Incorporating Time Delay and Magnification Distributions Predicted by Strong Lens Models into Ranking Possible Sub-threshold, Strongly Lensed Candidates

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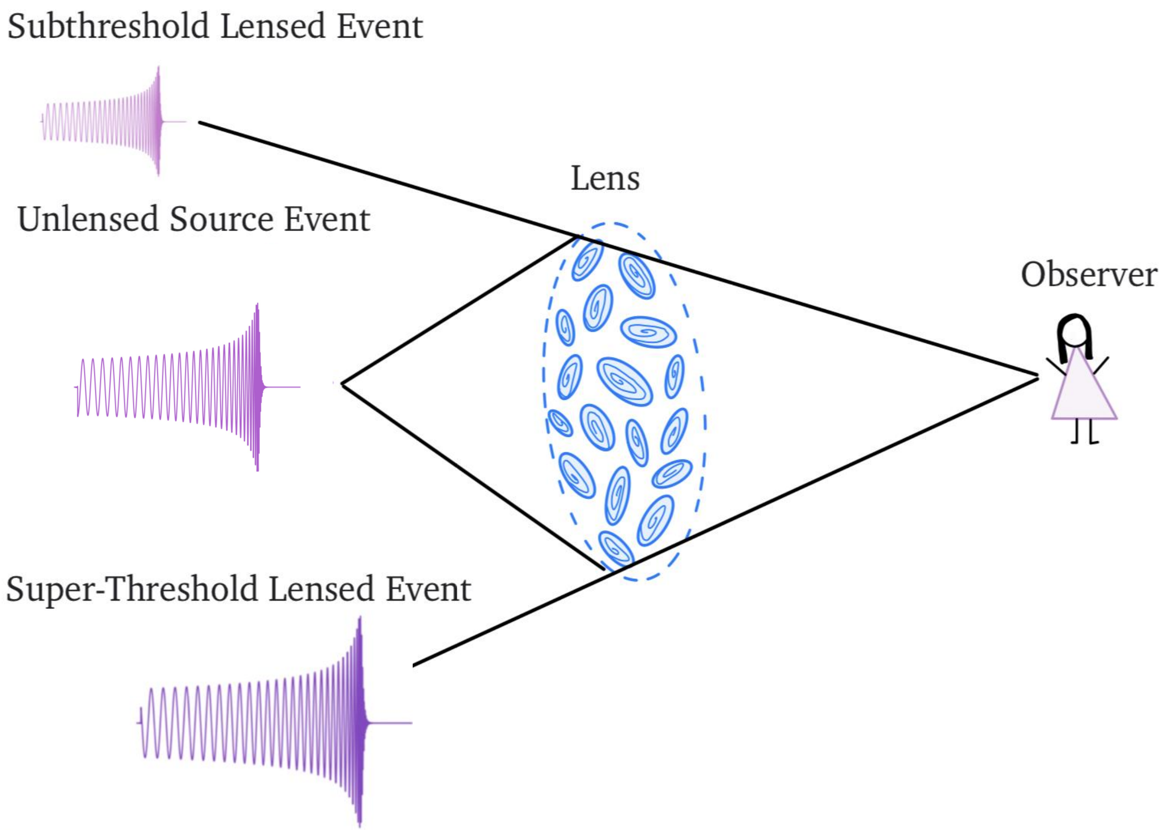
Roadmap

- What is Gravitational Lensing?
- Why do we study this?
- How do we study this?
- What is our goal?
- What are the results?

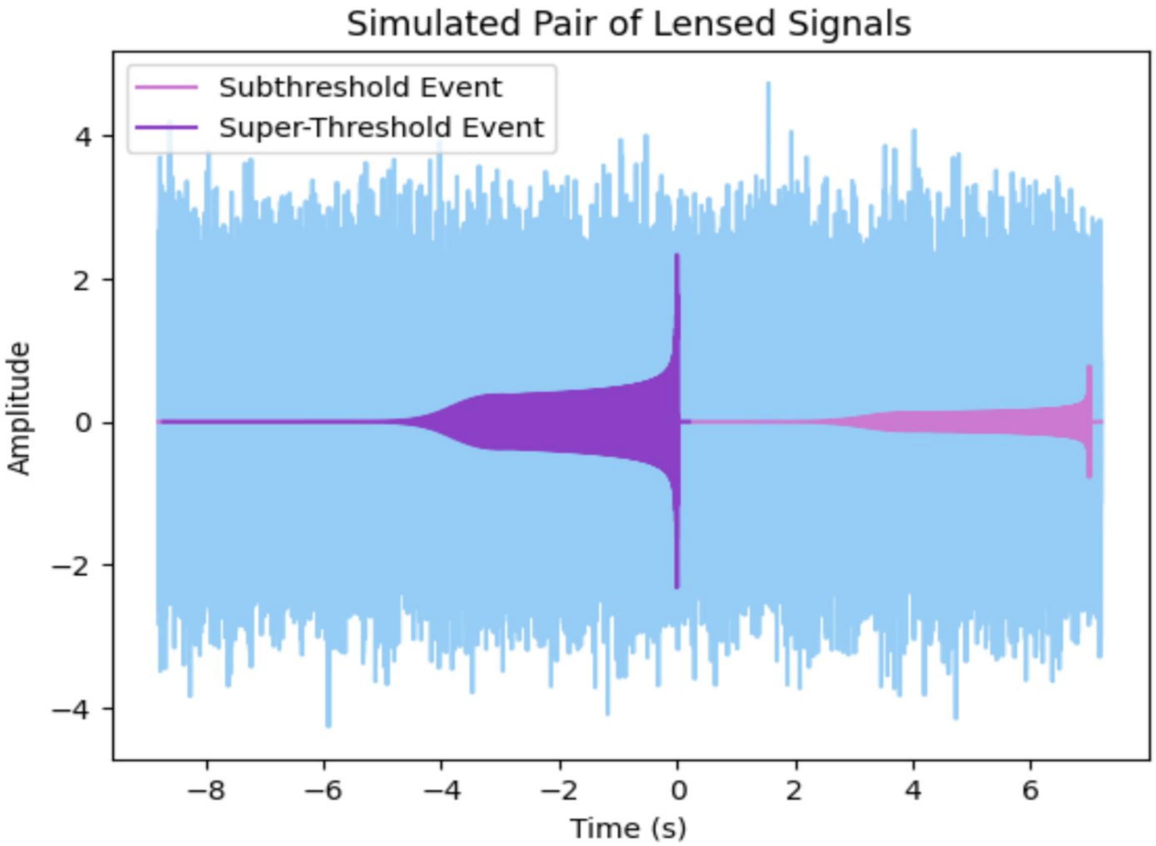
Strong Gravitational Lensing of Gravitational Waves



Key Points:

1. Results in a pair of lensed gravitational waves (GWs)
2. These have different paths/path-lengths:
   a. Causes some variation in measurements

What the Detectors See

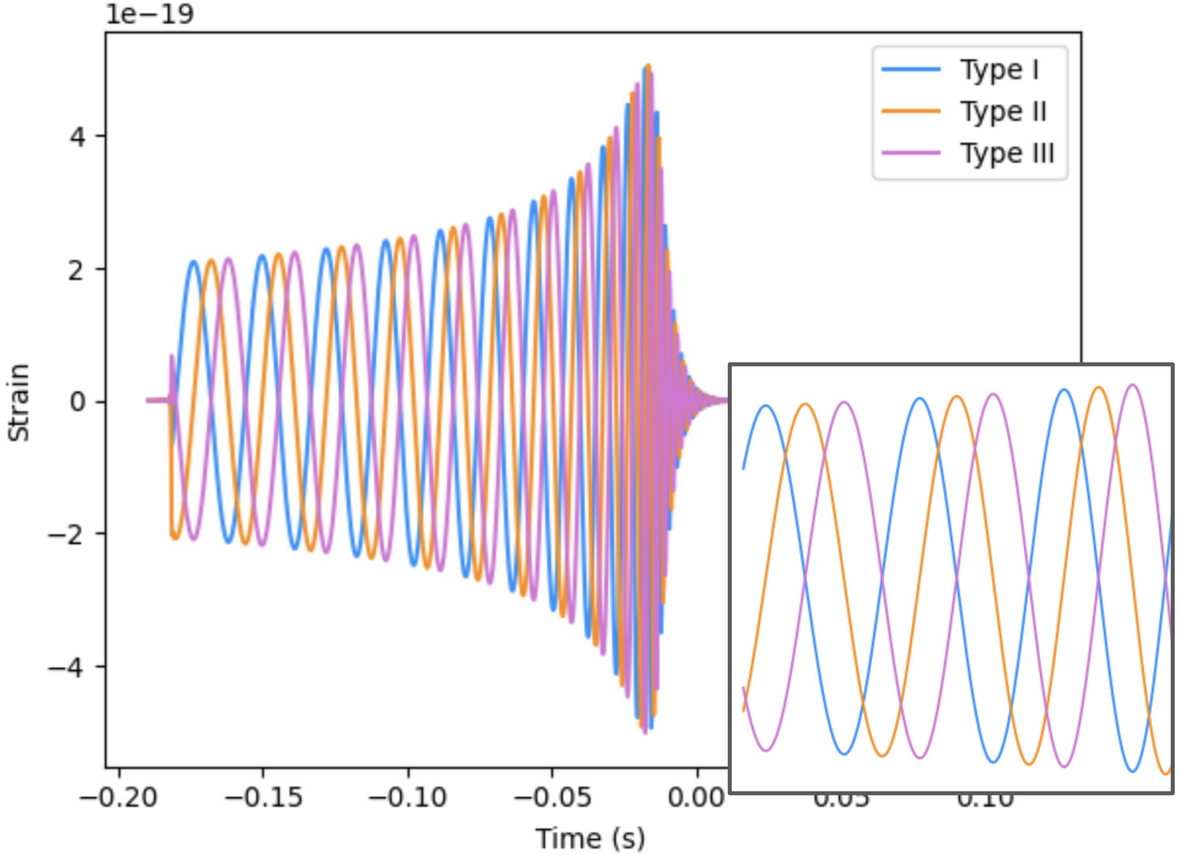


Key Points:

1. Pair of lensed GWs identical apart from:
   a. Amplitude (Signal-to-Noise Ratio, SNR)
   b. Arrival Time
   c. Morse Phase (not shown)

2. Only detect GW with larger amplitude (super-threshold)

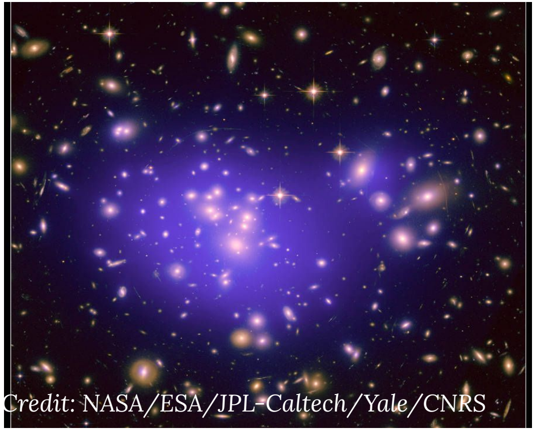
Morse Phase

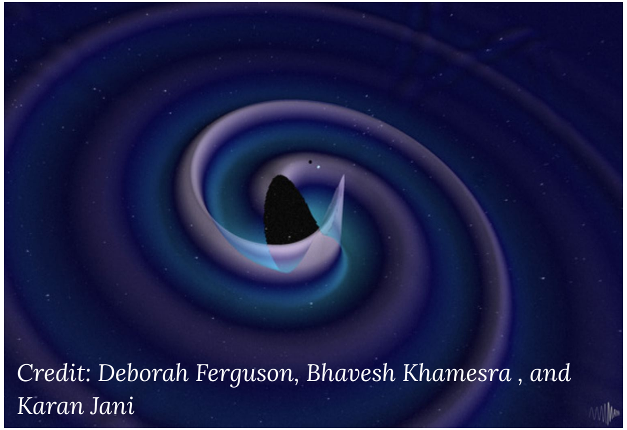


Key Points:

1. Determines overall phase shift
   a. Type I = 0; Minimum
   b. Type II = -π/2; Saddle
   c. Type III = -π; Maximum
2. Fermat Potential

Why Search for Subthreshold Events?





Must exist given equivalence principle

New way of charting the universe's dark matter distribution; Constrains Hubble constant

Constrain GW parameters like redshift and chirp mass, Joint parameter estimation

Finding Gravitational Wave Signal in the Noise

$$\mathcal{L} = \frac{\mathrm{P}(\vec{O}, \vec{D}_H, \vec{\rho}, \vec{\xi^2}, [\Delta\vec{t}, \Delta\vec{\phi}]|\mathrm{signal})}{\mathrm{P}(\vec{O}, \vec{D}_H, \vec{\rho}, \vec{\xi^2}, [\Delta\vec{t}, \Delta\vec{\phi}]|\mathrm{noise})} \times \frac{\mathrm{P}(\vec{\theta}|\mathrm{signal})}{\mathrm{P}(\vec{\theta}|\mathrm{noise})}$$

$\vec{O}$: participating detectors

$\vec{D}_H$: horizon distances for each detector (or sensitivity)

$\vec{\rho}$: matched-filter signal-to-noise ratio

$\vec{\xi^2}$: auto-correlation based signal consistency test values

Δt and $\Delta\vec{\phi}$:time and phase delay between coincident events

Bayesian Hypothesis Testing

$$\mathrm{P}(A|B) \propto \mathcal{L}(B|A) \times \mathrm{p}(A)$$

Posterior Likelihood Prior

Conditional Probability

$$\frac{\mathrm{P}(A_1|B)}{\mathrm{P}(A_2|B)} = \frac{\mathcal{L}(B|A_1) \times \mathrm{p}(A_1)}{\mathcal{L}(B|A_2) \times \mathrm{p}(A_2)}$$

Lens Model Dependent Parameters

Relative Magnification → $\mu = \frac{\rho_{\text{trigger}}}{\rho_{\text{target}}}$

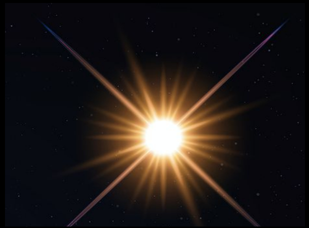
Arrival Time Delay → $\Delta t = t_{\text{trigger}} - t_{\text{target}}$

Morse Phase → $\boldsymbol{\Delta\phi}$, determined after search

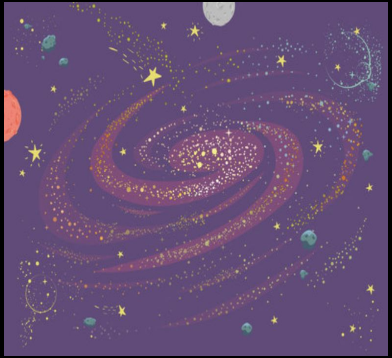
$$\mathrm{h}^{lensed}(f, \bar{\theta}, \mu, \Delta t, \Delta\phi) = \sqrt{\mu} \times \mathrm{h}^{original}(f, \bar{\theta}, \Delta t) \times \exp(i\mathrm{sign}(f)\Delta\phi)$$

Types of Lens Models

Point Mass

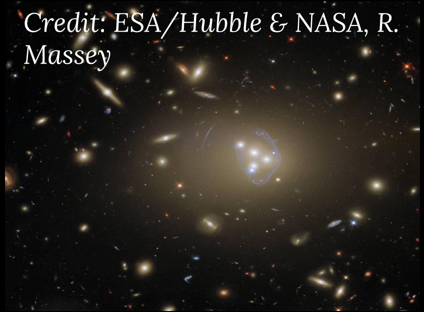


Singular Isothermal Sphere (SIS)



Singular Isothermal Ellipsoid (SIE)

Navarro-Frenk-White (NFW)



Credit: ESA/Hubble & NASA, R. Massey

Credit: NASA/CXC/UCI/A Lewis et al

…Plus many more!

Finding Lensed Signals using Registered Gravitational Waves

$$\frac{\mathrm{P}(\text{lensed}|\mu, \Delta t)}{\mathrm{P}(\text{not lensed}|\mu, \Delta t)} = \frac{\mathcal{L}(\mu, \Delta t|\text{lensed})}{\mathcal{L}(\mu, \Delta t|\text{not lensed})} \times \frac{\mathrm{p}(\text{lensed})}{\mathrm{p}(\text{not lensed})}$$

Posterior Odds

Tells us which hypothesis better explains the data

Likelihood Ratio, $\mathscr{L}$

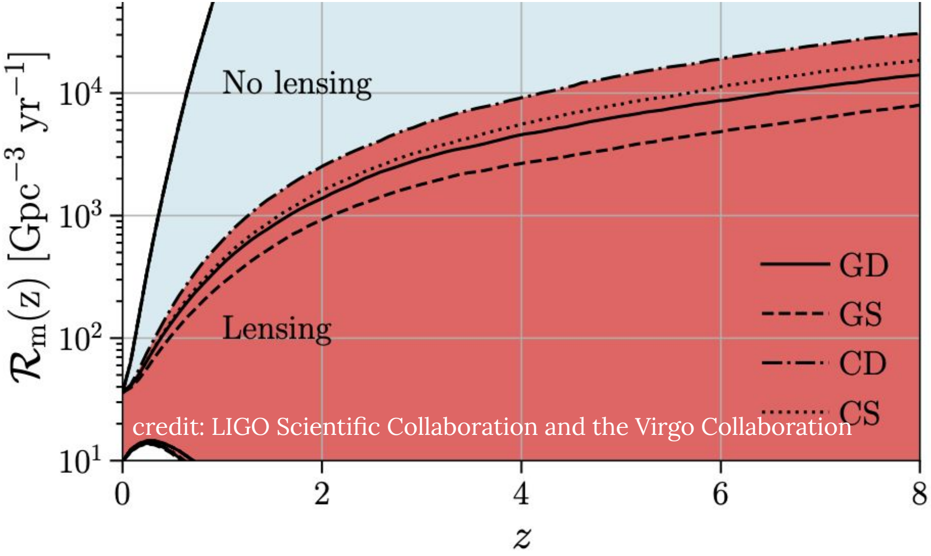
Tells us how well the data supports each hypothesis

Prior Odds, O

Measure of how strongly we believe our hypotheses

The Prior Odds

$$O^{L}_{NL} = \frac{\text{p(lensed)}}{\text{p(not lensed)}}$$



Key Points:

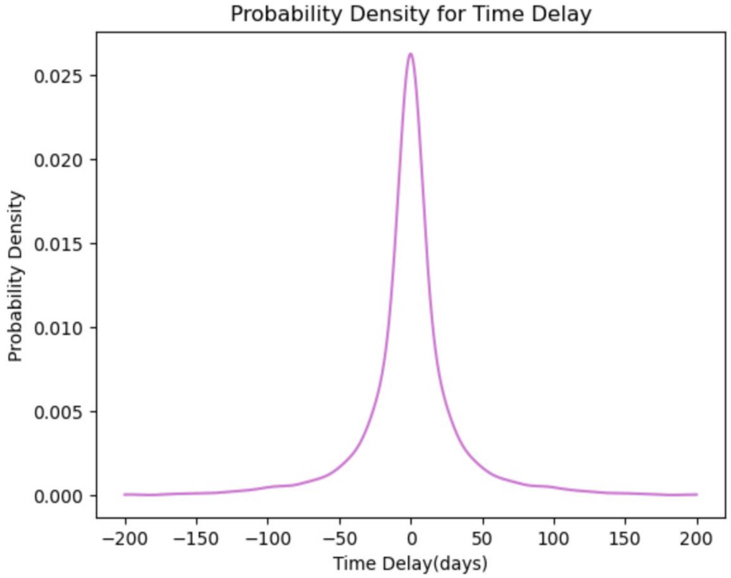
1. Prior Odds = Expected Rate of Strong Lensing
2. Have informed priors
   a. Lensing rate depends on redshift
3. Value very small, between 10^{-3} and 10^{-4}

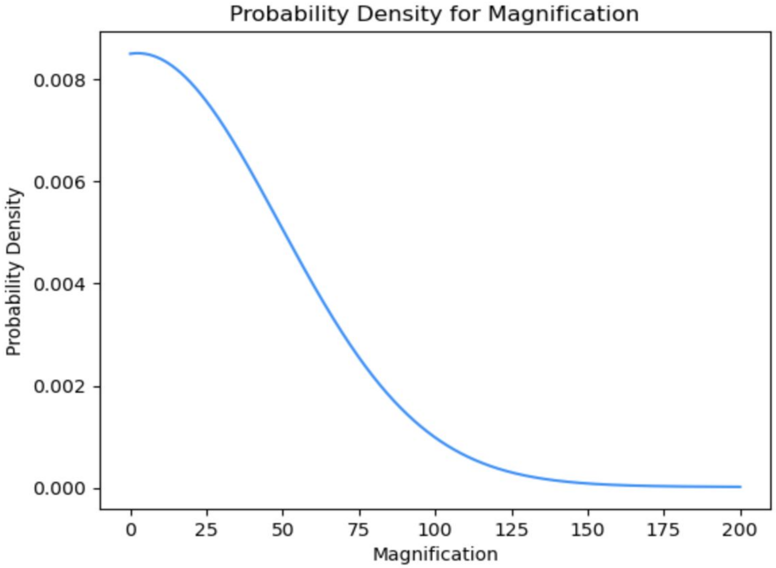
The Likelihood Ratio (or Bayes Factor)

$$\mathcal{L}^{\mathrm{L}}_{\mathrm{NL}} = \frac{\mathcal{L}(\Delta t, \mu|\mathrm{lensed})}{\mathcal{L}(\Delta t, \mu|\mathrm{not\ lensed})}$$

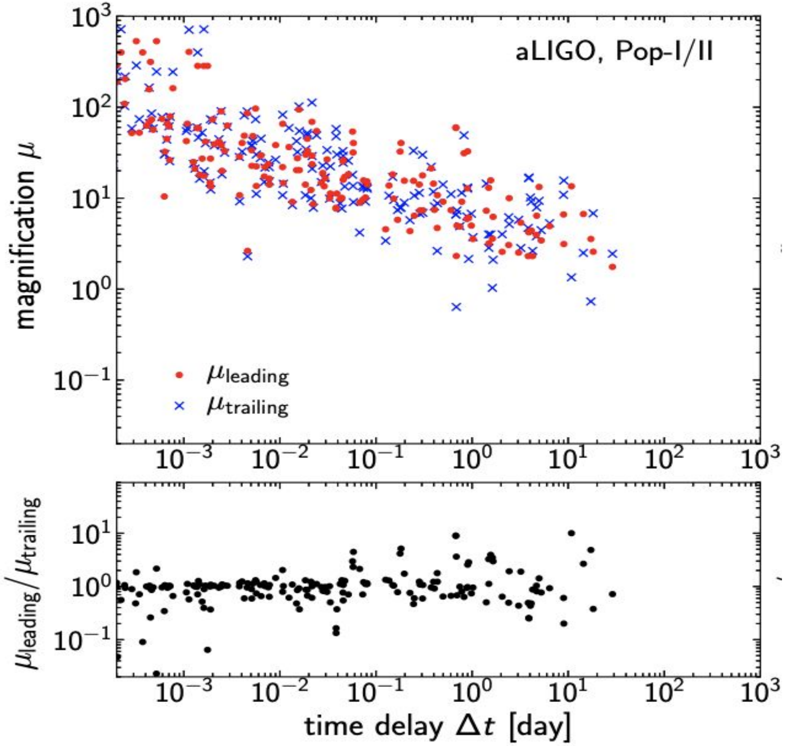
<u>Key Point(s):</u>

1. Likelihood = probability density





Finding the Time Delay/Magnification Probability Density Function



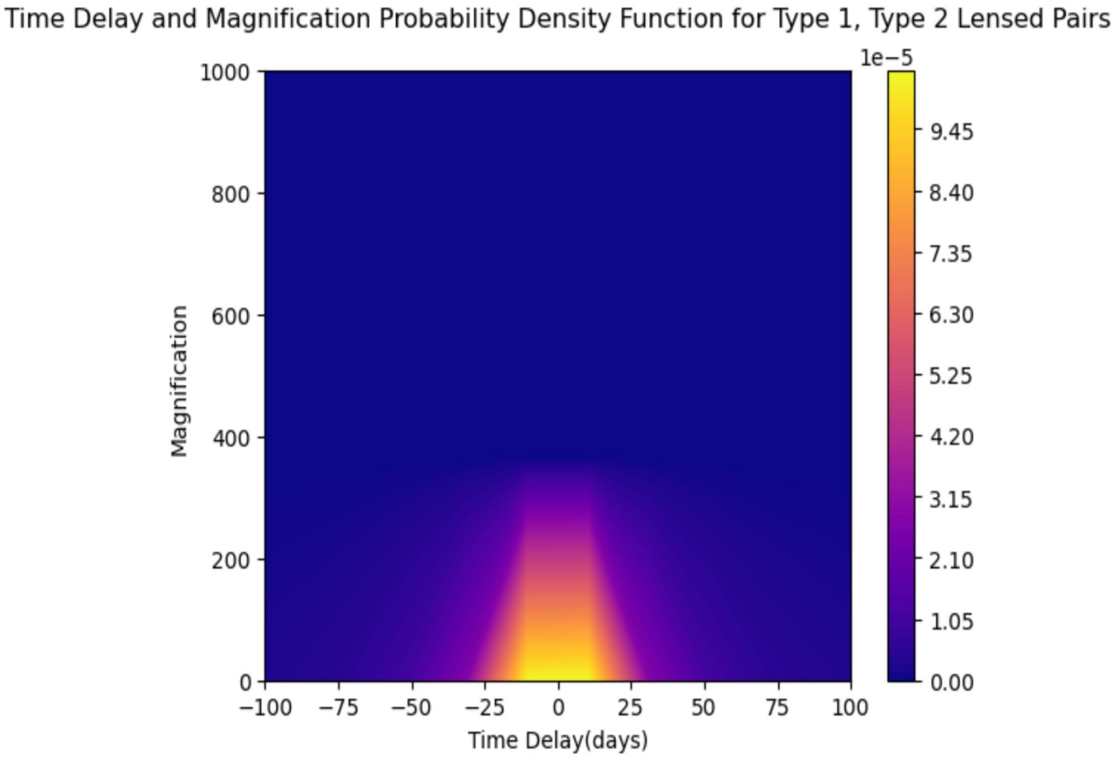
Key Points:

1. Use sample data to obtain probability density functions
   a. Gaussian kernel density estimation

2. Depends on lens model

Image credit: Masamune Oguri

Combined Probability Density Function



<u>Key Points:</u>

1. Smooth, bivariate probability density function

2. Gives the numerator of the likelihood ratio

Understanding the denominator

$$\mathrm{P}(\Delta t, \mu|\text{not lensed}) = \mathrm{P}(\Delta t|\text{not lensed}) \times \mathrm{P}(\mu|\text{not lensed})$$

$$\mathrm{P}(\Delta t|\text{not lensed}) = \frac{2}{T_{\text{obs}}}\left(1 - \frac{\Delta t}{T_{\text{obs}}}\right), \ \Delta t = t_b - t_a$$

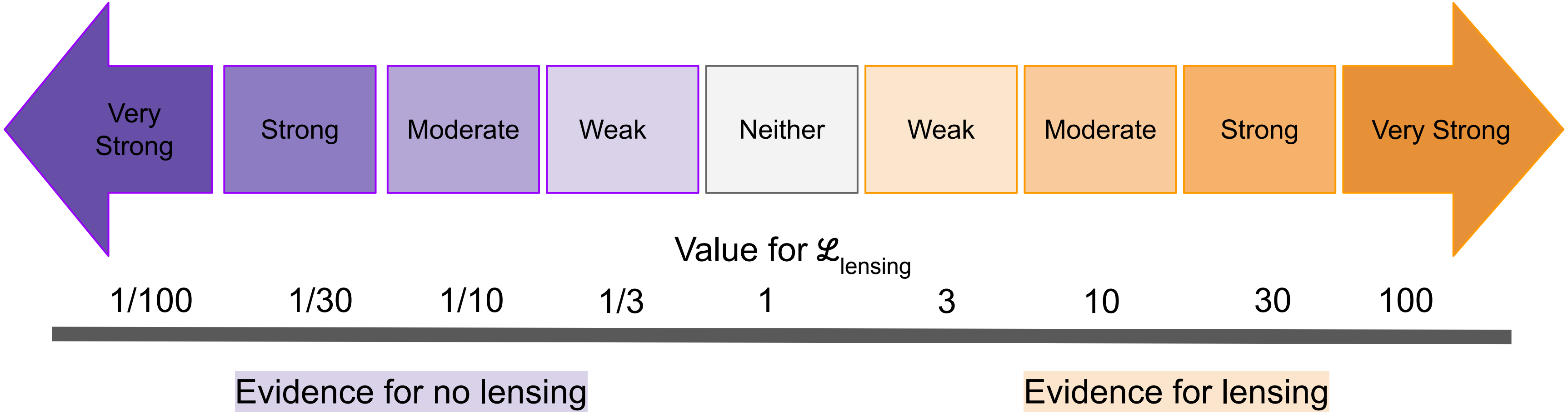
1. T_{obs} = live time of the detector(s)

$$\mathrm{P}(\mu|\text{not lensed}) = \left(\frac{d_L^a}{d_L^b}\right)^2$$

1. Rejection sampling used with redshift as a model
2. Uses relationship between magnification and luminosity distance

When does the Likelihood become Significant?

Here is the traditional standard:



Future injections will help us determine an appropriate threshold.

TESLA: TargetEd Subthreshold Lensing SeArch Pipeline

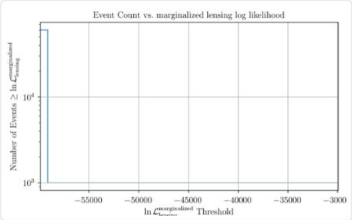
1. Posterior samples from parameter estimation
2. 1 injection with original SNR / 9 injections with lower SNRs
3. Keep templates recovered by gstLAL
4. Make reduced template bank
5. Run gstLAL, searching for sub-threshold counterparts
6. Rank candidate templates using their FAR
7. Further analysis in post-processing

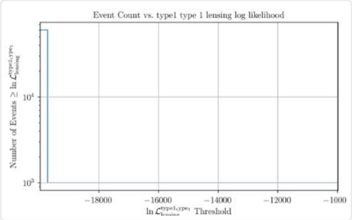
gstlal_inspiral_evaluate_lensing_likelihoods

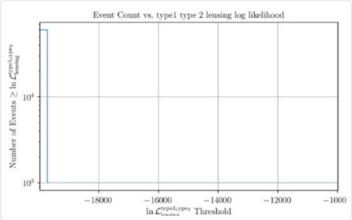
Post-Processing Code

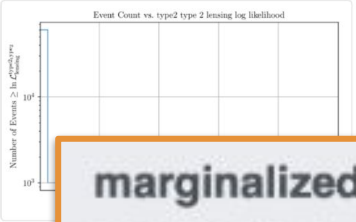
gstlal | Summary | Injection Parameters | Injection Recovery | Search Sensitivity | Background | Money Plots | About | Schedule

Lensing Log Likelihoods









Distribution of the candidates' lensing log likelihoods.

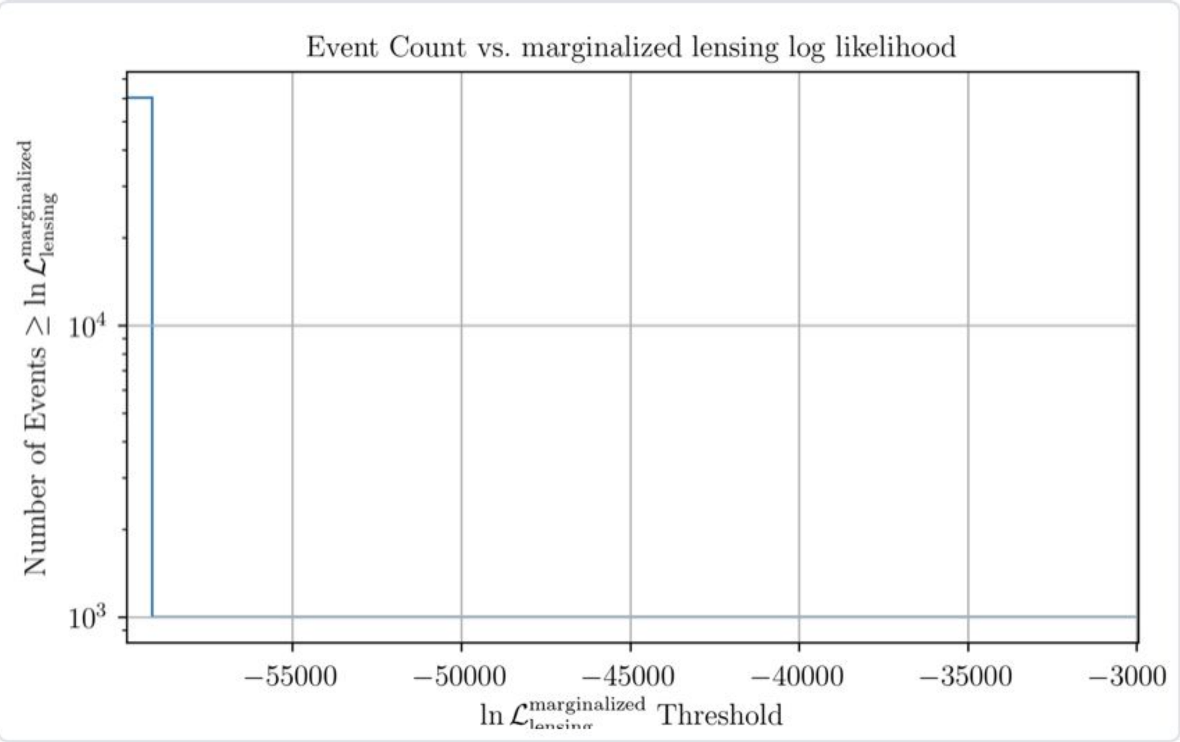
Open Box Summary Table

H1 bankchisq	L1 SNR	L1 chisq	L1 bankchisq	V1 SNR	V1 chisq	V1 bankchisq	marginalized lensing ln L	type1 type 1 lensing ln L	type1 type 2 lensing ln L	type2 type 2 lensing ln L
).8382	6.01	0.9521	0.6754	-	-	-	-29955.335908516092	-9983.415544254296	-9985.38369485338	-9986.536669408413
).9658	5.775	1.142	1.292	-	-	-	-29954.74990084154	-9983.169581671116	-9985.21358380782	-9986.366735362602

marginalized lensing ln L

-29955.335908516092

Results



<u>Key Point:</u>

1. Goal: Find events with higher lensing likelihoods

What's Next?

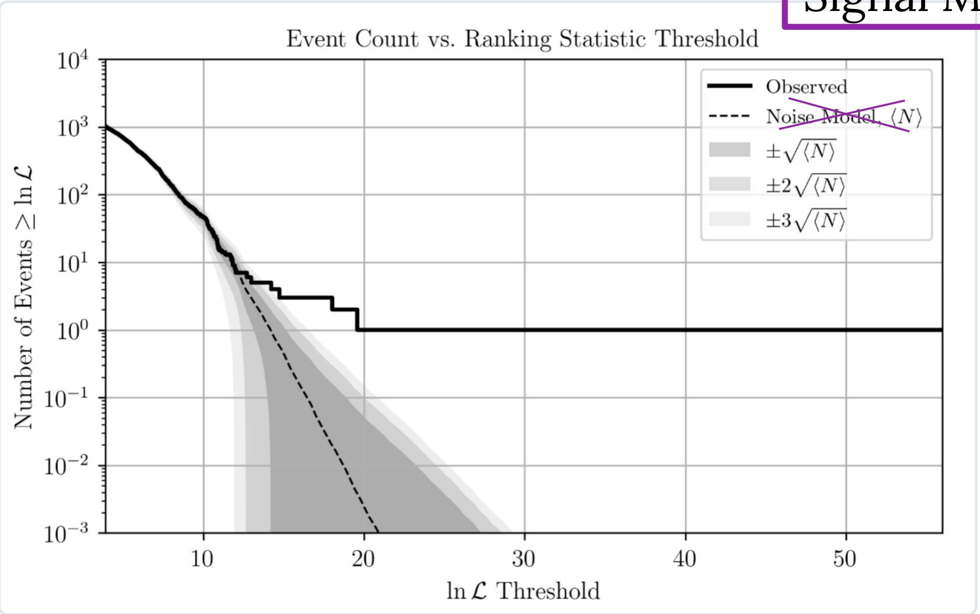
Determine threshold with non-lensed injections

Better Calculations

Implement into O4b

New "noise" curve:

Non-lensed Signal Model



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- Alan Weinstein
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- Everyone involved! :)

References

- “Search for lensing signatures in the gravitational-wave observations from the first half of LIGO-Virgo's third observing run” , **arXiv:2105.06384**
- “Bayesian statistical framework for identifying strongly lensed gravitational-wave signals”, Rico K. L. Lo, Ignacio Magana Hernandez , **arXiv:2104.09339**
- “Effect of gravitational lensing on the distribution of gravitational waves from distant binary black hole mergers”, Masamune Oguri, **arXiv:1807.02584**