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Updating Pcal combined displacement correction factors and estimating Pcal systematic errors.

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Abstract. We detail the calculation of updated Pcal combined displacement correction factors when end station responsivity and optical efficiency values are updated. We then describe how the updated factors can be used to estimate systematic errors in the Pcal fiducial displacement calibrations when the updated factors are not implemented in the front-end code.

1. Parameters and formalism

The Pcal calibrations are implemented via the values for a series of EPICS records that are implemented in the front-end code and displayed on Pcal MEDM screens such as the one shown in figure 1. Comparisons of the X-end and Y-end Pcal calibrations are tracked by calculations executed in real time in front-end code†, by calculations using the Spectral Line Monitoring (SLM) Tool (see LIGO-P1300120), and via Grafana pages generated by the LSC group at Kenyon College. When using interferometer signals to evaluate the Pcal X/Y calibration comparison factor, χ_{XY}, one must take into account any difference in the response of the interferometer signal to displacements at the frequencies of the two Pcal excitations. When using signals such as DELTA_L_EXTERNAL or GDS_CALIB_STRAIN in which the action of the DARM loop is ideally compensated, the discrepancy should be negligible at the hundredth of a percent level.‡ However, when using DARM_ERR the difference in the DARM loop responsivity must be taken into account.

We use R_{XY} to denote the ratio of the DARM_ERR signal response (in units of ct/m) to external differential length variations at the modulation frequencies used for

† See, for instance, the `H1:CAL-CS_TDEP_PCAL_COMPARE_XY_LIVE_OVER_REF` signal on the `CAL_CS_PCAL_COMPARISON.adl` MEDM screen. Note that because this comparison calculate in the front-end code uses the DARM_ERR signal, the reference value, `H1:CAL-CS_TDEP_PCAL_COMPARE_CHI_XY_REF`, (when $\chi_{XY} = C_X C_Y$) should be $1/R_{XY}$.

‡ For a treatment of the X/Y comparison calculated using the DELTA_L_EXTERNAL signal, see LIGO-T2400032.

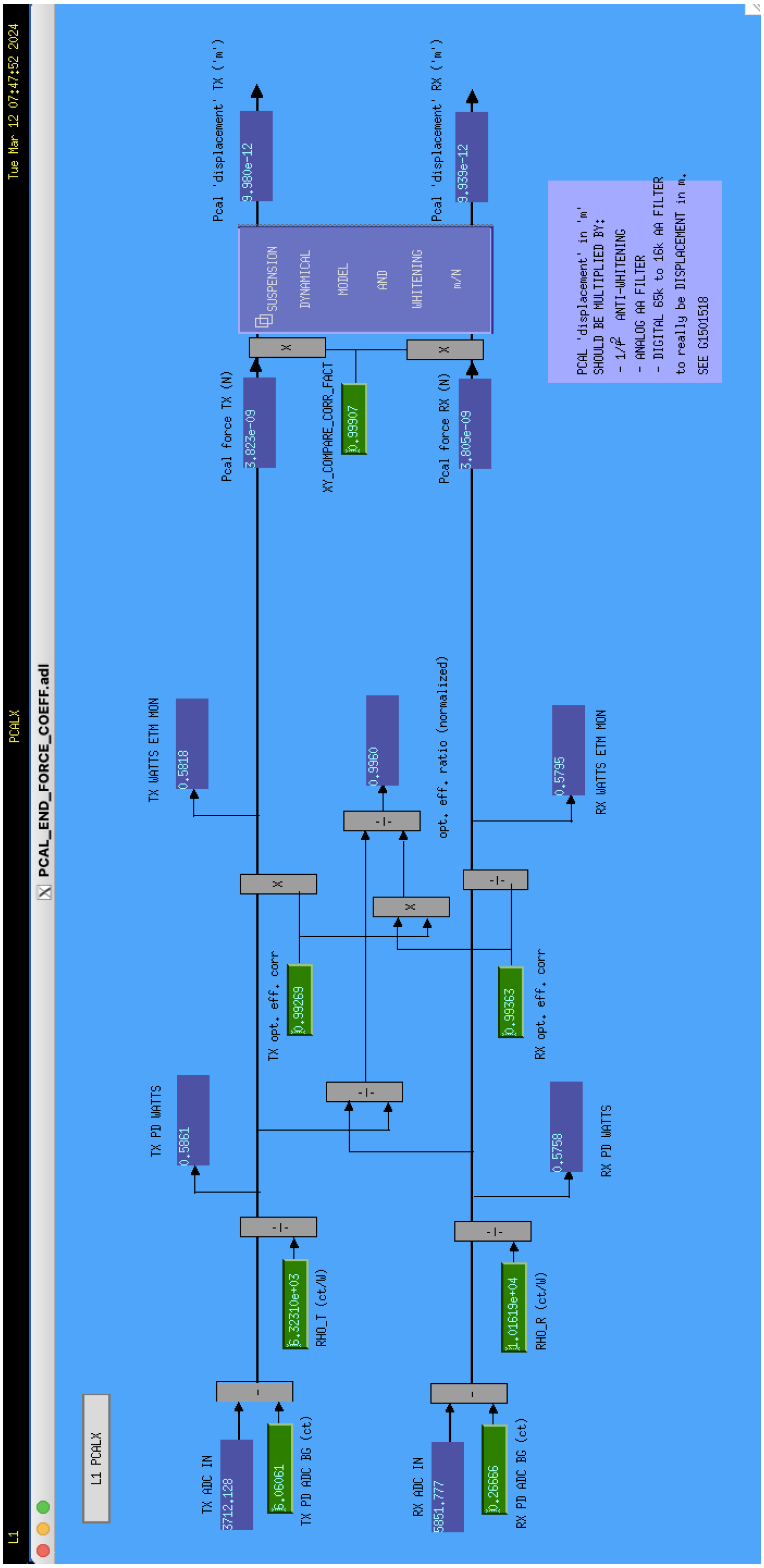

Figure 1. Pcal MEDM screen for calibrating the Pcal end station sensor output and for implementing X/Y comparison correction factors in the front-end code during the O4a observing run.

the X/Y comparison by X-arm and Y-arm Pcal systems. It is given by

$$R_{XY} = \frac{R(\omega_X)}{R(\omega_Y)} = \left.\frac{C}{1+G}\right|_{\omega=\omega_X} \left[\left.\frac{C}{1+G}\right|_{\omega=\omega_Y}\right]^{-1}. \tag{1}$$

where C is the DARM loop sensing function and G is the DARM loop overall gain.

As originally described in LIGO-P2000113, the Pcal X/Y calibration comparison factors are determined by comparing the X-end and Y-end calibrated Pcal displacements with amplitudes of induced lines in an interferometer output signal. When using the DARM_ERR signal, it is given by

$$\chi_{XY} = \frac{x(\omega_X)|_{Pcal} \,/\, x(\omega_X)|_{Derr}}{(x(\omega_Y)|_{Pcal} \,/\, x(\omega_Y)|_{Derr}} \tag{2}$$

where x denotes induced displacements reported by either the Pcal end station sensors or the DARM_ERR signal, ω_X and ω_Y are the frequencies of the Pcal excitations the X-end and Y-end. The *independent* (i.e. not estimated by combining the X- and Y-end calibrations) displacements derived from the calibrated Rx sensor outputs are given by

$$\begin{aligned} x(\omega_X)|_{Pcal} &= PcalX(\omega_X)\, C_X \quad \text{and} \\ x(\omega_Y)|_{Pcal} &= PcalY(\omega_Y)/C_Y \end{aligned} \tag{3}$$

The calibration correction factors, C_X and C_Y, appear here because they are applied by the front-end code (see figure 1) and must thus be divided out to obtain the displacements. The displacements derived from the DARM_ERR signal are given by

$$\begin{aligned} x(\omega_X)|_{Derr} &= Derr(\omega_X)/R(\omega_X) \quad \text{and} \\ x(\omega_Y)|_{Derr} &= Derr(\omega_Y)/R(\omega_Y) \end{aligned} \tag{4}$$

The Pcal X/Y calibration comparison factor in (2) can thus be written in terms of the Pcal and Derr signals as

$$\begin{aligned} \chi_{XY} &= \frac{PcalX(\omega_X)\, C_X}{PcalY(\omega_Y)/C_Y}\; \frac{Derr(\omega_Y)/R(\omega_Y)}{Derr(\omega_X)/R(\omega_X)} \\ &= \frac{PcalX(\omega_X)}{PcalY(\omega_Y)}\; \frac{Derr(\omega_Y)}{Derr(\omega_X)}\, C_X\, C_Y\, R_{XY} = CC_{Derr}\, C_X\, C_Y\, R_{XY}. \end{aligned} \tag{5}$$

Here C_X and C_Y are the values used in the front-end code (see, for example, figure 1) and R_{XY} is the DARM_ERR responsivity ratio given by (1) and CC_{Derr}. The ratio of the amplitudes of the four lines in the Pcal and DARM_ERR signals is given by

$$CC_{Derr} = \frac{PcalX(\omega_X)}{PcalY(\omega_Y)}\; \frac{Derr(\omega_Y)}{Derr(\omega_X)} = \frac{\chi_{XY}}{C_X\, C_Y}\; \frac{1}{R_{XY}}. \tag{6}$$

Ideally, $C_X\, C_Y = \chi_{XY}$ and $CC_{Derr} = 1/R_{XY}$.

The frequencies of the Pcal excitations used for the X/Y comparison at the LIGO end stations during the O4 observing run and the calculated (by JoeB) values for R_{XY} during O4a are shown in table 1.

Table 1. Calculated responsivity ratios, R_{XY}, for the O4 observing run.

Observatory	f_X **(Hz)**	f_Y **(Hz)**	R_{XY}
LLO	283.41	283.31	0.999456
LHO	283.91	284.01	1.00035

2. Calculating updated Pcal X/Y comparison factors after end station calibrations

To describe schemes for updating the calibration correction factors and to estimate systematic errors , we define the parameters listed in table 2. The superscript $*$ denotes parameters values after a change has been made and the supersctips 0 denotes parameter values before the changes were made.

Table 2. Parameters for systematic error estimation and tracking.

Parameter	**Nominal**	**Updated**
Rx, Tx sensor responsivity	ρ_R^0, ρ_T^0	ρ_R^*, ρ_T^*
Rx-side / Tx-side optical efficiency ratio	β	
Overall optical efficiency	η^0	η^*
Tx-, Rx-side optical efficiency	η_R^0, η_T^0	η_R^*, η_T^*
X/Y calibration comparison factor	χ_{XY}^0	χ_{XY}^*
X-end/Y-end DARM_ERR responsivity ratio	R_{XY}	
X/Y comparison using DARM_ERR	CC_{Derr}^0	CC_{Derr}^*
X-end, Y-end combined displacement correction factors	C_X^0, C_Y^0	C_X^*, C_Y^*
Relative systematic error	U_X^{sys}, U_Y^{sys}	

Pcal end station measurements yield updated values for the Rx sensor responsivity and the overall optical efficiency. These two new numbers, ρ_R^* and η^*, together with the measured value of X/Y comparison before completing the end station measurements, CC_{Derr}^0 can be used to calculate updated combined displacement correction factors C_X^* and C_Y^*.

Before updating the calibration correction factors, the calibration comparison is given by

$$CC_{Derr}^0 = \frac{\chi_{XY}^0}{C_X^0 C_Y^0} \frac{1}{R_{XY}}. \tag{7}$$

χ_{XY} is related to the Rx sensor responsivities and the optical efficiencies as

$$\chi_{XY} \propto \frac{\eta_{RY} \rho_{RY}}{\eta_{RX} \rho_{RX}}. \tag{8}$$

If, for instance, the X-end values for ρ_R and η are updated (and η_R and η_T are calculated using β), then the resulting calibration comparison factor would be given by

$$\chi^*_{XY} = \chi^0_{XY} \frac{\eta^0_{RX}\rho^0_{RX}}{\eta^*_{RX}\rho^*_{RX}} = CC^0_{Derr}\, C^0_X\, C^0_Y\, R_{XY} \frac{\eta^0_{RX}\rho^0_{RX}}{\eta^*_{RX}\rho^*_{RX}} = C^*_X C^*_Y. \tag{9}$$

If updated values η^*_{RX} and ρ^*_{RX} are entered into the front-end code via the EPICS records, then the calibration comparison would be expected to yield

$$CC^t_{Derr} = \frac{\chi^0_{XY}}{C^0_X C^0_Y} \frac{\eta^0_{RX}\rho^0_{RX}}{\eta^*_{RX}\rho^*_{RX}} \frac{1}{R_{XY}} = CC^0 \frac{\eta^0_{RX}\rho^0_{RX}}{\eta^*_{RX}\rho^*_{RX}}. \tag{10}$$

Here, the superscript t indicates that this calibration comparison is a *temporary* value based on updated responsivity and optical efficiency values but using the original combined displacement correction factors.

We can use (9) to calculate updated combined displacement correction factors based on the calibration comparison results before the responsivity and optical efficiency values were updated. Once the EPICS records for these parameters are updated, we expect that the calibration comparison would yield

$$CC^*_{Derr} = \frac{\chi^*_{XY}}{C^*_X C^*_Y} \frac{1}{R_{XY}} = \frac{1}{R_{XY}} \tag{11}$$

Note that if the Y-end Rx sensor responsivity and optical efficiency are updated, (9) becomes

$$\chi^*_{XY} = \chi^0_{XY} \frac{\eta^*_{RY}\rho^*_{RY}}{\eta^0_{RY}\rho^0_{RY}} = CC^0_{Derr}\, C^0_X\, C^0_Y\, R_{XY} \frac{\eta^*_{RY}\rho^*_{RY}}{\eta^0_{RY}\rho^0_{RY}} = C^*_X C^*_Y. \tag{12}$$

The ratio of the updated to initial values is inverted compared with (9).

3. Estimation of systematic error incurred by not updating combined displacement correction factors in the front-end code

Calculation of updated combined displacement correction factors C^*_X and C^*_Y when either end station measurements indicate revised values for Pcal power sensor responsivities and optical efficiency is detailed in section 2, equations (9) and (12). If measurements at both end stations are performed updated combined displacement correction factors can be calculated by combining (9) and (12) as

$$\chi^*_{XY} = \chi^0_{XY} \frac{\eta^0_{RX}\rho^0_{RX}}{\eta^*_{RX}\rho^*_{RX}} \frac{\eta^*_{RY}\rho^*_{RY}}{\eta^0_{RY}\rho^0_{RY}} = CC^0_{Derr}\, C^0_X\, C^0_Y\, R_{XY} \frac{\eta^0_{RX}\rho^0_{RX}}{\eta^*_{RX}\rho^*_{RX}} \frac{\eta^*_{RY}\rho^*_{RY}}{\eta^0_{RY}\rho^0_{RY}} = C^*_X C^*_Y. \tag{13}$$

Here CC^0_{Derr} is the calibration comparison calculated using the DARM_ERR signal with end station values in the front-end code that have not been updated (values with superscript 0). The product of the updated correction factors C^*_X and C^*_Y is calculated using (13) and the individual updated combined displacement corrections factors are calculated using the procedure detailed in section 2 of LIGO-P2000113. They are calculated using the weighted geometrical mean, μ_g, of 1 and χ^*_{XY} with the weighting factors, w_X and w_Y, dictated by the inverse of the estimated variances in the end station displacement factors due to uncertainty contributions that are not common to both end stations. w_X is applied to 1 and w_Y is applied to χ^*_{XY} to calculate μ_g. Thus

$$\mu^*_g = \chi^{*w_Y/(w_X+w_Y)}_{XY} \tag{14}$$

$$C^*_X = \mu^*_g \text{ and } C^*_Y = \chi^*_{XY}/\mu^*_g\,. \tag{15}$$

The relative systematic errors incurred by not implementing the updated end station parameters in the front-end code are given by

$$U^{sys}_X = \frac{C^0_X}{C^*_X} \text{ and } U^{sys}_Y = \frac{C^*_Y}{C^0_Y}\,. \tag{16}$$

Values used during the O4b observing run to estimate calculate the weighting factors w_X and w_Y are listed in table 3.

Table 3. Weighting factors for calculating combined displacement correction factors and contributing parameter values.

	LHO		LLO	
Parameter	**X-end**	**Y-end**	**X-end**	**Y-end**
Non-common rel. unc. (1-σ)	0.0042	0.0037	0.0014	0.0016
w_X, w_Y	56689.34	73046.02	510204.08	390625.0
$w_Y/(w_X+w_Y)$	0.563039		0.433628	
χ^0_{XY}	1.0027		1.0061	
μ^0_g	1.0015		1.0026	
C^0_X, C^0_Y	1.0015	1.0012	1.0026	1.0035

4. O4 EPICS variables update

4.1. Start of O4a

At the beginning of O4a run, the EPICS variables were calculated using the weighted mean of TSA, TSB and WSS responsivities, the weighted mean of WS/GS responsivity ratios and the mean of the Rx/WS responsivity ratios. The following "caput" commands were used to update the EPICS records for the four Pcal systems:

```
caput H1:CAL-PCALX_FORCE_COEFF_RHO_T 8334.52
caput H1:CAL-PCALX_FORCE_COEFF_RHO_R 10692.3
```

caput H1:CAL-PCALX_FORCE_COEFF_TX_PD_ADC_BG 8.8131
caput H1:CAL-PCALX_FORCE_COEFF_RX_PD_ADC_BG 0.2599
caput H1:CAL-PCALX_FORCE_COEFF_TX_OPT_EFF_CORR 0.9935
caput H1:CAL-PCALX_FORCE_COEFF_RX_OPT_EFF_CORR 0.9946
caput H1:CAL-PCALX_XY_COMPARE_CORR_FACT 0.9988

caput H1:CAL-PCALY_FORCE_COEFF_RHO_T 7157.80
caput H1:CAL-PCALY_FORCE_COEFF_RHO_R 10652.5
caput H1:CAL-PCALY_FORCE_COEFF_TX_PD_ADC_BG 17.5161
caput H1:CAL-PCALY_FORCE_COEFF_RX_PD_ADC_BG -0.8150
caput H1:CAL-PCALY_FORCE_COEFF_TX_OPT_EFF_CORR 0.9920
caput H1:CAL-PCALY_FORCE_COEFF_RX_OPT_EFF_CORR 0.9931
caput H1:CAL-PCALY_XY_COMPARE_CORR_FACT 1.0015

caput L1:CAL-PCALX_FORCE_COEFF_RHO_T 6323.1
caput L1:CAL-PCALX_FORCE_COEFF_RHO_R 10161.9
caput L1:CAL-PCALX_FORCE_COEFF_TX_PD_ADC_BG 6.06061
caput L1:CAL-PCALX_FORCE_COEFF_RX_PD_ADC_BG 0.266662
caput L1:CAL-PCALX_FORCE_COEFF_TX_OPT_EFF_CORR 0.992692
caput L1:CAL-PCALX_FORCE_COEFF_RX_OPT_EFF_CORR 0.993626
caput L1:CAL-PCALX_XY_COMPARE_CORR_FACT 0.99907

caput L1:CAL-PCALY_FORCE_COEFF_RHO_T 6624.89
caput L1:CAL-PCALY_FORCE_COEFF_RHO_R 10384.1
caput L1:CAL-PCALY_FORCE_COEFF_TX_PD_ADC_BG 17.6538
caput L1:CAL-PCALY_FORCE_COEFF_RX_PD_ADC_BG 1.32913
caput L1:CAL-PCALY_FORCE_COEFF_TX_OPT_EFF_CORR 0.992957
caput L1:CAL-PCALY_FORCE_COEFF_RX_OPT_EFF_CORR 0.995775
caput L1:CAL-PCALY_XY_COMPARE_CORR_FACT 1.00121

4.2. Start of O4b

For the O4b observing run, the EPICS record values were calculated using *local* (in time) values for the TSA transfer standard responsivity, WS/GS responsivity ratios and Rs/Ws responsivity ratios. We updated the combined displacement correction factors based on equation (13) in this document. The calculated EPICS record values for the start of the O4b observing run are given by the following "caput" commands (however, as noted below, there was an error in the calculations):

caput H1:CAL-PCALX_FORCE_COEFF_RHO_T 8305.09

```
caput H1:CAL-PCALX_FORCE_COEFF_RHO_R 10716.60
caput H1:CAL-PCALX_FORCE_COEFF_TX_PD_ADC_BG 9.6571
caput H1:CAL-PCALX_FORCE_COEFF_RX_PD_ADC_BG 0.7136
caput H1:CAL-PCALX_FORCE_COEFF_TX_OPT_EFF_CORR 0.9938
caput H1:CAL-PCALX_FORCE_COEFF_RX_OPT_EFF_CORR 0.9948
caput H1:CAL-PCALX_XY_COMPARE_CORR_FACT 0.9979

caput H1:CAL-PCALY_FORCE_COEFF_RHO_T 7145.62
caput H1:CAL-PCALY_FORCE_COEFF_RHO_R 10649.59
caput H1:CAL-PCALY_FORCE_COEFF_TX_PD_ADC_BG 18.2388
caput H1:CAL-PCALY_FORCE_COEFF_RX_PD_ADC_BG -0.2591
caput H1:CAL-PCALY_FORCE_COEFF_TX_OPT_EFF_CORR 0.9923
caput H1:CAL-PCALY_FORCE_COEFF_RX_OPT_EFF_CORR 0.9934
caput H1:CAL-PCALY_XY_COMPARE_CORR_FACT 1.0013


caput L1:CAL-PCALX_FORCE_COEFF_RHO_T 6337.88
caput L1:CAL-PCALX_FORCE_COEFF_RHO_R 10186.85
caput L1:CAL-PCALX_FORCE_COEFF_TX_PD_ADC_BG 6.8032
caput L1:CAL-PCALX_FORCE_COEFF_RX_PD_ADC_BG 0.0997
caput L1:CAL-PCALX_FORCE_COEFF_TX_OPT_EFF_CORR 0.9926
caput L1:CAL-PCALX_FORCE_COEFF_RX_OPT_EFF_CORR 0.9942
caput L1:CAL-PCALX_XY_COMPARE_CORR_FACT 0.9985

caput L1:CAL-PCALY_FORCE_COEFF_RHO_T 6620.70
caput L1:CAL-PCALY_FORCE_COEFF_RHO_R 10410.65
caput L1:CAL-PCALY_FORCE_COEFF_TX_PD_ADC_BG 17.2872
caput L1:CAL-PCALY_FORCE_COEFF_RX_PD_ADC_BG 1.4549
caput L1:CAL-PCALY_FORCE_COEFF_TX_OPT_EFF_CORR 0.9929
caput L1:CAL-PCALY_FORCE_COEFF_RX_OPT_EFF_CORR 0.9937
caput L1:CAL-PCALY_XY_COMPARE_CORR_FACT 1.0013
```

The X/Y comparison was monitored for two weeks after the new EPICS record values were updated in the front-end code. At LHO, we expected a X/Y ratio of 0.99965 when calculating CC using the DARM_ERR signal, denoted CC^0_{Derr}. Using the GDS_CALIB_STRAIN channel to calculate the calibration comparison, as is the case for the Grafana pages, we would expect CC_{GDS} to be 1.0 since R_{XY} should be unity in this case (see 13). As shown in figure 2, we observed $CC^0_{GDS} \simeq 0.9979$, a 0.21% discrepancy. We traced this discrepancy to incorrect calculation of the combined displacement correction factors.

RS Need to continue editing here.In equation (13), instead of using the $CC^0_{GDS} = 1.0012$, the value after implementing the O4a combined displacement correction factors,

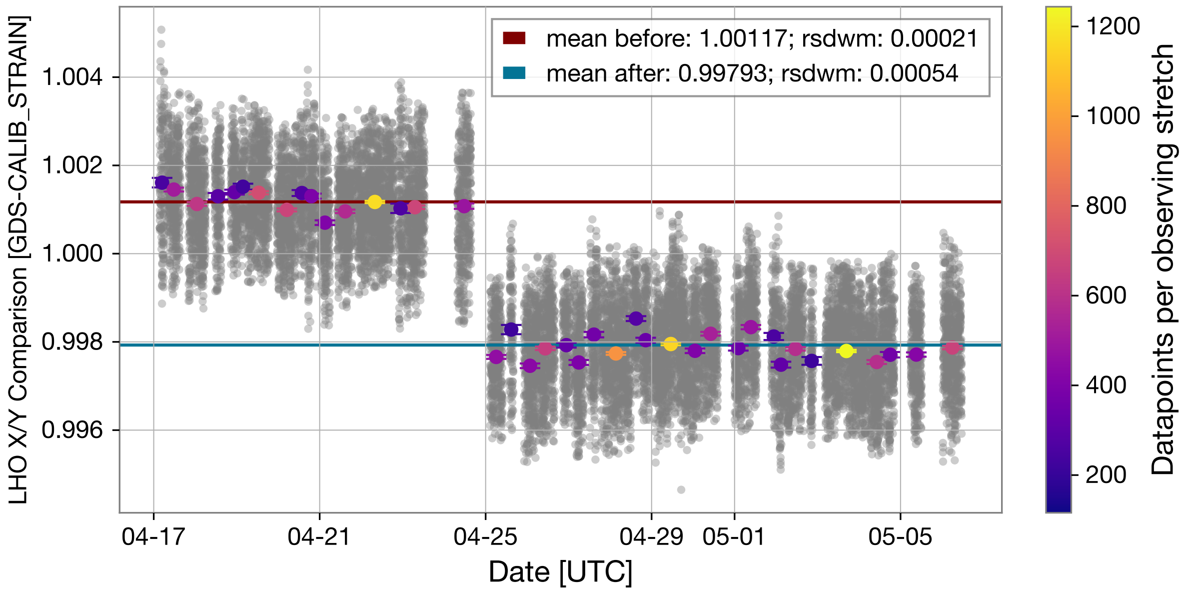

Figure 2. X/Y calibration comparison for LHO

we used $CC^0_{Derr} = 1.0027$, the value without any correction implemented. Since we are now using Grafana data to calculate X/Y calibration comparison factor, $R_{XY} = 1$ and $CC_{GDS} = 1$. We intend to update the EPICS records values in the front-end code to the following.

caput H1:CAL-PCALX_XY_COMPARE_CORR_FACT 0.9991

caput H1:CAL-PCALY_XY_COMPARE_CORR_FACT 1.0005

This update would result in a relative change in the combined fiducial displacement factors of 0.12 % for X-End, and - 0.08 % for Y-End from the previous update. The changes in the EPICS variables and the contributing parameter are tabulated in table 4

LLO EPICS variables updates were done correctly, so there is no plan to further update them. As shown in figure 3, no significant change in the X/Y calibration comparison factor before and after the EPICS upgrades on April 25th, indicating that the upgrades were done correctly.

Table 4. Changes in LHO EPICS variables and the parameters that contribute to their calculation. There was an error in the calculation of the initial values that we are correcting by making these updates.

Parameter	Updated values 5/07/2024		Initial O4b values 4/25/2024	
	X-End	Y-End	X-End	Y-End
X/Y cal. comp., CC^0	1.0012 (GDS)		1.0027 (Derr)	
X/Y cal. comp. fact., χ^*_{XY}	1.0014		1.0034	
Combined disp. correction factors, $1/C^*_X$, C^*_Y	0.9991	1.0005	0.9979	1.0013
Change in calibration (%)	0.12	-0.08		

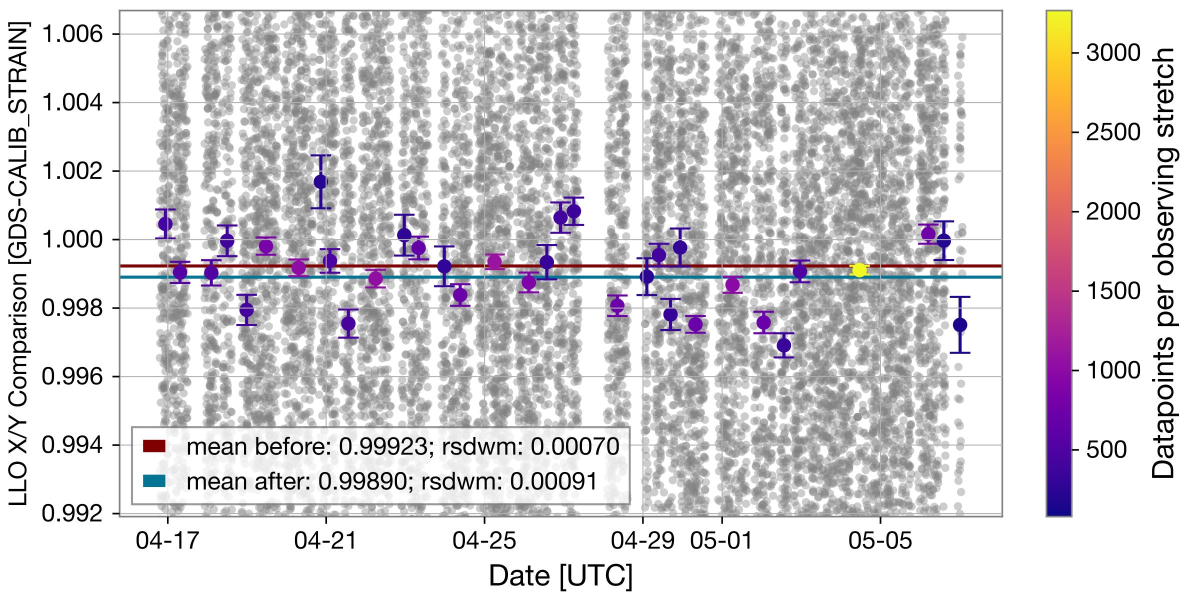

Figure 3. X/Y calibration comparison for LLO