Modeling Gravitational-wave (GW) waveforms based on Machine learning (ML)

Expanding GW waveforms of generic-spin binary black hole (BBH) mergers to new physical parameter coverage using Deep Neural Network (DNN)

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What does it mean NEW physical parameter coverage?

What is generic-spin?

What GW waveforms?

What parameter are you extending?

Why binary black hole?

Why expanding?

Why Deep Learning?

Why Machine learning?

Choose approximants for training

GWSurrogate

NRSur7dq4

A surrogate model for **generic spin numerical relativity** waveforms

Training Set: 1528 NR simulation from Spectral Einstein Code (SpEC) developed by the SXS collaboration (1464 precessing NR simulations + 64 aligned-spin simulations)

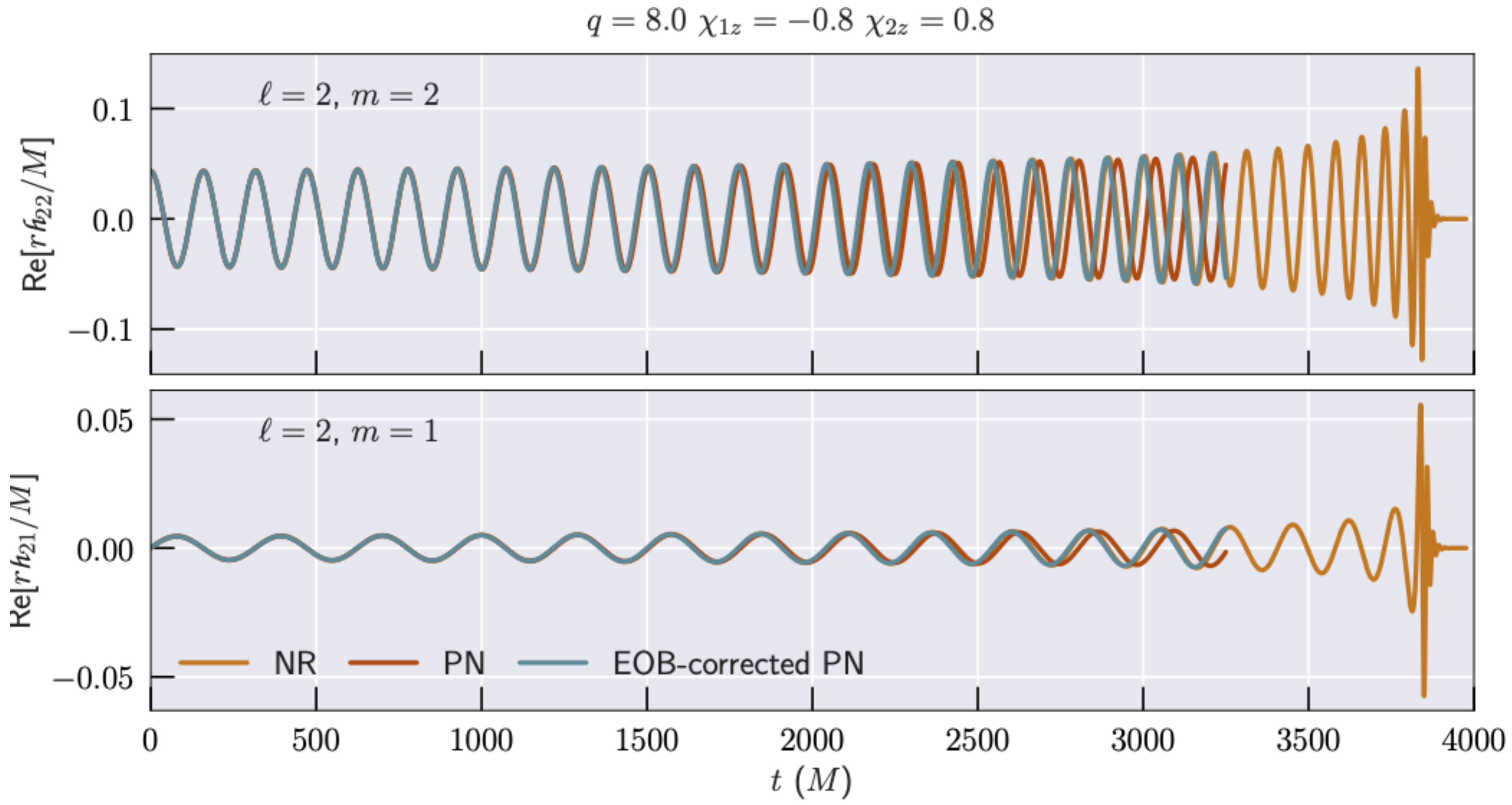
Free parameters:

$\chi_{x1}, \chi_{y1}, \chi_{z1}, \chi_{x2}, \chi_{y2}, \chi_{z2}, q(mass\ ratio\ q \leq 4)$

NRHybSur3dq8

A surrogate model for **hybridized nonprecessing numerical relativity** waveforms, that is valid for the entire LIGO band (starting at 20 Hz).

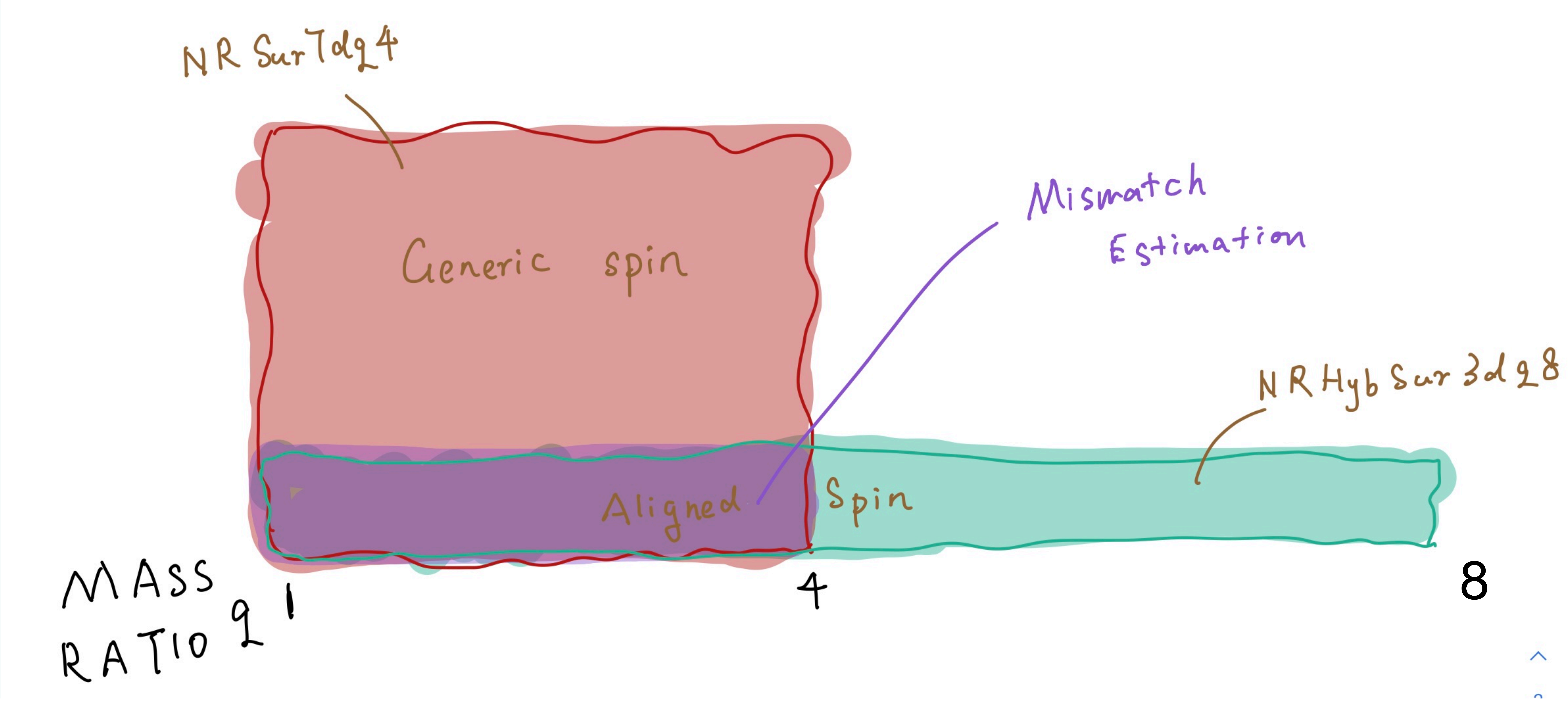
Free parameters: $\chi_{z1}, \chi_{z2}, q(mass\ ratio\ q \leq 8)$



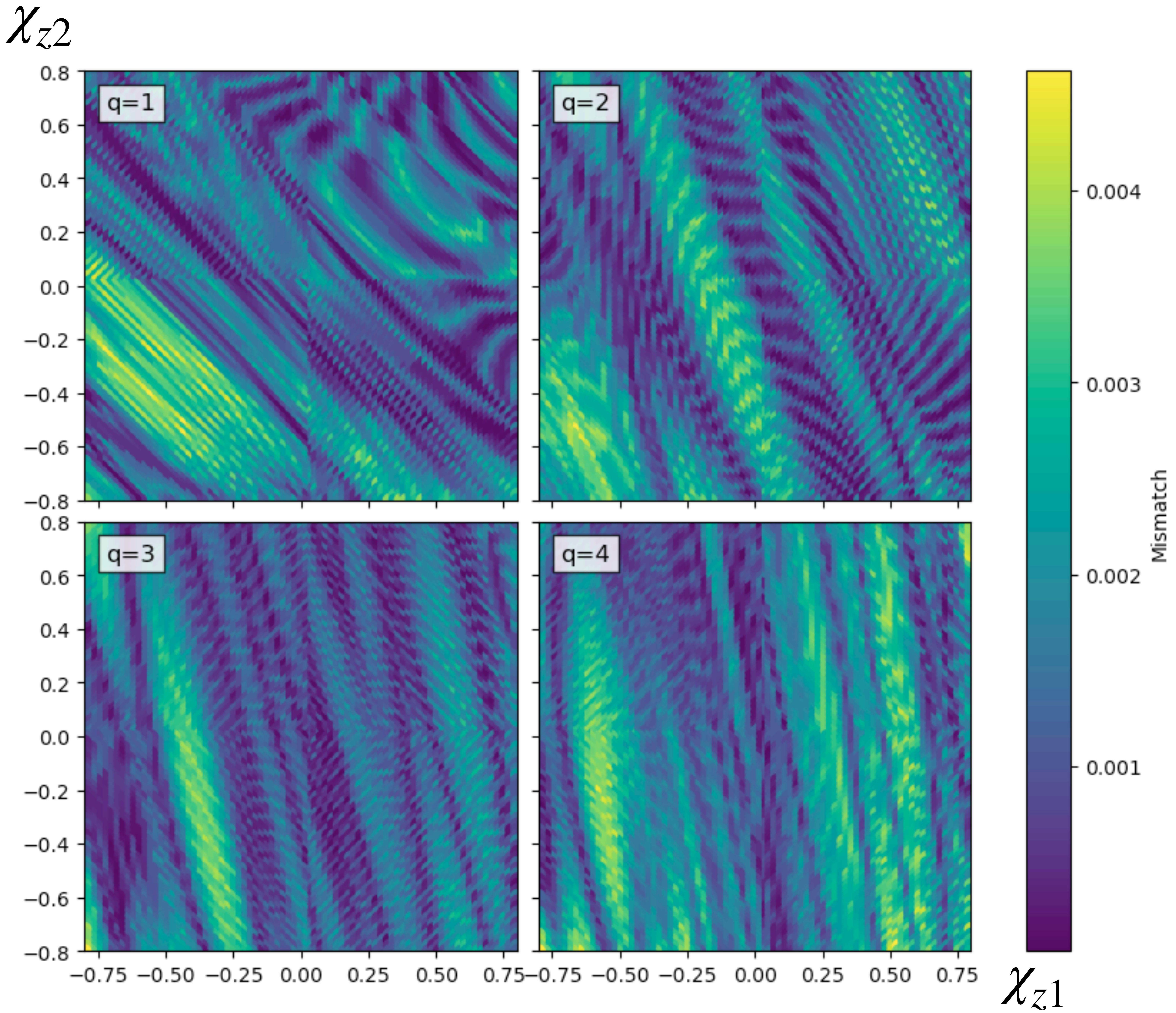
Reference: https://pypi.org/project/gwsurrogate/

Data preparation

Is that reasonable to use two models in one training?



Mismatch Analysis of Aligned-Spin BBHs for Various q



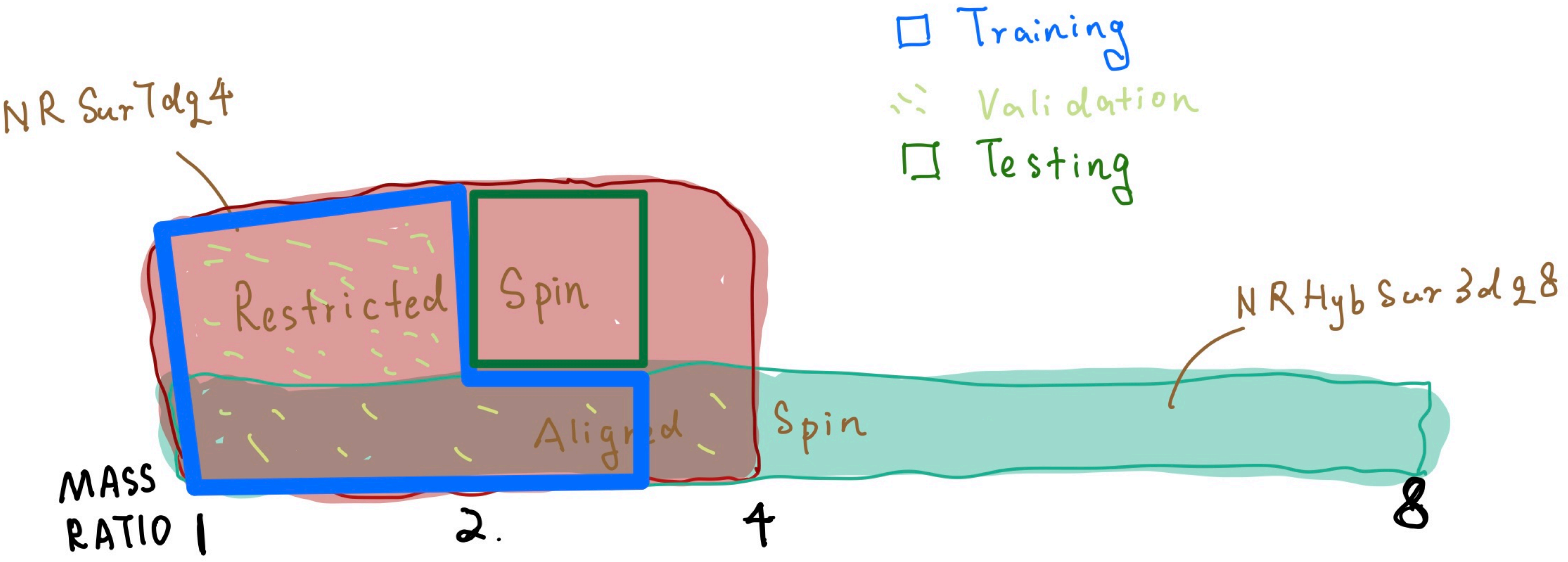
Mean Mismatch:
1.53e-03

Standard Deviation:
8.92e-05

Max Mismatch:
5.52 e-03

Training plan

From big picture to practical first step



Data preparation

Seeding, train/test in/out

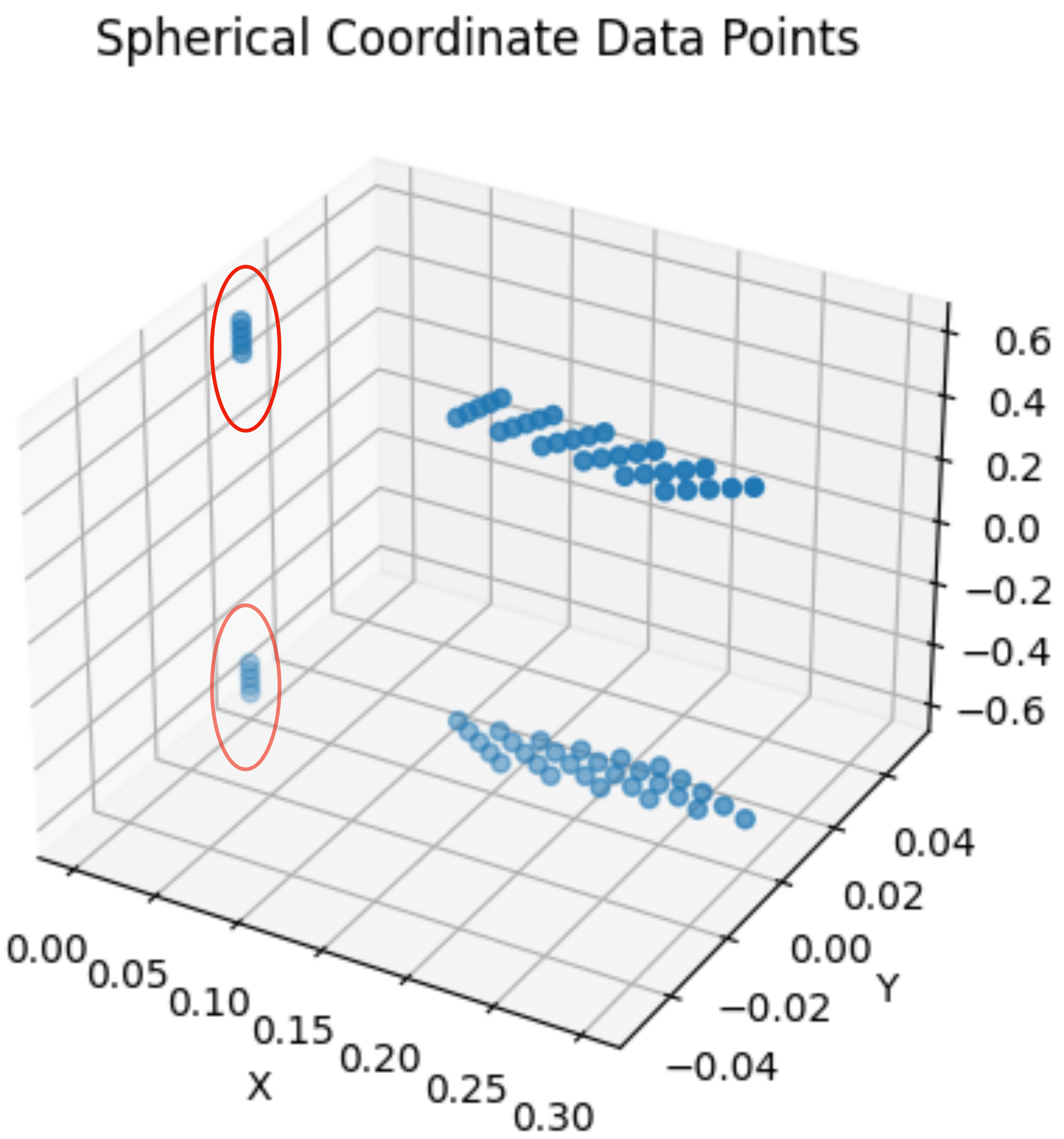
NRSur7dq4: $\pi/12 \leq \theta \leq \pi/6, \theta = 0, \pi$; $0.4 \leq r \leq 0.5$; $q = 1.0, 1.5, 2.0$ $for\ train$ $2.5, 3.0$ $for\ test$, Mode: (2,2)

NRHybSur3dq8: $\theta = 0, \pi$; $0.4 \leq r \leq 0.5$; $q = 1.0, 1.5, 2.0, 2.5, 3.0$, Mode:(2,2)

Train/Test in: $\chi_{x1}, \chi_{y1}, \chi_{z1}, \chi_{x2}, \chi_{y2}, \chi_{z2}, q$

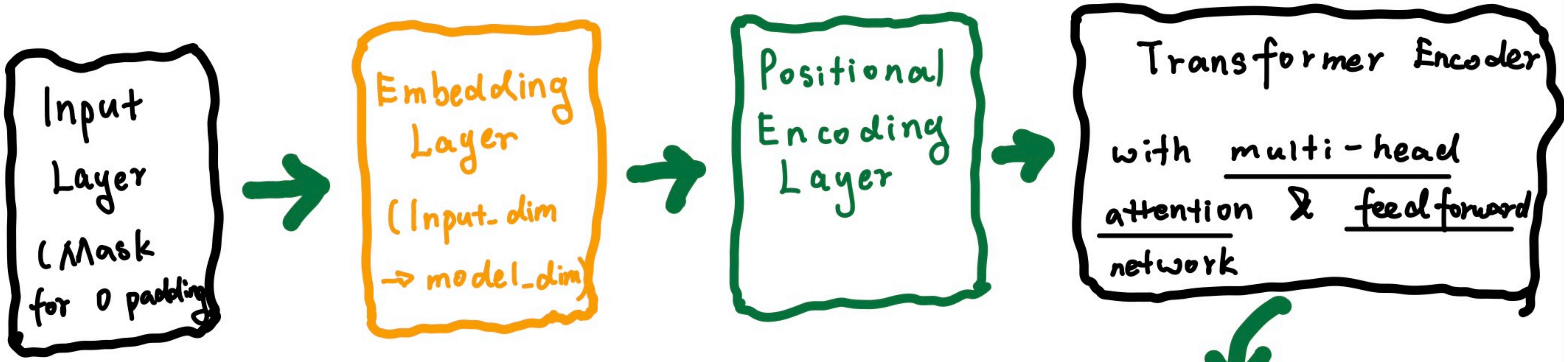
Train/Test out: amplitudes and phases

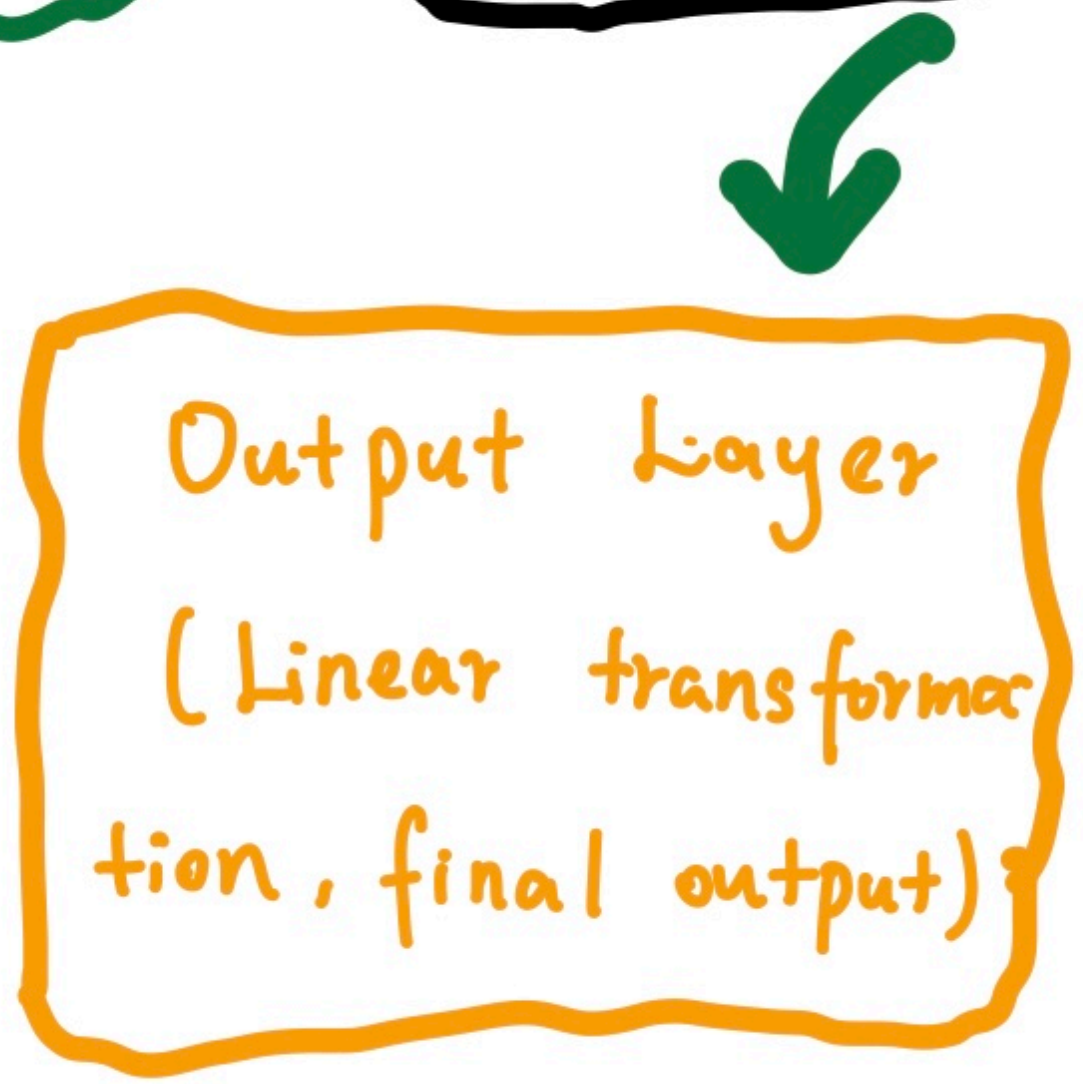
Aligning, Normalization, Zero padding…



Deep Neural Network

An attention-based transformer deep neural network

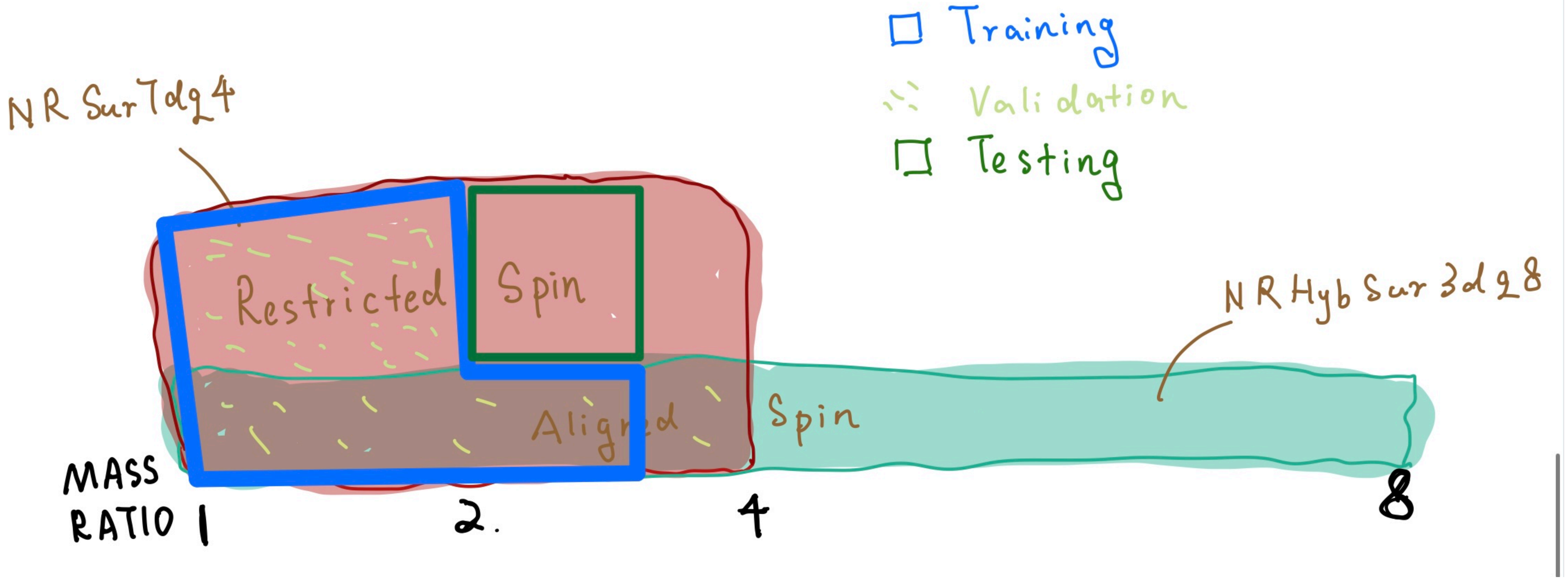


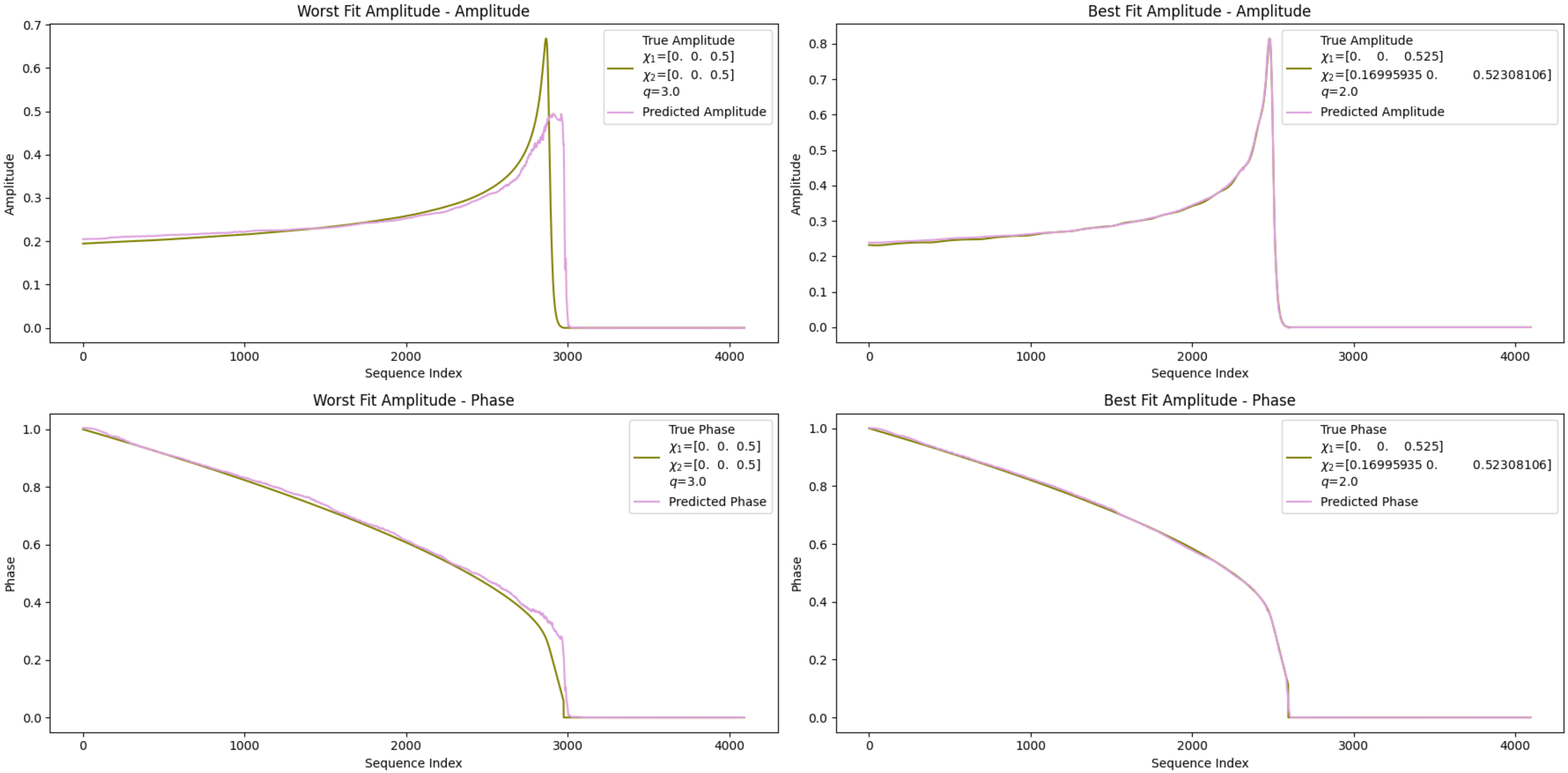


training epoch = 16
batch size = 10
model dim = 32
num of head = 8
num of encoder layer = 4
num of decoder layer = 4
feed forword dim = 128

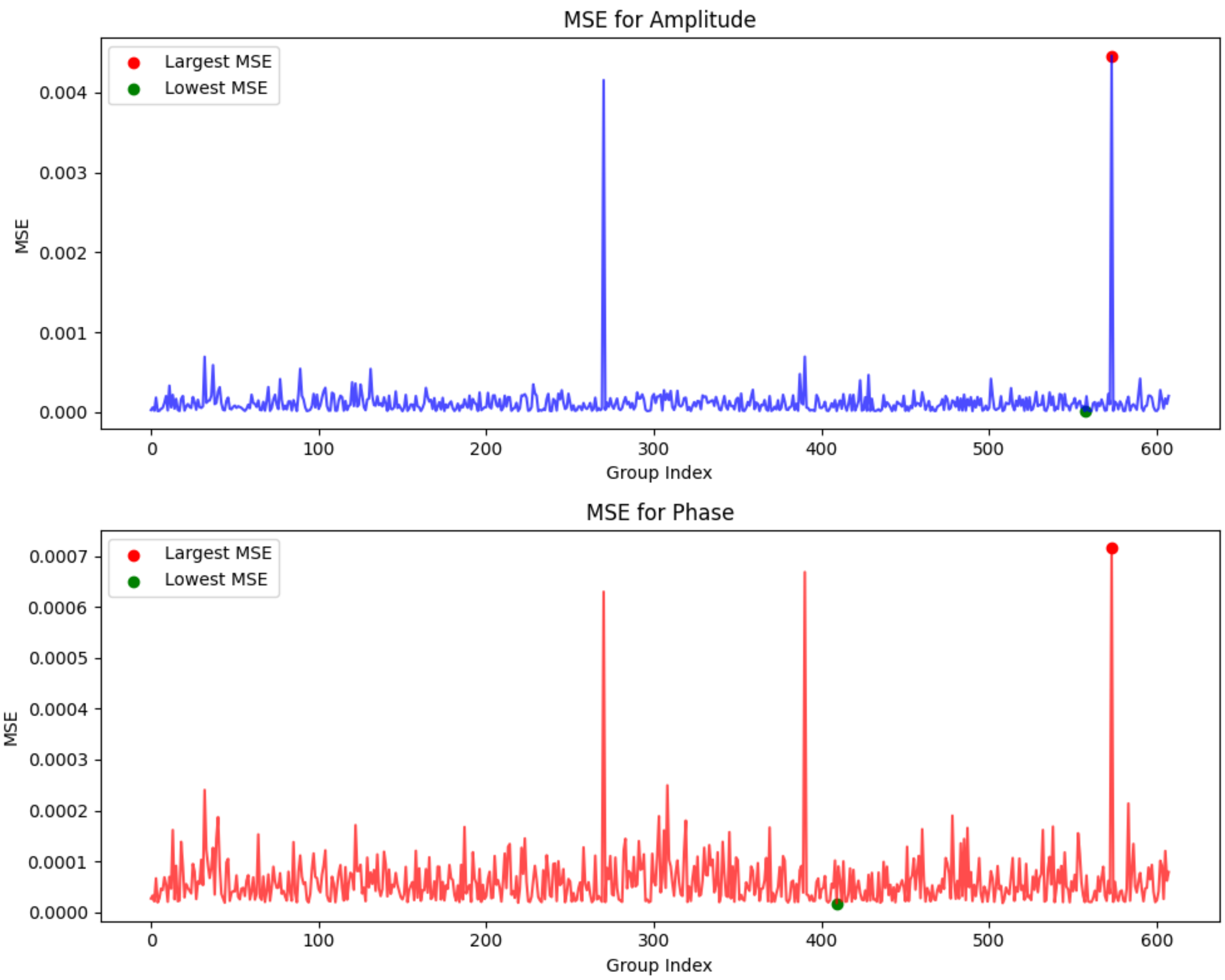
Validation

Interpolation: testing within training physics parameter coverage





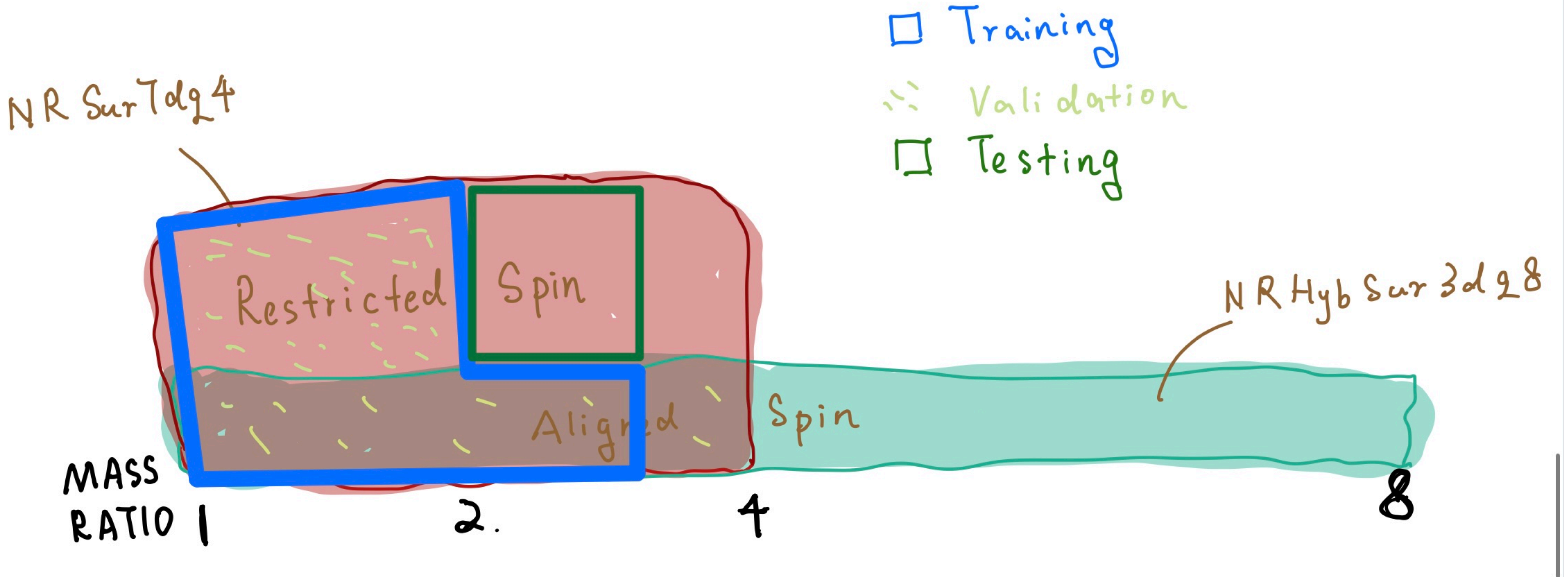
MSE loss for amplitude: 0.0001
MSE loss for phase: 0.0001

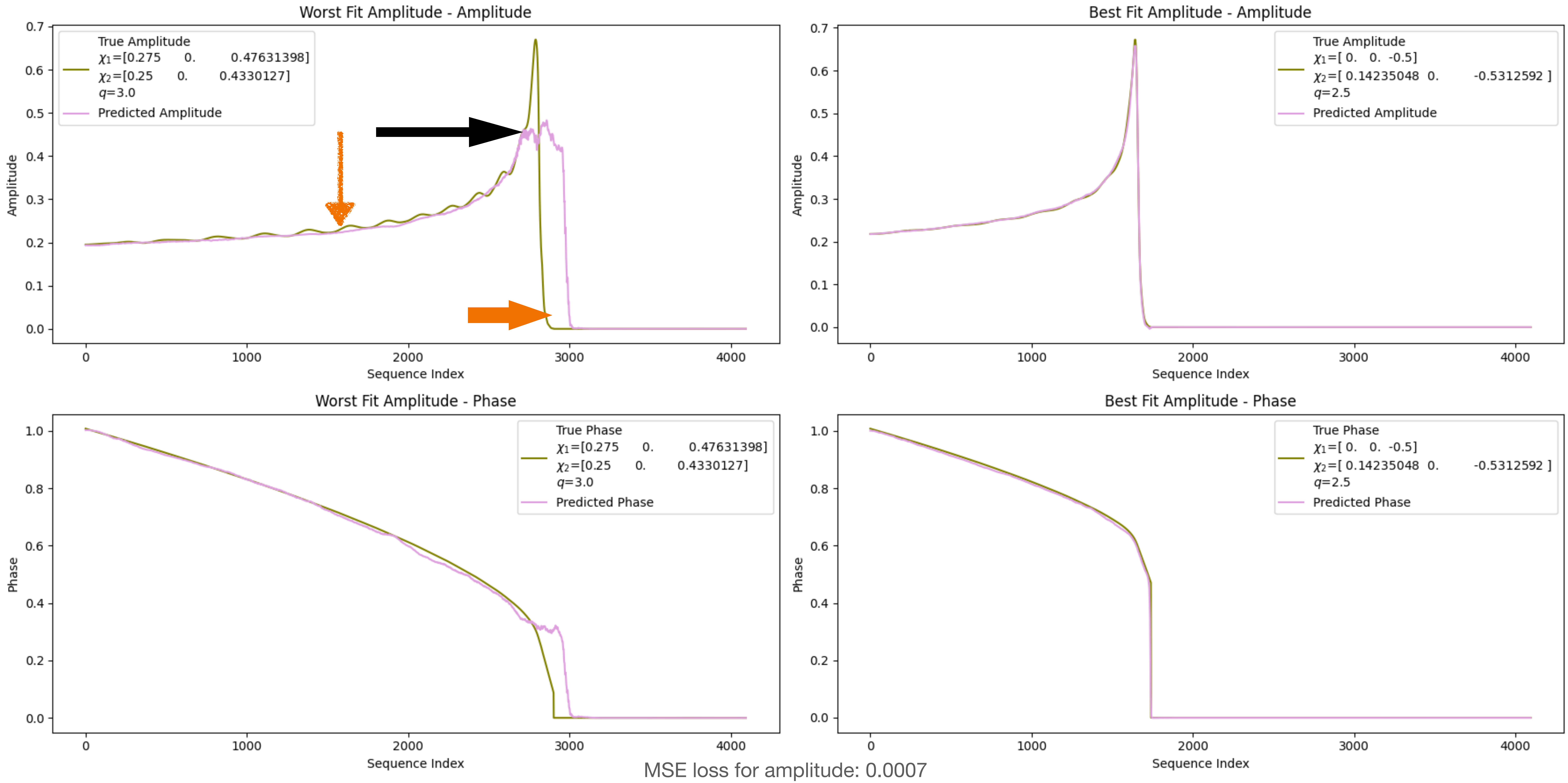


MSE for Amplitude
Largest MSE
Lowest MSE
MSE
0.004
0.003
0.002
0.001
0.000
0
100
200
300
400
500
600
Group Index
MSE for Phase
Largest MSE
Lowest MSE
MSE
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
0
100
200
300
400
500
600
Group Index

Testing

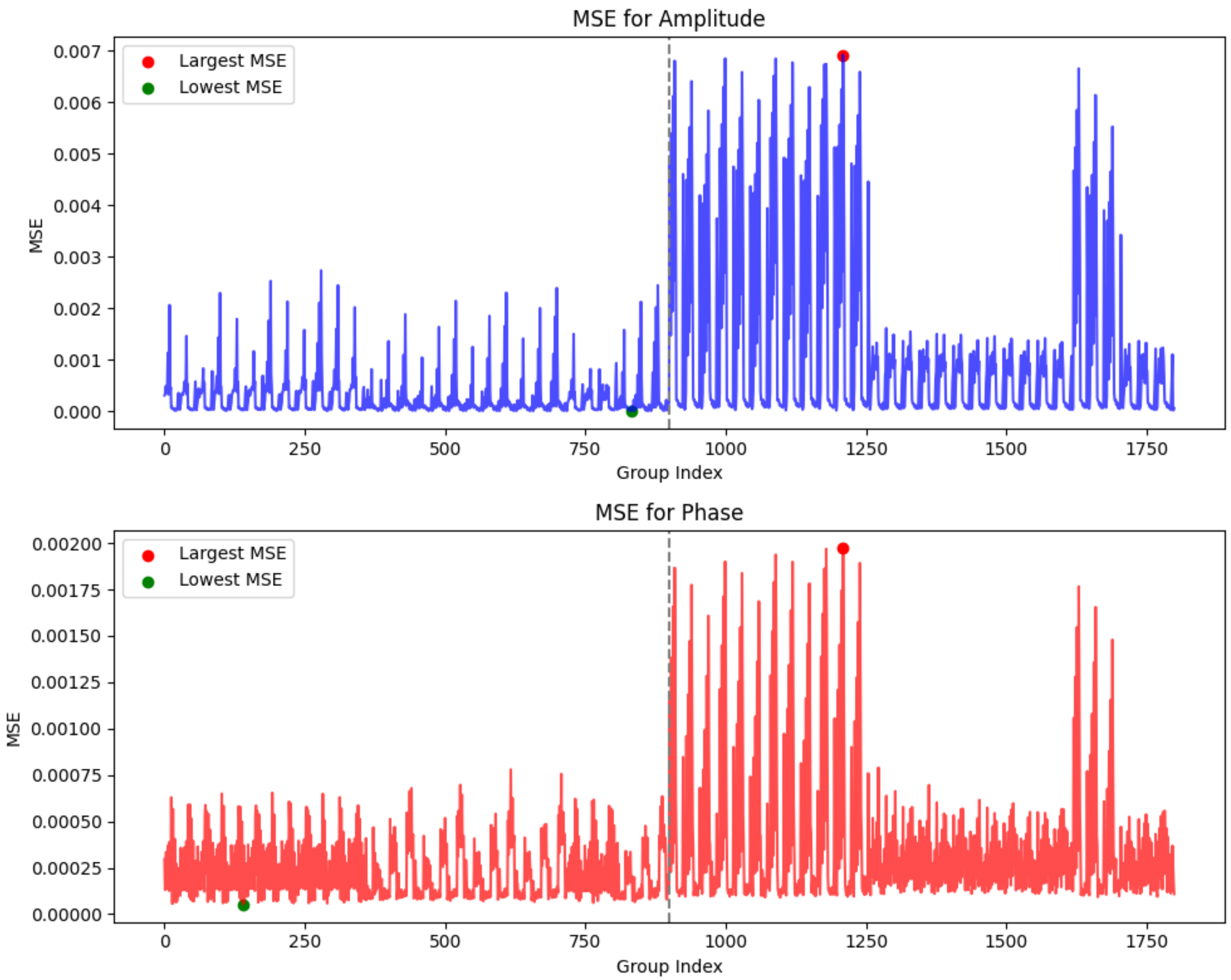
Extrapolation: testing within new physics parameter coverage





MSE loss for amplitude: 0.0007

MSE loss for phase: 0.0003



MSE for Amplitude
Largest MSE
Lowest MSE
MSE
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0.000
0
250
500
750
1000
1250
1500
1750
Group Index
MSE for Phase
Largest MSE
Lowest MSE
MSE
0.00200
0.00175
0.00150
0.00125
0.00100
0.00075
0.00050
0.00025
0.00000
0
250
500
750
1000
1250
1500
1750
Group Index

What is the next step?

Computer scientist VS Physicist

Computer Science

Improve data preparation and NN architecture

Fine-tune hyperparameter

Switch study strategy: predict length of waveforms/ merging moment…

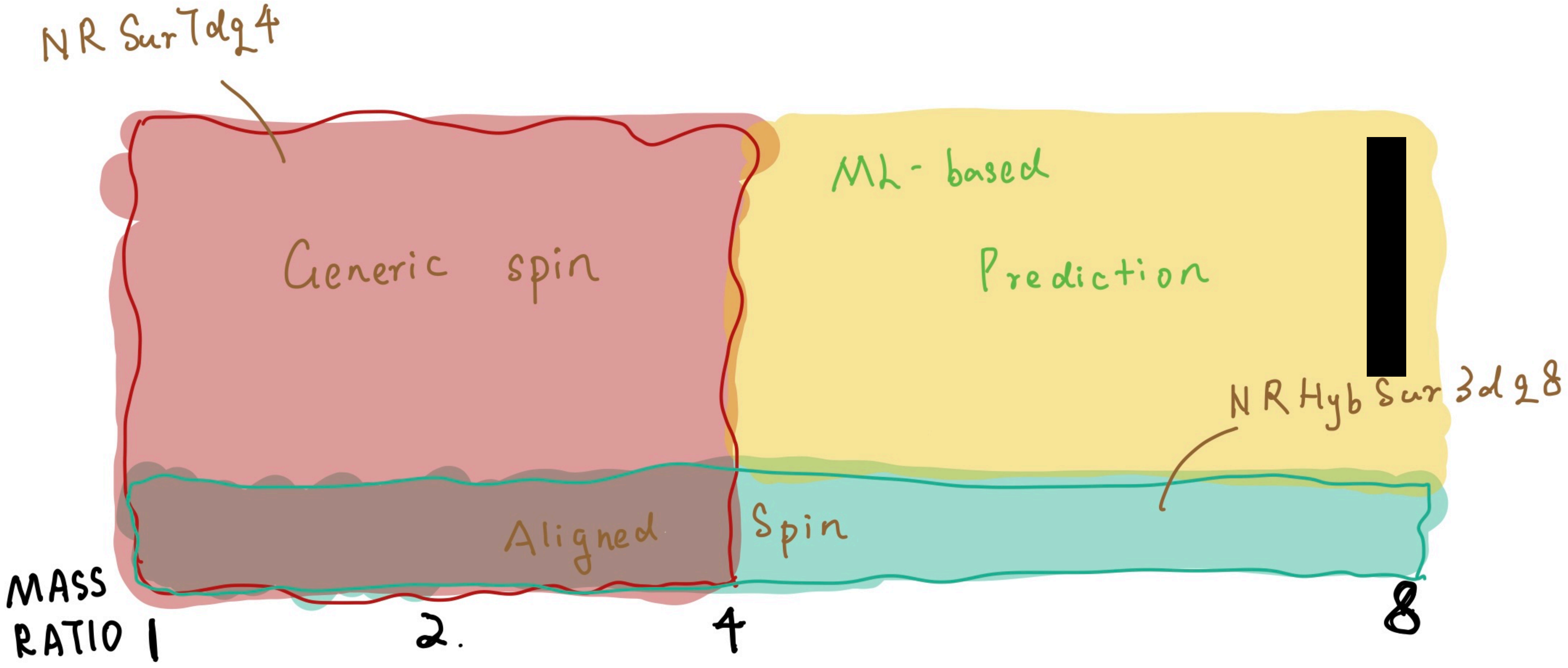
When data become larger, find the proper way for training

Physics

Include total mass as a new parameter

Proper step to take

Using mismatch to evaluate test/validation result



Model Binary GW waveforms by Machine Learning

Machine learning architecture

Mismatch:

In order to estimate the difference between two complexified waveforms, h_1 and h_2, we use:

$$MM = 1 - \max_{t_{\min} \le t \le t_{\max}} \left| \frac{\langle h_1, h_2 \rangle}{\sqrt{\langle h_1, h_1 \rangle \langle h_2, h_2 \rangle}} \right|$$

$$\langle h_1, h_2 \rangle = 4\mathrm{Re} \int_{f_{\min}}^{f_{\max}} \frac{\tilde{h}_1(f) \tilde{h}_2^*(f)}{S_n(f)} df,$$