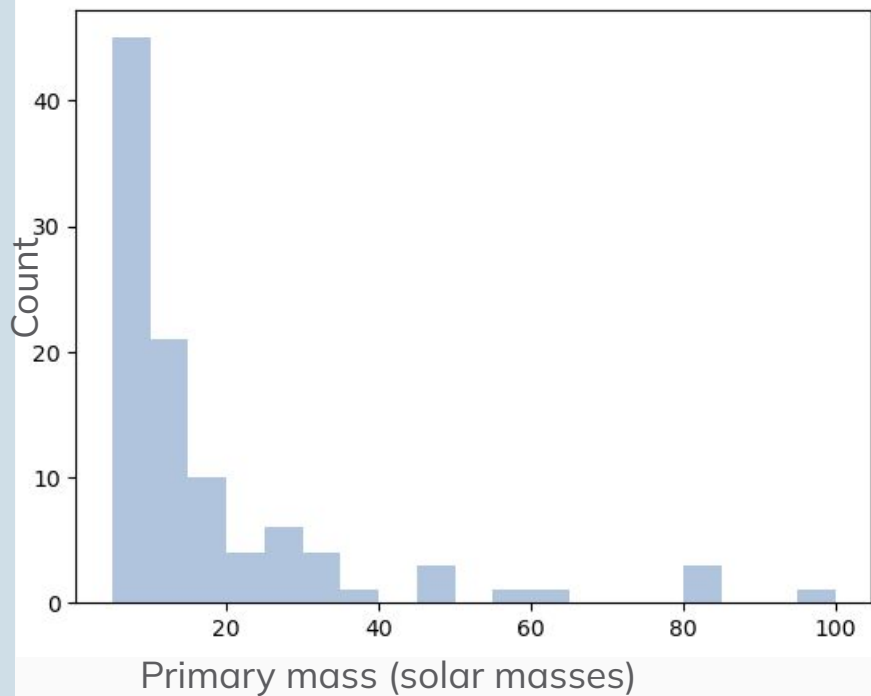
- [7] KG Arun, Bala R Iyer, Bangalore Suryanarayana Sathyaprakash, and Pranesh A Sundararajan. Parameter estimation of inspiralling compact binaries using 3.5 post-newtonian gravitational wave phasing: The nonspinning case. *Physical Review D—Particles, Fields, Gravitation, and Cosmology*, 71(8):084008, 2005.
- [8] Tessa Baker, Dimitrios Psaltis, and Constantinos Skordis. Linking tests of gravity on all scales: from the strong-field regime to cosmology. *The Astrophysical Journal*, 802(1):63, 2015.
- [9] Luc Blanchet and Thibault Damour. Post-newtonian generation of gravitational waves. In *Annales de l'IHP Physique théorique*, volume 50, pages 377–408, 1989.
- [10] Albert Einstein and Emil Warburg. *Die Relativitätstheorie*. Springer, 1911.
- [11] Maya Fishbach, Daniel E Holz, and Will M Farr. Does the black hole merger rate evolve with redshift? *The Astrophysical Journal Letters*, 863(2):L41, 2018.
- [12] Davide Gerosa, Geraint Pratten, and Alberto Vecchio. Gravitational-wave selection effects using neural-network classifiers. *Physical Review D*, 102(10):103020, 2020.
- [13] Maximiliano Isi, Katerina Chatziioannou, and Will M Farr. Hierarchical test of general relativity with gravitational waves. *Physical Review Letters*, 123(12):121101, 2019.
- [14] Ryan Magee, Maximiliano Isi, Ethan Payne, Katerina Chatziioannou, Will M. Farr, Geraint Pratten, and Salvatore Vitale. Impact of selection biases on tests of general relativity with gravitational-wave inspirals. *Phys. Rev. D*, 109(2):023014, 2024.
- [15] Simona Miller, Thomas A Callister, and Will M Farr. The low effective spin of binary black holes and implications for individual gravitational-wave events. *The Astrophysical Journal*, 895(2):128, 2020.



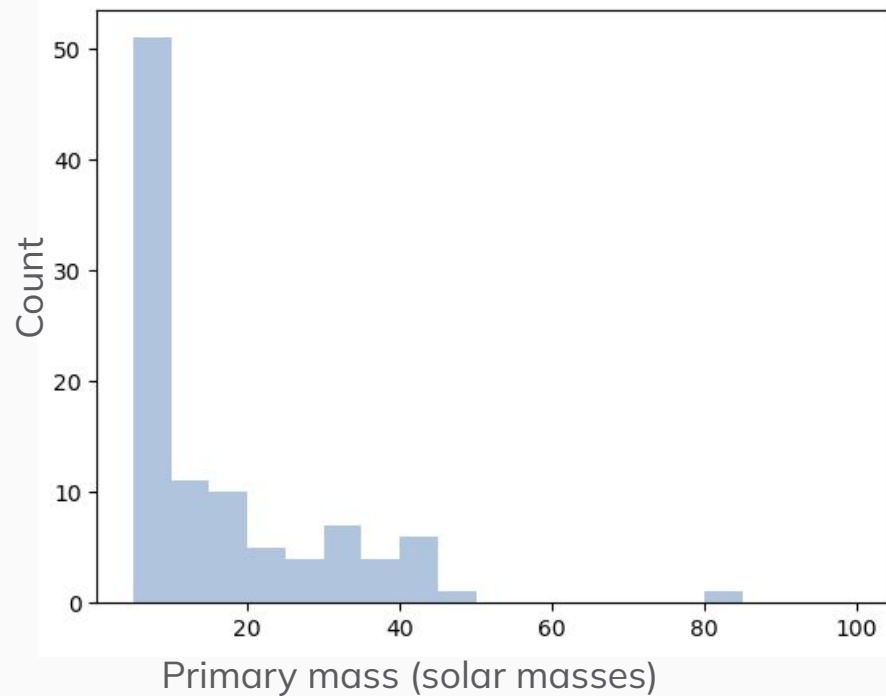
- [16] Ethan Payne, Maximiliano Isi, Katerina Chatziioannou, and Will M. Farr. Fortifying gravitational-wave tests of general relativity against astrophysical assumptions. *Phys. Rev. D*, 108(12):124060, 2023.
- [17] William H Press and Kip S Thorne. Gravitational-wave astronomy. *Annual Review of Astronomy and Astrophysics*, 10(1):335–374, 1972.
- [18] Dimitrios Psaltis, Colm Talbot, Ethan Payne, and Ilya Mandel. Probing the black hole metric: Black hole shadows and binary black-hole inspirals. *Physical Review D*, 103(10):104036, 2021.
- [19] Bangalore Suryanarayana Sathyaprakash and SV Dhurandhar. Choice of filters for the detection of gravitational waves from coalescing binaries. *Physical Review D*, 44(12):3819, 1991.
- [20] Hideyuki Tagoshi, Masaru Shibata, Takahiro Tanaka, and Misao Sasaki. Post-newtonian expansion of gravitational waves from a particle in circular orbit around a rotating black hole: Up to $\mathcal{O}(v^8)$ beyond the quadrupole formula. *Physical Review D*, 54(2):1439, 1996.
- [21] Colm Talbot and Eric Thrane. Determining the population properties of spinning black holes. *Physical Review D*, 96(2):023012, 2017.
- [22] Colm Talbot and Eric Thrane. Measuring the binary black hole mass spectrum with an astrophysically motivated parameterization. *The Astrophysical Journal*, 856(2):173, 2018.
- [23] Eric Thrane and Colm Talbot. An introduction to bayesian inference in gravitational-wave astronomy: parameter estimation, model selection, and hierarchical models. *Publications of the Astronomical Society of Australia*, 36:e010, 2019.
- [24] Daniel Wysocki, Jacob Lange, and Richard O’Shaughnessy. Reconstructing phenomenological distributions of compact binaries via gravitational wave observations. *Physical Review D*, 100(4):043012, 2019.



POWER LAW population



POWER LAW + PEAK population

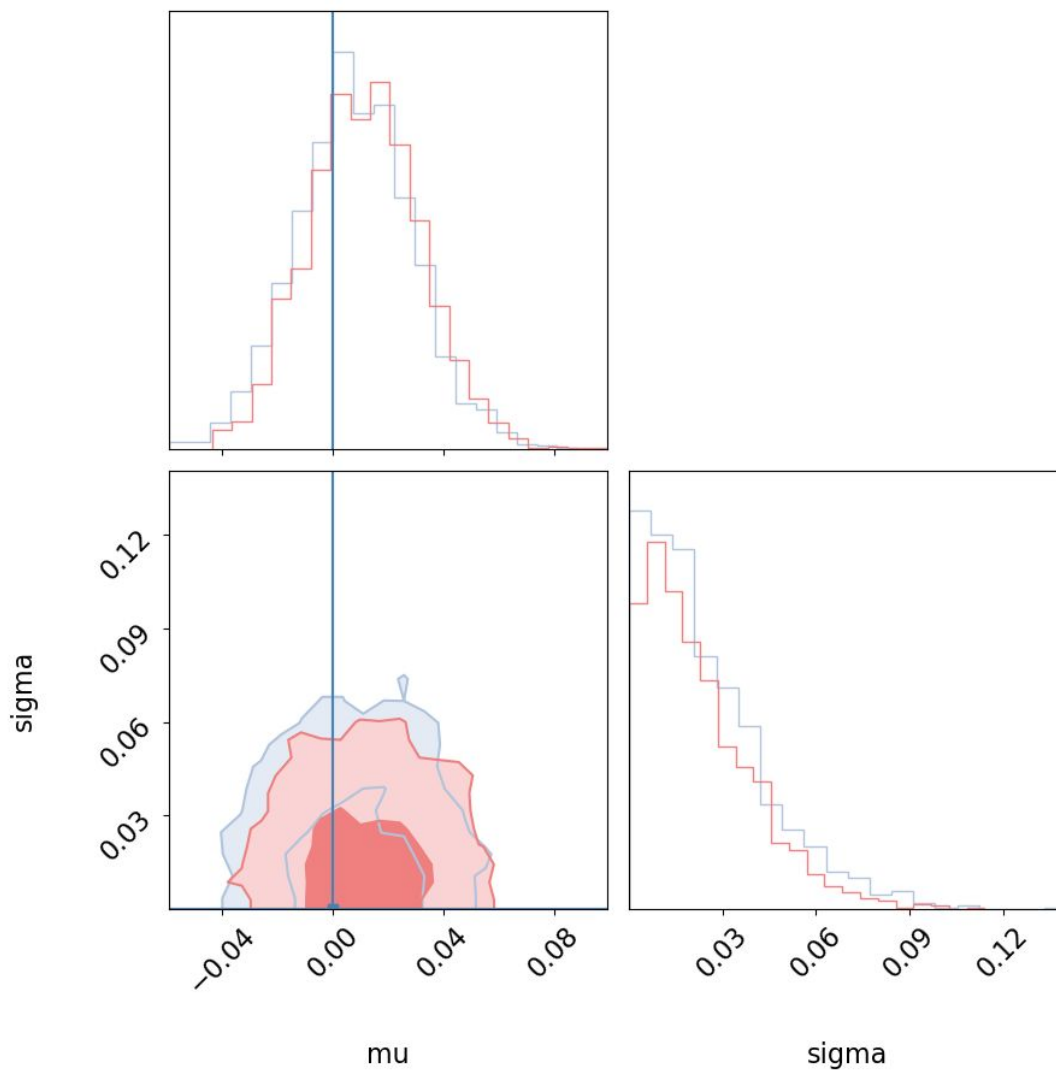


Injected:
POWER
LAW

Recovered:
POWER
LAW
POWER
LAW +
PEAK

GR is
injected to
be **True**

N = 250



Injected:
POWER
LAW

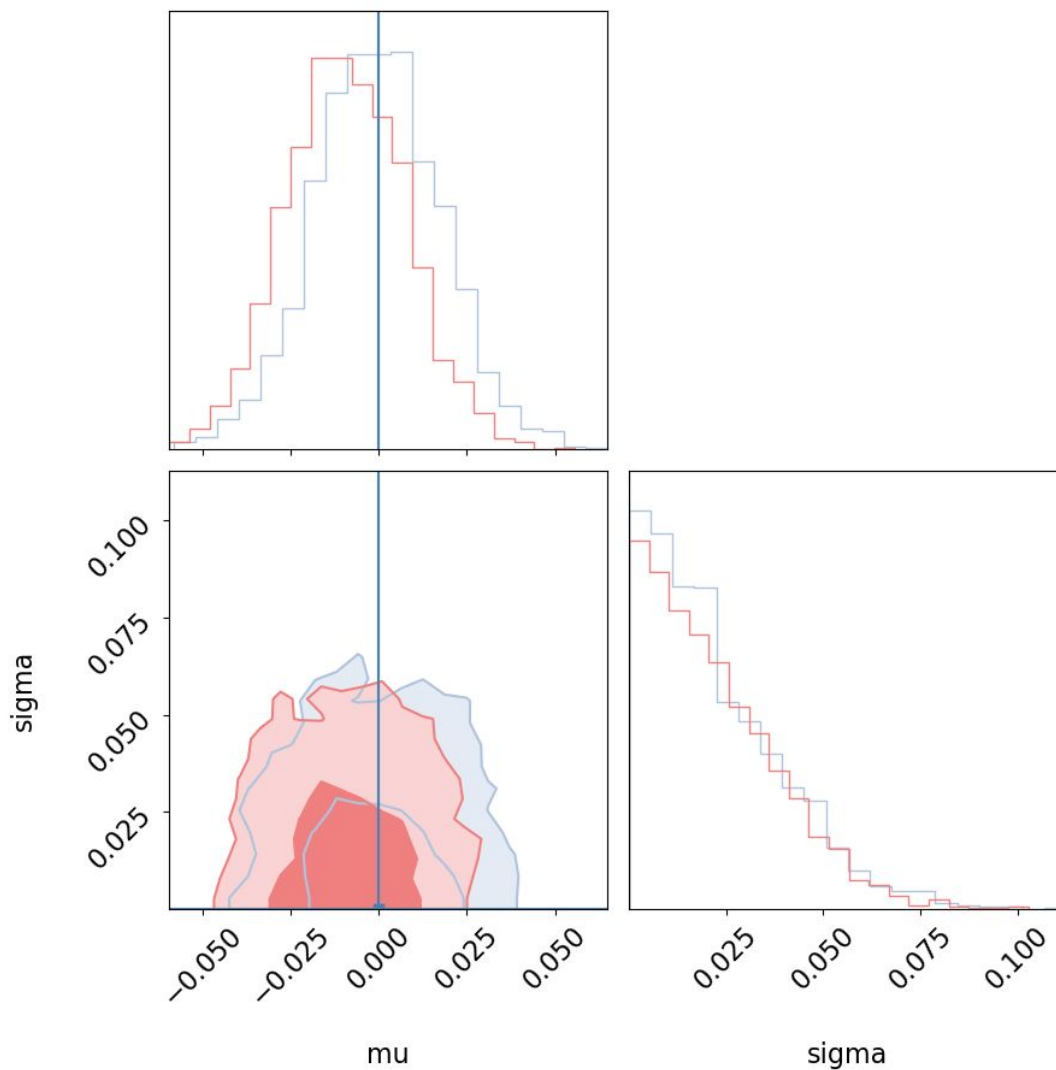
Recovered:

POWER
LAW

POWER
LAW +
PEAK

GR is
injected to
be **True**

$N = 500$



Injected:
GAUSSIAN
Recovered:
POWER
LAW

GR is
injected to
be **True**

$N = 500$

