Caden Swain

Demonstration of Bayesian Transfer Function Fitting Method – A Potential Tool for Estimating Interferometer Uncertainty

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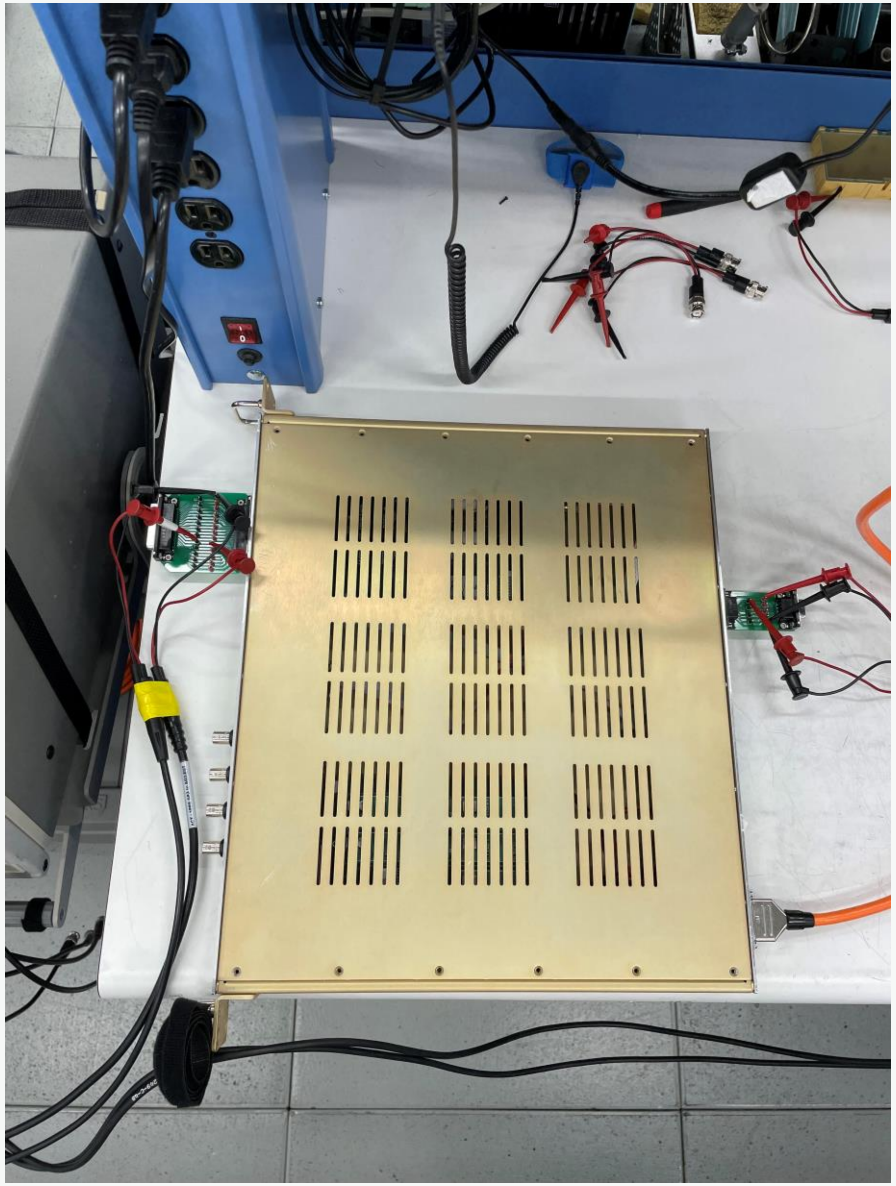
Project Goals

- Primary Goal:
  - Analyze Bayesian transfer function fitting method *(BayesianTF)* and compare results to previous transfer-function-fitting method *(IIRRational)*
    - Includes testing *IIRRational's* effectiveness at varying signal-to-noise ratios (SNR); predicted to be very accurate at SNR and to fail at low SNR
    - *BayesianTF* developed by Ethan Payne at Caltech
    - *IIRRational* developed by Lee McCuller at Caltech
- Secondary Goals:
  - Characterize spare OMC DCPD whitening chassis for use in the interferometer
  - Generally assist with Detector Calibration

Transfer Functions

- Function detailing a system's response to an input signal
  - Usually for electrical systems but includes any system that can be modelled with differential equations
  - Ex. Electronic filters, harmonic oscillators
  - Frequency dependent
  - Complex – includes magnitude and phase
- Transfer function is calculated with the ratio of the Output and Input signals
  - $TF = \frac{Output(f)}{Input(f)}, TF * Input(f) = Output(f)$
- To test *IIRRational* and *BayesianTF*, I gathered transfer function and noise data from a spare OMC DCPD whitening chassis



Response Function

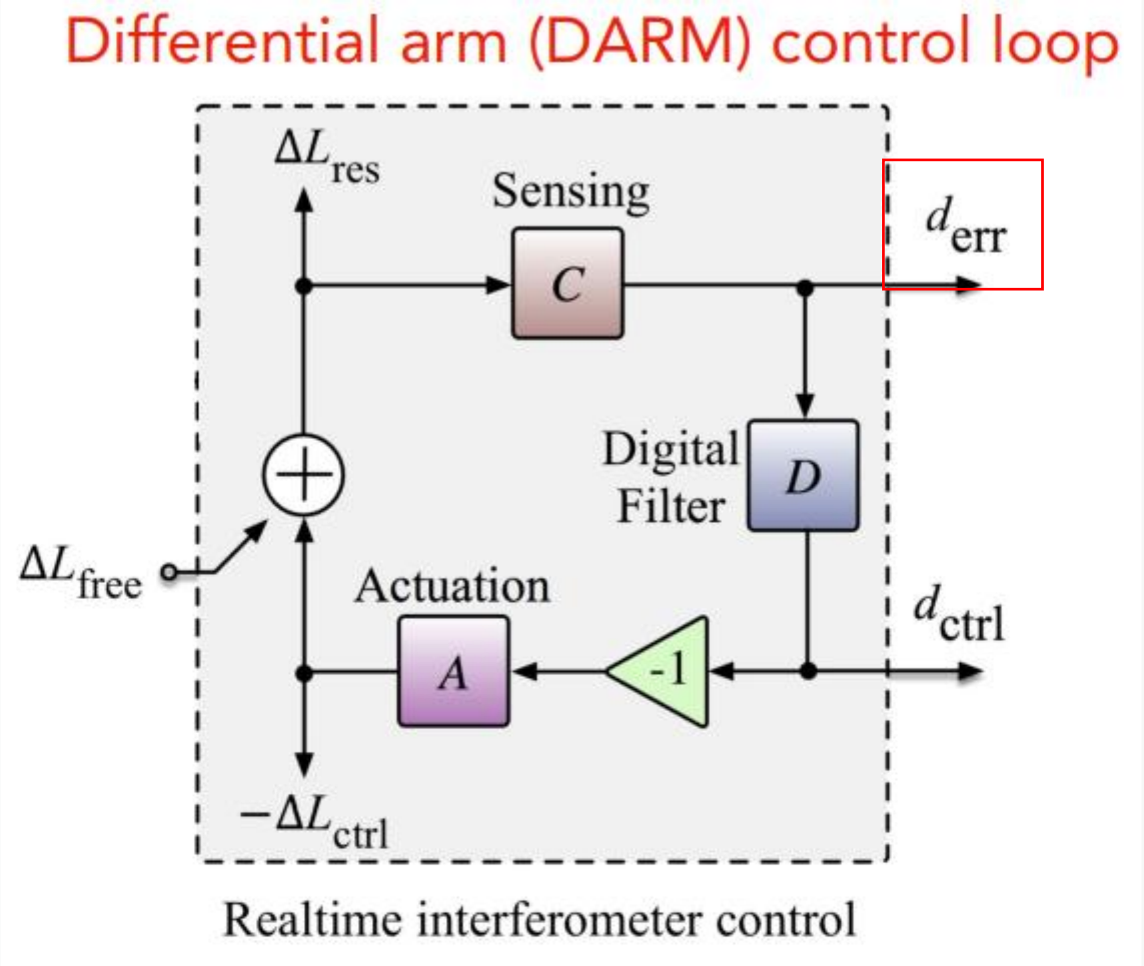
- Function of the interferometer's response to external stimuli (Ex. Gravitational Waves)
- Important because it directly propagates to the strain in the interferometer with

$$h(t) = \frac{R^{(model)} d_{err}}{L}$$

- Each part of the IFO (Sensing & Actuation Functions, Digital Filter) can be modelled with transfer functions

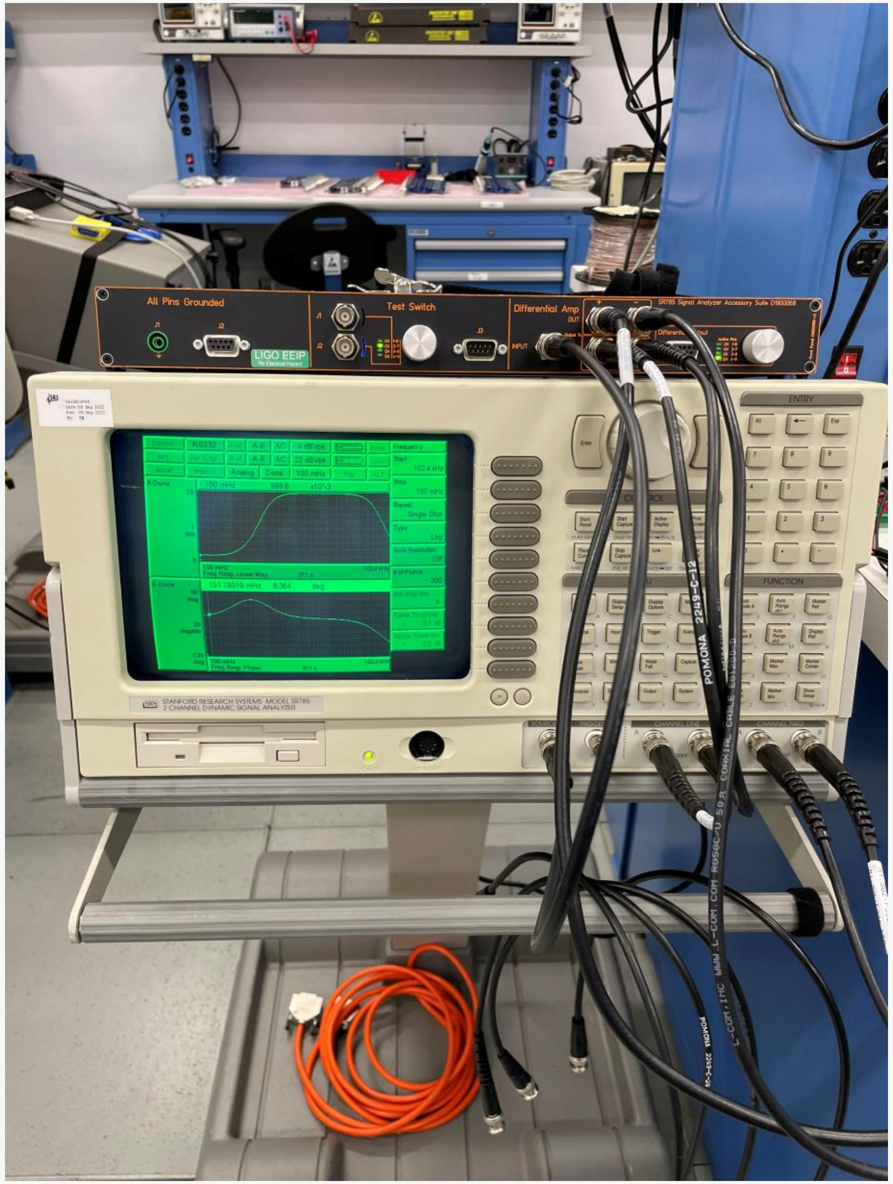
$$R^{(model)} = \frac{1 + A^{(model)} D C^{(model)}}{C^{(model)}}$$

- May fit a transfer function to the Response Function as a whole

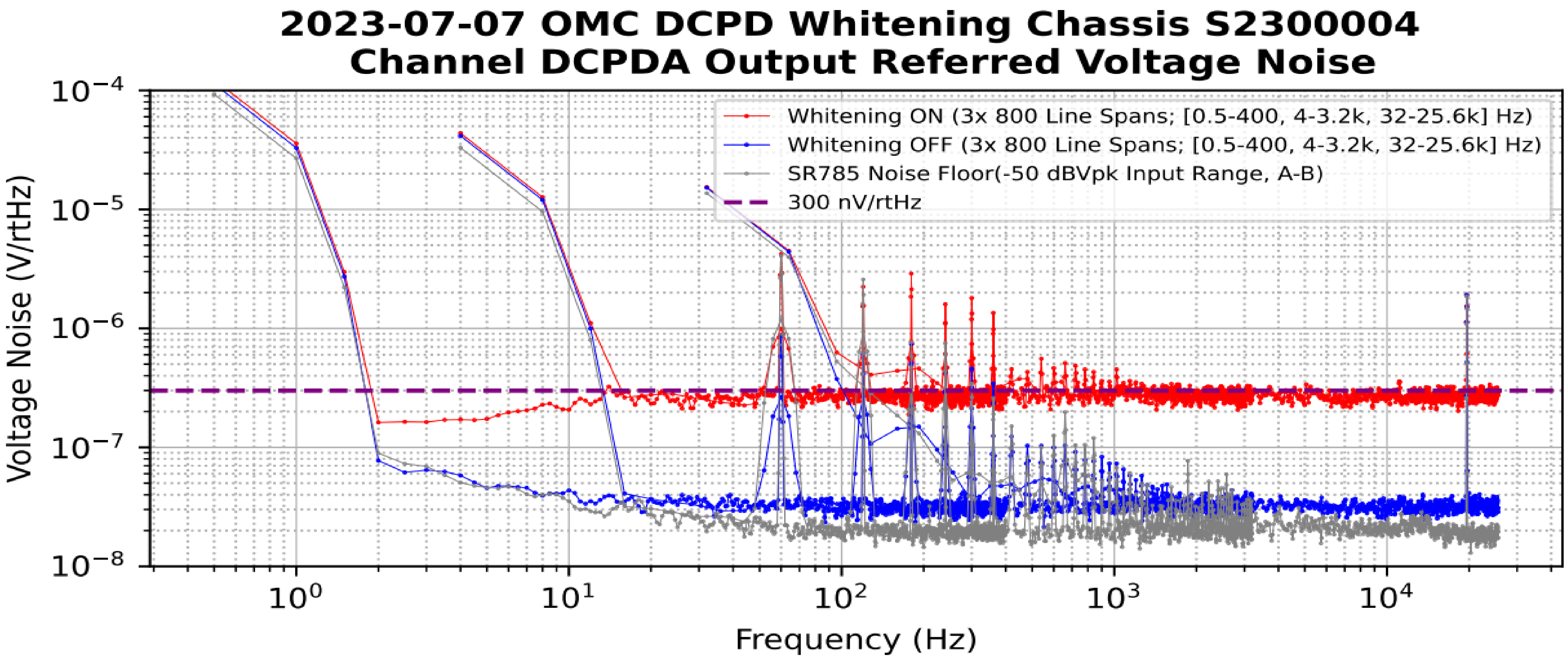


Noise Data

- Gathered noise data from a spare OMC DCPD whitening chassis – S2300004 – using the SR785 Dynamic Signal Analyzer and FFT measurements
    - No signal was inputted, only the inherent noise of the chassis measured
    - Plotted and compared to Jeff Kissel's previous noise data from S2300003 whitening chassis – noise data aligned very well – https://alog.ligo-wa.caltech.edu/aLOG/index.php?callRep=71117
- Noise data is used to calculate the signal-to-noise ratio (SNR) at varying input voltages
    - SNR is used to calculate the coherence of the input and output signals
    - Coherence in turn is used to calculate uncertainty in the measurements
        - Measurement uncertainty required to run *BayesianTF* – statistics
        - $\mathrm{SNR} = \frac{V_{(signal)}}{V_{(noise)}} = \frac{C(f)}{1-C(f)}$
        - $\sigma(f) = \sqrt{\frac{1-C^2(f)}{2N_{avg}C^2(f)}}$



https://arxiv.org/abs/2107.00129

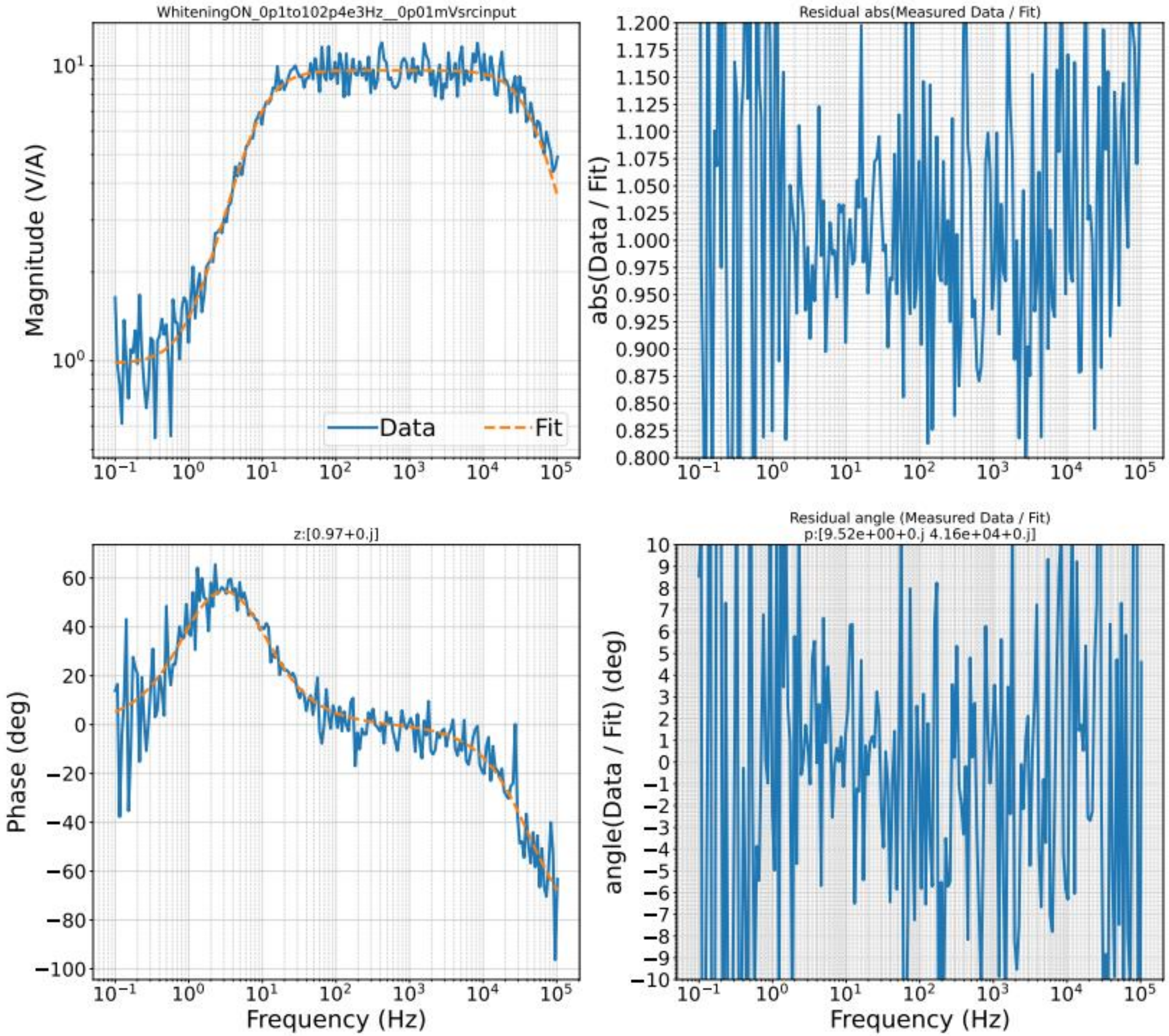


- Noise data gathered using plotted in ASD units (V/rtHz) – normalizes noise floor to a roughly flat value
- Total noise calculated by multiplying ASD noise taken with FFT by the sqrt of the frequency bin width of the FFT
  - Varies depending on the frequency range the FFT is taken within
- Total noise in my data increased as frequency increased
  - First measurement: 7.8mHz to 6.25mHz, 7.8mHz FFT bin width
  - Last measurement: 25.6kHz to 102.4kHz, 128Hz FFT bin width

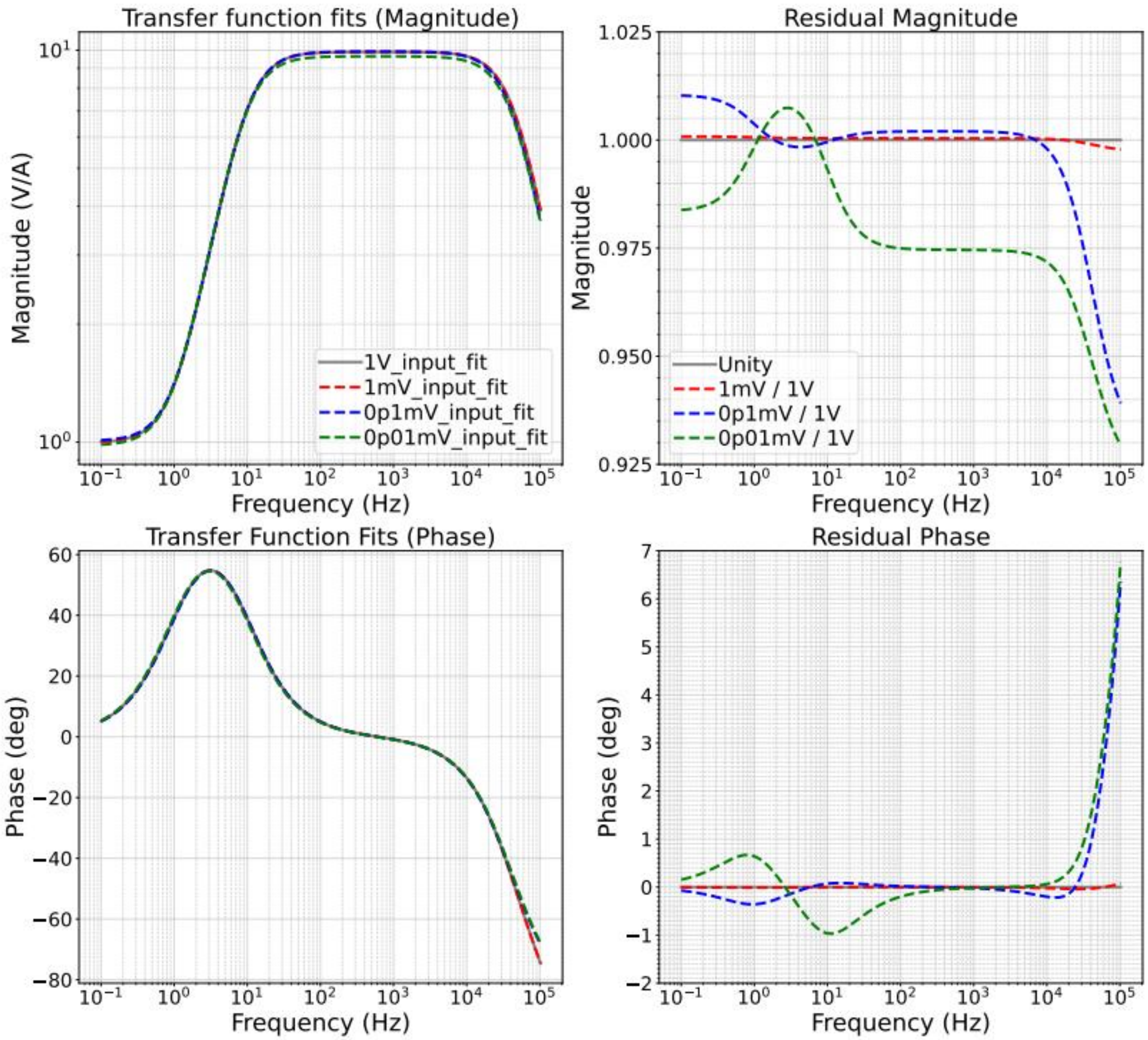
Transfer Function Data

- Gathered transfer function data from the whitening chassis at varying input voltages
  - Used a swept-sine measurement with the SR785 – signals are inputted over a broadband frequency range (100mHz to 102.4kHz) and the output is recorded
  - Measures the magnitude and phase of the transfer function
  - Various signal inputs (magnitude): 1V, 0.5V, 0.1V, 10mV, 1mV, 0.1mV, 0.05mV, 0.03mV, 0.01mV
- Used *IIRRational* to characterize the analytical transfer function at high SNR / low measurement uncertainty – 1V input, SNR = 11,306,947
- Calculated residual between various *IIRRational* transfer function models/fits at low and high measurement uncertainty – 1mV (SNR = 11,307), 0.1mV (SNR = 1131), and 0.01mV (SNR = 113)
  - *IIRRational* still very effective with all inputs above 1mV due to the high SNR / low measurement uncertainty

- Left plots are Bode plots displaying the transfer function fits from *IIRRational* using the TF data from various input voltages gathered using SR785
  - 1V (standard, lowest measurement uncertainty)
  - 1mV
  - 0.1mV
  - 0.01mV
- Right plots are residuals between measurement data and TF fits
- Want residuals to be at unity (Magnitude=1 and Phase=0)

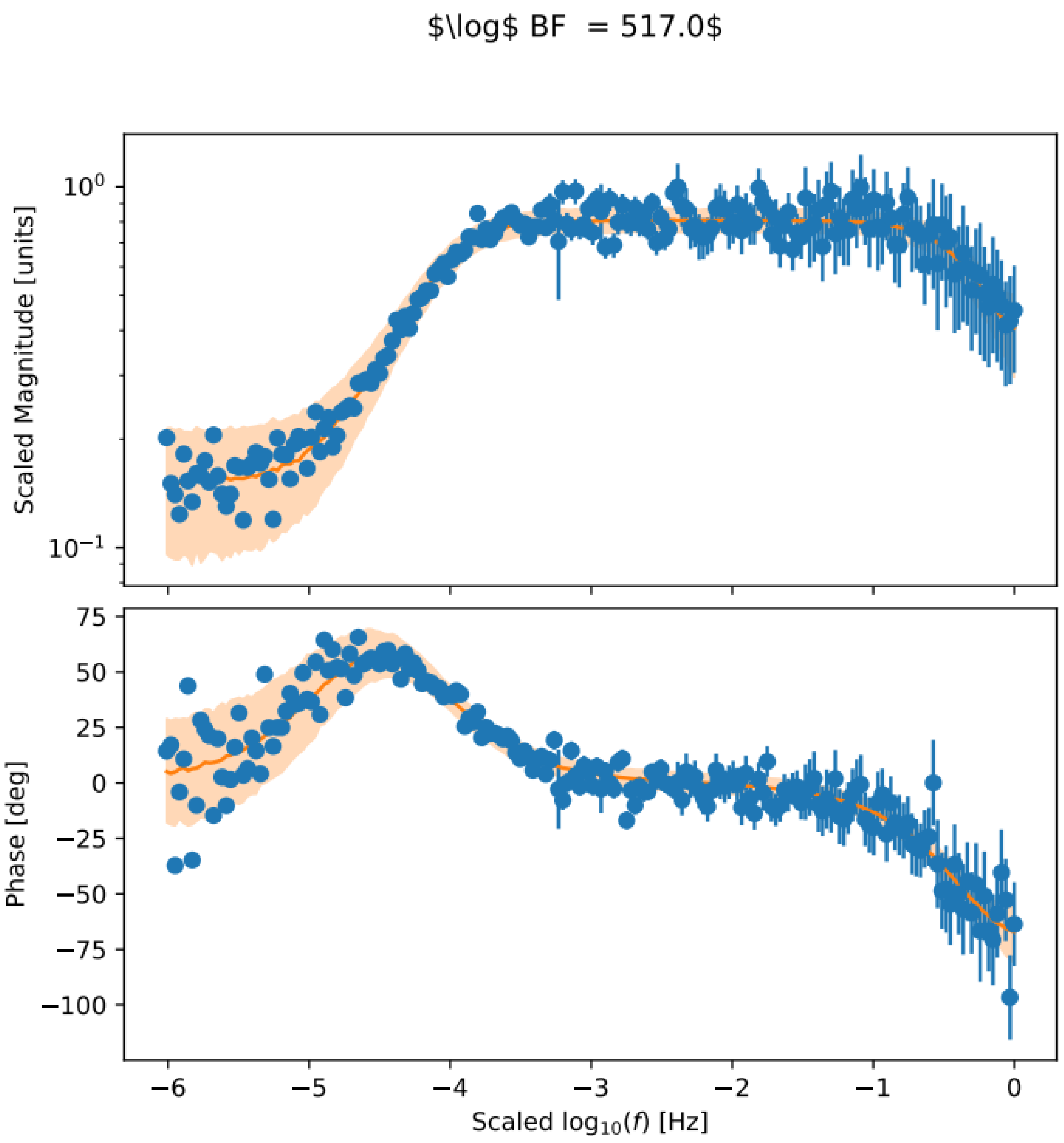


- Left plots are Bode plots displaying the transfer function fits from *IIRRational* using the TF data from various input voltages gathered using SR785
- Right plots are residuals between each of the TF fits and the standard 1V fit
- Want residuals to be at unity (Magnitude=1 and Phase=0)
- 1mV fit accurate, other fits deviate significantly from unity
- <u>Deviation shows that *IIRRational* is inaccurate at low SNR / high measurement uncertainty</u>
- <u>Heavy deviation begins somewhere between SNR ~ 10^4 and SNR ~ 10^3</u>



BayesianTF Results

- Currently only have results for 0.01mV and 0.1mV input datasets
- Limit ~ 10^3 SNR, 0.1mV input dataset
  - Fails with higher SNR datasets
- Takes measurement uncertainty into account – creates error bars for the fit
  - Very useful for application to the Response Function
  - *IIRRational* does not do this
- Transfer function fit from *BayesianTF*
  - 0.01mV input / ~10^2 SNR fit
- Still a work in progress
  - Fit was just gathered two days ago
  - More analysis must be done



BayesianTF vs *IIRRational*

- *BayesianTF* begins to fail around the same SNR ($\sim 10^3$) as *IIRRational*
    - *BayesianTF* handles low SNR datasets
    - *IIRRational* handles high SNR datasets
- More analysis must be done between $\sim 10^4$ and 10^3 SNR datasets to determine precise points of failure for each method
- Graphical comparisons between TF fits TBD

Summary

- Characterized a spare OMC DCPD whitening chassis with *IIRRational*
  - Gathered transfer function and noise data
- Tested *IIRRational's* effectiveness at varying SNR's
  - Collected transfer function data at varying input voltages
- Tested *BayesianTF* using the same datasets as used with *IIRRational*
- Compared results between the two transfer-function-fitting methods
  - Graphical comparisons TBD

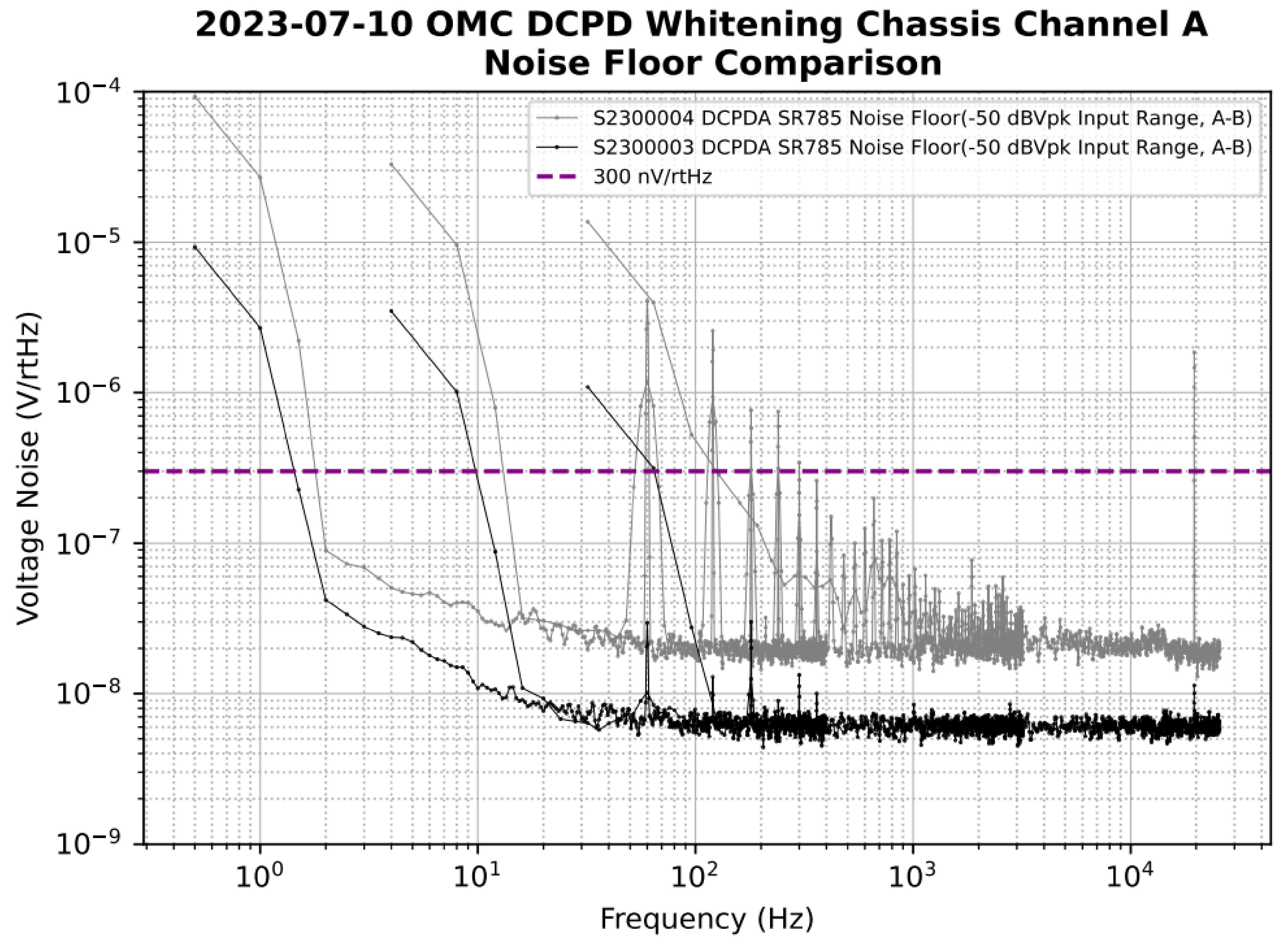
Acknowledgements

Special thanks to:

- My mentor, Louis Dartez, Ethan Payne, Marc Pirello, Jeff Kissel, and Rick Savage
- LHO SURF staff
- LIGO
- National Science Foundation
- California Institute of Technology

About Myself

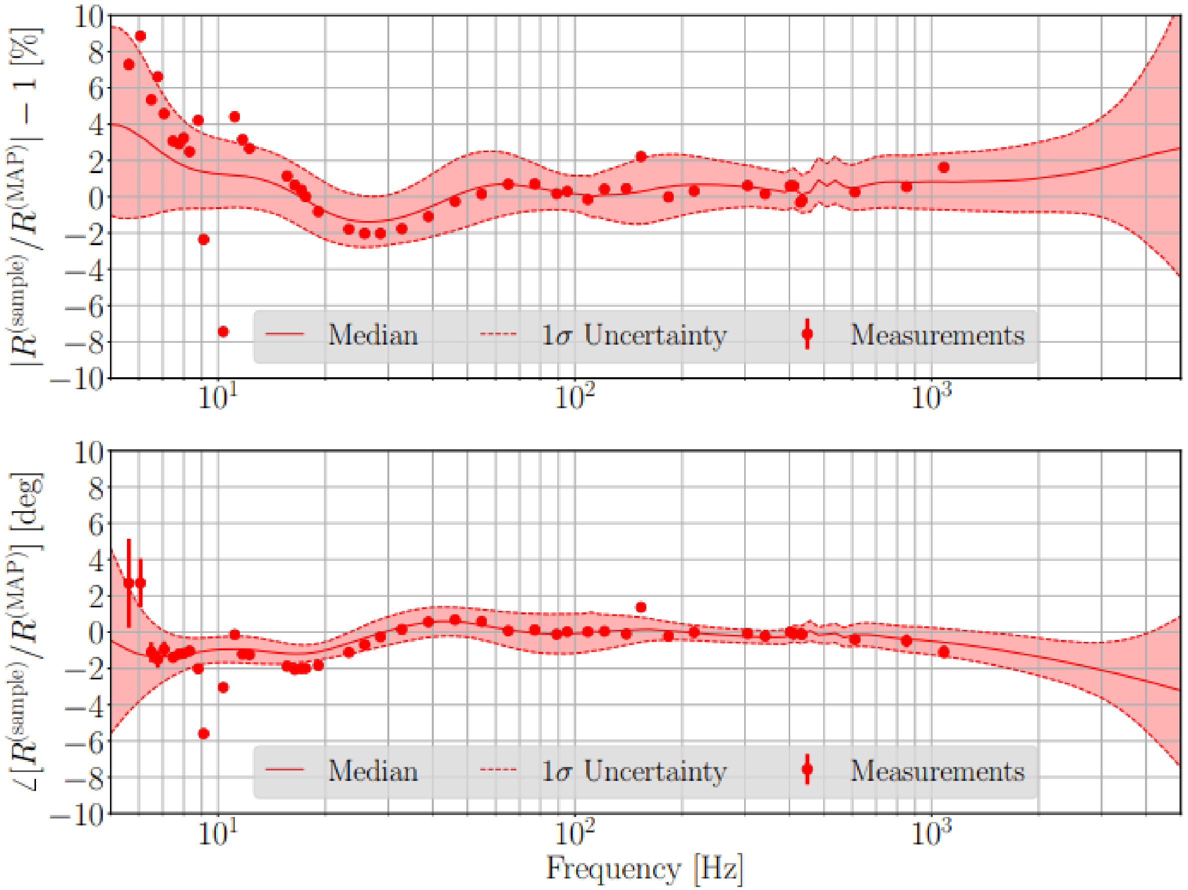
- Raised in New Orleans, Louisiana
  - Graduated from Jesuit High School in 2020
- Currently live in Los Angeles, California
  - Attending Loyola Marymount University (LMU)
  - Physics & Applied Math double major
  - Class of 2024 (TBD)

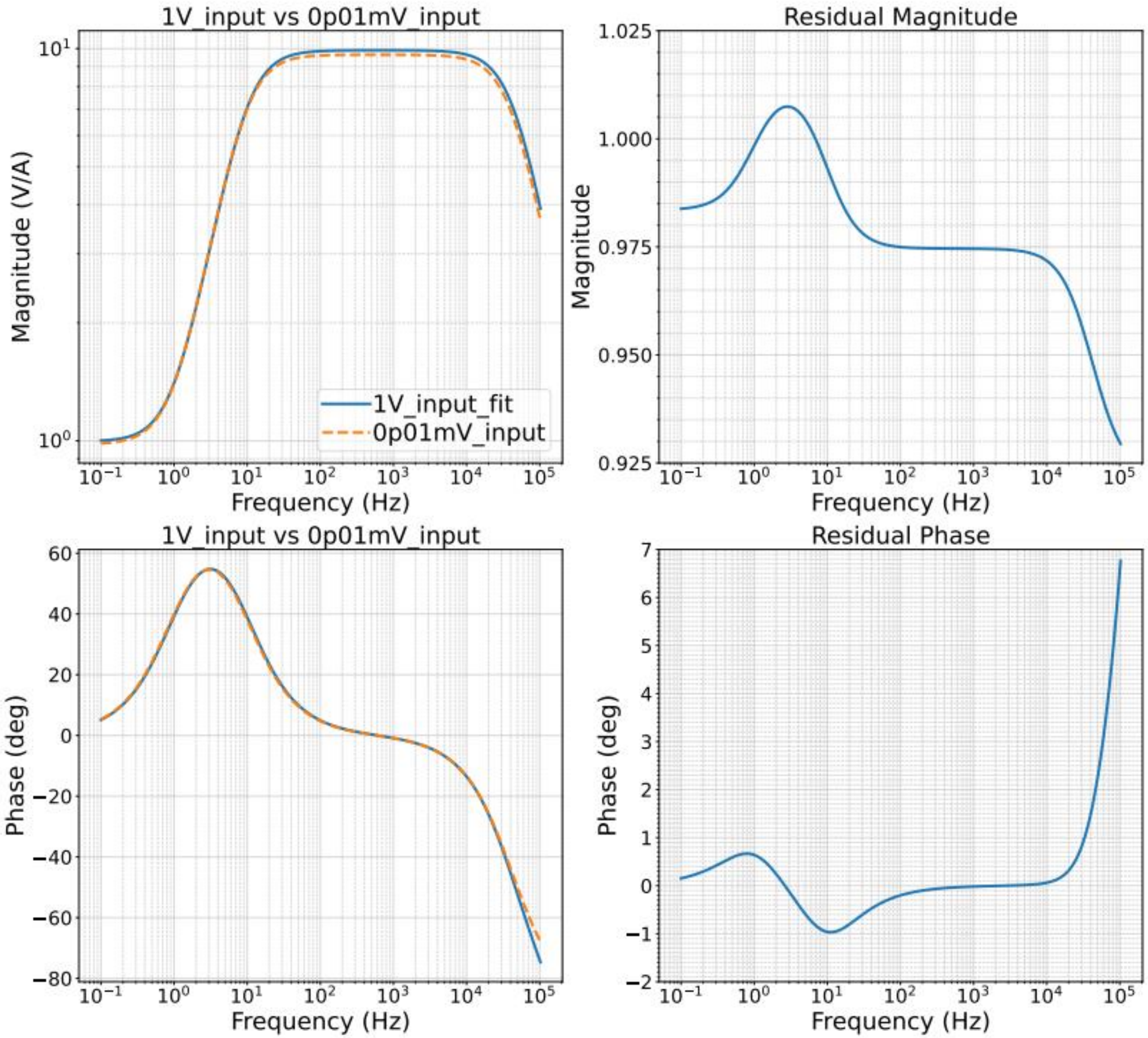


2023-07-10 OMC DCPD Whitening Chassis Channel A
Noise Floor Comparison
S2300004 DCPDA SR785 Noise Floor(-50 dBVpk Input Range, A-B)
S2300003 DCPDA SR785 Noise Floor(-50 dBVpk Input Range, A-B)
300 nV/rtHz
Voltage Noise (V/rtHz)
Frequency (Hz)
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^0
10^1
10^2
10^3
10^4

Response Function

- Measured at discrete frequencies using external excitations to the IFO (Pcal system)
- Uncertainty in these measurements is calculated and interpolated over a broadband frequency range using Gaussian Process Regression (GPR)
  - Effective, but could be improved
- Another strategy: Fit an analytical transfer function to the Response Function
  - *IIRRational* – does not capture uncertainty in measurements, very accurate at high SNR/low measurement uncertainty
  - *BayesianTF* – statistical method, encapsulates measurement uncertainty, (hopefully) effective at low SNR/high measurement uncertainty
- Goal: Compare results of *IIRRational* and *BayesianTF*





1V_input vs 0p01mV_input
Magnitude (V/A)
Frequency (Hz)
1V_input_fit
0p01mV_input
Residual Magnitude
Magnitude
Frequency (Hz)
1V_input vs 0p01mV_input
Phase (deg)
Frequency (Hz)
Residual Phase
Phase (deg)
Frequency (Hz)