Draft version June 9, 2023
Typeset using LaTeX default style in AASTeX631

Identifying Correlations in Precessing Gravitational-Wave Signals with Machine Learning

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ABSTRACT

Binary binary hole (BBH) spins provides important insights on the formation environments, evolutionary history, and dynamics of these objects, which could be of interest of the broader astrophysics community (Mandel & Farmer 2022). We would like to better measure signals for highly massive (total mass $> 100 M_{\odot}$, primary mass$\lesssim 500 M_{\odot}$), highly spinning BBH systems, which are subject to spurious measurements due to their very short duration and low bandwidth (Abbott et al. 2020). The astrophysical parameters of gravitational wave (GW) sources are extracted from match filtering observed signals to templated waveforms. The waveforms which include the most underlying physics are those generated with numerical relativity (NR). However, different parameters of NR simulation, such as mass and spin, can lead to extremely similar waveforms. In such cases, the analysis pipeline will not be able to distinguish potential sources. We are interested in constructing a neural network to study the correlations between different parameters of waveforms with spin precession and to identify potential ways to break such degeneracies. The results produced by this network would inform us the measurability of spin parameters from inferred waveform signals.

1. INTRODUCTION

GW190521 ($M \sim 150 \odot$) is the heaviest BH binary detected to date and one of the few BBHs measured to be highly precessing. It is the first strong observational evidence of intermediate-mass BH (IMBH), which is believed to be the missing link for explaining the formation of supermassive BHs (Abbott et al. 2020). The detected waveform is dominated by the merger phase where effects of precession remain elusive. In the upcoming LIGO's fourth observing run (O4) as well as in the advent of the space-based Laser-Interferometer Space Antenna (LISA) and the ground-based Einstein Telescope (ET), we expect to observe large sample of events similar to GW190521 as a result of increased detector sensitivity. Spin configurations are indicative of the compact progenitor's orbital dynamics and therefore help illuminate the formation channels of BH mergers in the pair-instability (PI) mass gap and aid population modeling of BBHs (Abbott et al. 2020; Mandel & Farmer 2022). We would like to respond to signals from such systems with maximal accuracy, which requires a thorough phenomenological understanding of the measurability of spin parameters from inferred waveforms.

2. OBJECTIVES

The objective of the project is to identify degeneracies in the parameter space for highly massive, precessing BBH systems using machine learning. I will 1) determine whether a certain set of parameters can be recovered from detected waveform, 2) investigate the correlations between degenerate parameters, and 3) quantify and produce visualizations of such correlations. Understanding the degeneracies and correlations between spin measurements will inform us as to which spin paramameters are actually independently measurable in GW data.

3. BACKGROUND & APPROACH

3.1. *Theoretical Modeling*

My project will be largely based on the neural network `mismatch_prediction` presented in (Ferguson 2023). The model is currently trained on the SXS GW catalog. For this project, I will make use waveform model `NRSur7dq4`, which is the only model calibrated to numerical simulations of precessing BH binaries. The network predicts the mismatch of the GW emitted by two BBH systems with initial input parameters $\lambda = \eta, \mathbf{a_1}, \mathbf{a_2}$, consisting of the symmetric mass ratio η

$$\eta = m_1 m_2 / \left(m_1 + m_2\right)^2 \quad 0 \leq \eta \leq 0.25$$

and the dimensionless spin vectors $\mathbf{a_1}, \mathbf{a_2}$, which are 3-dimensional vectors with x, y, z components.

$$\mathbf{a} = \frac{\mathbf{J}}{m^2} \quad 0 < a < 1$$

The model defines a mismatch metric $\mathcal{MM}$ to assess how different a resulting waveform is from an existing waveform.

$$\mathcal{MM} = 1 - \max_{t,\phi} \mathcal{O}\left[h_1, h_2\right] \equiv 1 - \max_{t,\phi} \frac{\langle h_1 \mid h_2 \rangle}{\sqrt{\langle h_1 \mid h_1 \rangle \langle h_2 \mid h_2 \rangle}}$$

where

$$\langle h_1 \mid h_2 \rangle = 2 \int_{f_0}^{\infty} \frac{h_1^* h_2 + h_1 h_2^*}{S_n} df$$

h_1, h_2 are the frequency domain strain of the waveforms, and S_n is the one-sided power spectral density of the detector. $\mathcal{MM}$ is normalized to 1 with $\mathcal{MM} = 0$ corresponding to two identical waveforms. The mismatch is computed on a flat noise curve at f_{ref}, where the spin vectors are defined. The existing network is able to identify degenerate regions in the parameter space with $l = 2, m = 2$ modes for systems of $M_{tot} = 1 M_{\odot}$, corresponding to $f_{\mathrm{ref}} = 1840$ Hz. I will be adjusting the frequency cutoff to adapt the network specifically to highly massive, precessing BBHs with fewer observable cycles.

3.2. *Degeneracy Mapping*

Since it is difficult to infer component spin parameters from GW, we use $\chi_{\mathrm{eff}}, \chi_p$ to characterize the signal detected. The effective inspiral spin χ_{eff} is the mass-weighted average of the components of the BBH's spins aligned with the system's angular momentum

$$\chi_{\mathrm{eff}} = \frac{m_1 a_1 \cos\left(\theta_1\right) + m_2 a_2 \cos\left(\theta_2\right)}{m_1 + m_2} \quad -1 \leq \chi_{eff} \leq 1$$

The effective precession spin can be modeled with a single parameter χ_p, which is defined to be the mass-weighted in-plane spin component that contributes to precession of the orbital plane at some (arbitrary) instant during the inspiral phase (Schmidt et al. 2015).

$$\chi_p = \max\left(a_1 \sin\left(\theta_1\right), \frac{4m_2^2 + 3m_1 m_2}{4m_1^2 + 3m_1 m_2} a_2 \theta_2\right) \quad 0 < \chi_p < 1$$

Expectedly, characterizing an extremely complex GW waveform with few parameters results in degeneracy in the parameter space (i.e. $\mathcal{MM} \ll 0.2$). Fig. 1 provides a visualization of the degeneracies between the parameters χ_{eff} and η in the case where BH spins are aligned with the orbital angular momentum.

I will be making similar plots with different parameters and reference data point. If time permits, I will investigate whether such degeneracies are broken at higher/different modes.

4. WORK PLAN

Before arrival: Start background reading on project.
Download required software and libraries
Familiarize myself with the mismatch-prediction model by Ferguson (2023) and understand working principles of the APIs used (TensorFlow and Keras).

Week 1-2: Orientation; Reproducing existing results
Write plotting routines.

Week 3-4: Modify framework for higher mass systems; report I

Week 5-8: Experiment with different parameters; report II
Automate algorithm (Hessian) for identifying degeneracies in parameter space.

Week 9-10: Follow-up with interesting results; final report

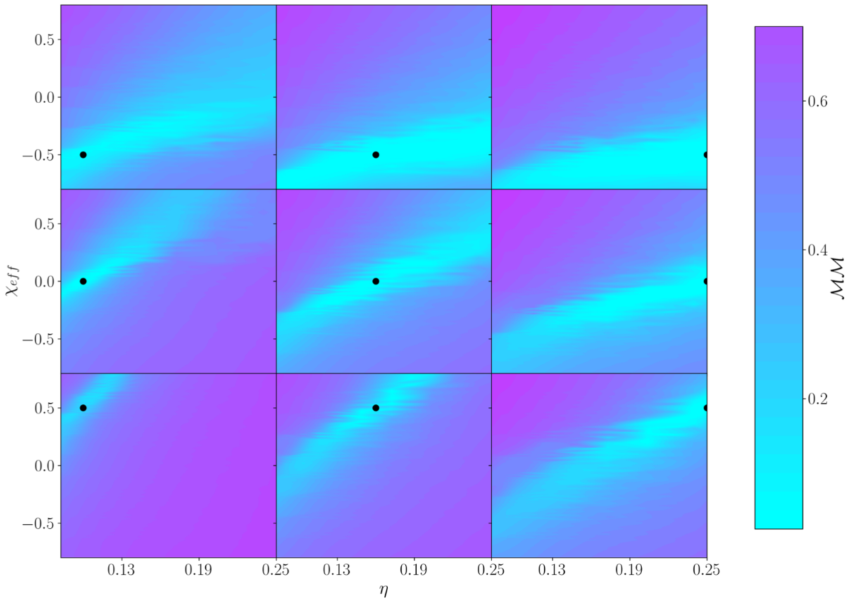

Figure 1. Mismatch between reference binary (black dot) and other binaries throughout the parameter space plotted as χ_{eff} vs. η. From left to right, reference simulation has $\eta = 0.1, 0.16, 0.25$. From top to bottom, reference has $\chi_{eff} = -0.5, 0, 0.5$. Figure from Ferguson (2023)

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