Taking it to the next level: Searching for gravitational waves with eccentricity from compact binary coalescences

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Gravitational waves (GWs)[1, 2] is a fundamental prediction of general relativity. Detecting GW has introduced a novel window into the universe and is revolutionizing ~~the potential to revolutionize~~ our understanding of astrophysics. The motion of two massive objects in an eccentric orbit emits GWs which carry information about the eccentricity of the binary black hole (BBH) source. These waveforms are characterized by their eccentricity, which measures the deviation of the orbit from a quasi-circular orbit. Studying eccentric binary orbit is crucial because eccentric binaries provide evidence for dynamic formation of the binary system. In this proposal, the study of GW waveforms from eccentric binaries and its implications for detecting and analyzing GW will be proposed. I will develop eccentric waveform models and parameter estimation frameworks for eccentric BBH and use these tools to analyze the data from current and upcoming GW observations. Since eccentric waveforms are predicted to have similar waveforms with GWs from BBH systems with non-aligned spin, I will try to distinguish eccentric waveforms and non-aligned spin waveforms by investigating their differences. The minimum eccentricity that the waveform can no longer find out is eccentric will also be studied.

I. INTRODUCTION AND MOTIVATION

A. Background

The discovery of GW, initially proposed by Einstein's theory of general relativity[3–6], has brought a new outlook on the cosmos. Exploring the properties of GW can provide us with fresh perspectives into the activities of massive objects in the universe, including the formation and progression of galaxies.

B. Gravitational Wave

GW are ripples in space-time, which propagate outward at the speed of light, generated by the acceleration of massive objects, such as binary black holes or neutron stars. Einstein's theory of general relativity first predicted them. These ripples are called GW. GW has a property called polarization, which describes the orientation of the ripples. Just as electromagnetic waves have different polarizations (linear, circular, or), GW can also have different polarizations. GW has two transverse polarization modes: plus-polarization (h_+) and cross-polarization (h_+). The terms plus and the cross will be collectively known as the linear polarization basis. They stretch and compress the spacetime in the two directions orthogonal to the direction of propagation.[7] h_+ is like the stretching and squeezing of spacetime in GW with a 45-degree angle. The impact on test particles in a h_x GW would be similar to that of a regular polarized GW but with a 45-degree rotation.

C. Gravitational Wave Detectors

Nowadays, gravitational waves can be detected by gravitational waves observatories[8], including Advanced LIGO (aLIGO)[9], VIRGO[10], and Kagra[11], which already have three observing runs[12–14] in total. BBH mergers and binary neutron star (BNS) mergers are common compact binary coalescences that generate gravitational waves. Figure 1 shows the configuration of the laser interferometer at the heart of the LIGO detectors and the laser mirrors (test masses) on their quadruple-pendulum suspensions. Since the two arm lengths have the same length, the split laser beams will have destructive interference at the output port when joined, and the detector will detect no signal. When GW passes through the detector, the arms will be stretched or compressed, resulting in length differences. The interference pattern will then be partially constructive such that a weak signal can be detected.

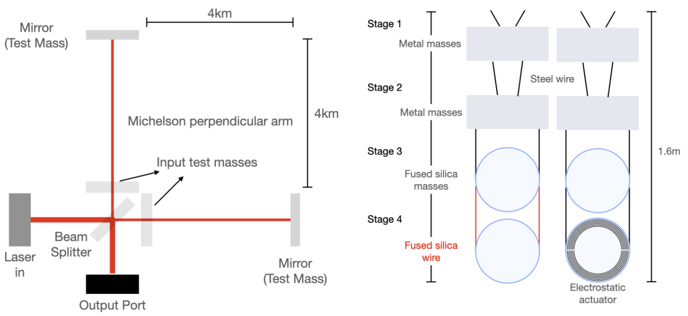

Figure 1. Interferometer configuration (Left) and test mass setup (Right)

D. Eccentric Waveforms

When two massive objects move in an eccentric orbit, they generate GW waveforms that encode the eccentricity - eccentric waveforms. The extent of orbit deviation from a perfect circle is reflected by the eccentricity of these waveforms. For a quasi-circular orbit (since eccentricity = 0), there are fifteen parameters (sixteen for BNS) to determine a GW, including the masses of the two mergers, the spin of the two mergers in three different directions (x, y, and z directions for both mergers), source distance, sky location (right ascension and declination), coalescence time, coalescence phase, inclination and polarization (with tidal deformability for BNS), in which eccentricity is not one of the parameters. Our detectors cannot detect GW when the orbital frequency is lower than 10Hz. The problem is that the orbit could be eccentric initially but become less eccentric or nearly circular when the two sources are getting close with an orbital frequency higher than 20Hz. If the eccentricity is high at 10Hz, it is predicted to be zero, which is unpredictable, by the time the orbital frequency reaches 20Hz.

Compact binary stars, black holes, and supernovas are common in the universe. Commonly GW is emitted due to a BBH system formed from two supernovas. There also exists dynamical capture in which the binary system is formed by capturing other massive objects.

It is essential to investigate eccentric gravitational waveforms as they can offer valuable information about the formation characteristics of BBH. There has been growing interest in studying gravitational eccentricity waveforms in recent years. They can provide new insights into the behavior of binary systems with highly eccentric orbits, helps to distinguish different formation scenarios of BBH (see next subsection, and Figure 2). Several studies[15, 16] have been conducted on eccentric gravitational waveforms, base on the standard approach and a novel method suggested by Stefano Schmidt[17], which uses parameter estimation. These investigations offer new perspectives into the characteristics of these waveforms and their possible uses in examining the cosmos.

E. BBH Formation in Isolation and Dynamic Capture

One of our main goals is to understand how compact binary systems form. The first is the BBH Formation in Isolation, in which the two massive stars' binary system undergoes Roche lobe overflow and common envelope. One of the stars will directly turn into a black hole. If a common envelope occurs, the giant envelope will surround the orbit of the system. Thermal energy is transferred to the envelope and may trigger the ejection of the envelope. Once the ejection of the envelope occurs, the massive star will directly turn into a black hole, leading to the inspiral of the two black holes and merging into a single one at the end. This BBH formation and merger is a common evolution in which the orbit is not eccentric. The second is BBH formation in star clusters (e.g. at the center of galaxies or in globular clusters) through dynamic capture. The binary massive star system undergoes a similar process as the BBH formation in isolation unless another massive BH is captured by the cross-section area of the two stars, ejecting the massive stars out of the original orbit and forming a new BBH system.[18] Since the new-coming BH removed some GW energy from the original orbit, the period can be decreased from ten trillion to less than ten minutes, leading to a highly eccentric orbit. Figure 2 shows the evolution of the two BBH formation mechanisms. The probability of dynamic capture depends on the original orbit's cross-section. The study of eccentric GWs is vital as we would like to determine which form of BBH system dominates. We also want to look for dynamic capture with a very small cross-section such that the time of the merger is within seconds.

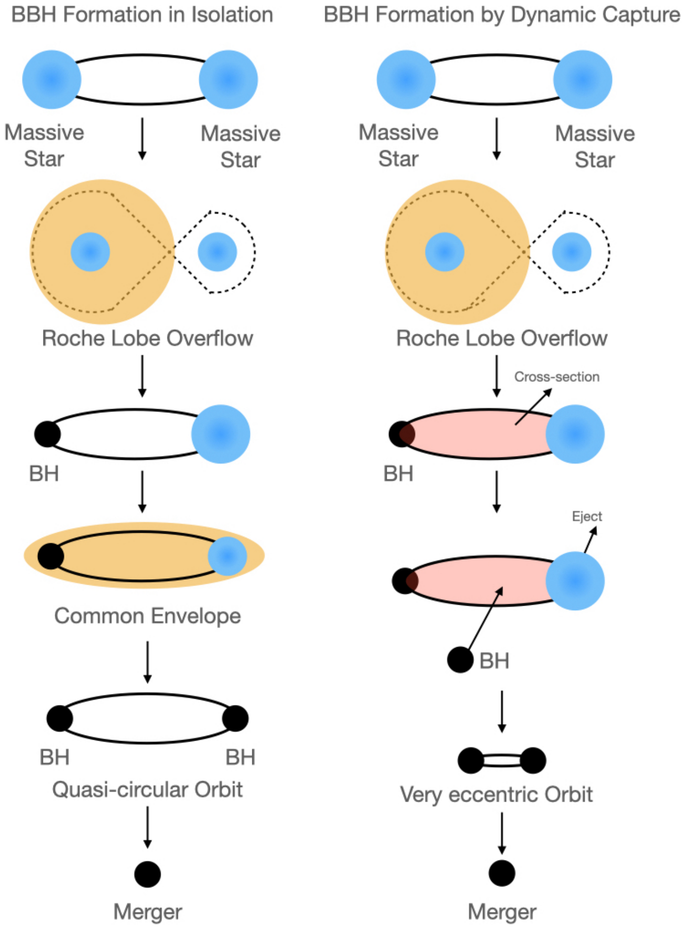

Figure 2. Following Michela Mapelli et al., BBH Formation in Isolation (Left) and by Dynamic Capture (Right)

F. Matched-filtering

Matched-filtering[19], with PyCBC[20–22] search pipeline is a technique that can detect numerous possible GW candidates from a given period with similar shapes. It can detect signals from stationary Gaussian noise. Suppose $n(t)$ is the stationary Gaussian noise process, $S_n(f)$ is the one-sided power spectral density (PSD), the matched-filtering output of a data stream is

$$x(t_0) = 2\int_{-\infty}^{\infty} \frac{\tilde{s}(f)\tilde{h}^*_{template}(f)}{S_n(f)} df \tag{1}$$

which may only contain noise $s(t) = n(t)$, or signal with noise $s(t) = n(t) + h(t)$ where $h(t)$ is the signal. Denote $h_{template}(t)$ as the filter template. Since there are unknown parameters (amplitude, coalescence phase, and binary com[anion masses) in the waveform, the "best match" unknown phase ϕ_0 has to be found by maximizing $x(t_0)$ over ϕ_0.

$$x(t_0) = x_{re}(t_0)cos2\phi_0 + x_{im}(t_0)sin2\phi_0, \tag{2}$$

where $x_{re,im}$ can be found by using Eq. 1 or Eq. 2 with $\phi_0 = 0$. Therefore, the maximum can be found in

$$x^2(t_0)|_{\hat{\phi_0}maximum} = x^2_{re}(t_0) + x^2_{i,}(t_0) \tag{3}$$

at $2\hat{\phi_0} = arg(x_{re} + ix_{im})$. The modulus of complex filter output then gives the maximum:

$$z(t_0) = x_{re}(t_0) + ix_{im}(t_0) \tag{4}$$

$$= 4\mathbb{R}\int_0^{\infty} \frac{\tilde{s}(f)(\tilde{h}^*_{template}(f))_0}{S_n(f)} e^{2\pi i f t_0} df, \tag{5}$$

where $(\tilde{h}^*_{template}(f))_0 = (\tilde{h}^*_{template}(f))_{t_0=0,\phi_0=0}$. The normalization constant for each template is calculated by

$$\sigma^2_m = 4\int_0^{\infty} \frac{|\tilde{h}_{1Mpc,m}(f)|^2}{S_n(f)} df, \tag{6}$$

such that the signal-to-noise ratio can be calculated afterward. the Amplitude signal-to-noise ratio of the quadrature matched-filtering is given by

$$\rho_m(t) = \frac{|z_m(t)|}{\sigma_m}. \tag{7}$$

If the signal is absent, then

$$< \rho^2_m >= 2. \tag{8}$$

Since for purely static and Gaussian noise, it is improbable to obtain $\rho_m >> 1$, a lower threshold on ρ_m is often used to identify event candidates.

Finally, for each trigger, a False-Alarm-Probability (FAP), which is the probability that noise can produce a trigger with a ranking statistic $\ln\mathcal{L} \geq \ln\mathcal{L}^*$, will be calculated as

$$\mathbf{FAP} = P(\ln\mathcal{L} > \ln\mathcal{L}^*|\mathbf{noise}) = \int_{\ln\mathcal{L}^*}^{\infty} P(\ln\mathcal{L}|\mathbf{noise}) d\ln\mathcal{L} \tag{9}$$

The lower the FAP, the more likely the trigger comes from an actual GW signal.

II. OBJECTIVE

In this context, the proposed research aims to evaluate the properties of existing and newly-developed waveforms. The existing data will be analyzed to compare eccentric and non-eccentric waveforms, and more profound studies regarding this parameter will be conducted. The analytical method for constructing high-accuracy templates for the GW signals emitted by compact binaries moving in inspiralling eccentric orbits proposed by Thibault Damour's team can also be tested in this research.

The study by E.A. Huerta is significant as it replicates a time-domain waveform with small eccentricity for binaries, which can provide a better understanding of the behavior of highly eccentric binary systems. The study[23] focuses on binaries with a total mass of less than or equal to 12M$_\odot$ and considers eccentricity values between 0 and 0.4. To achieve this, Huerta used a spectral approximation of the PN Kepler problem, which modeled the orbital phase as a frequency-dependent function and considered eccentricity effects up to $O(e^8)$ for each post-Newtonian order. This approach allows for a more accurate representation of the waveforms generated by highly eccentric binary systems.

Another study, conducted by Tanay et al., calculated a comprehensive analytical inspiral waveform in the frequency domain, which incorporated eccentric impacts up to the sixth order at each post-Newtonian order, with Newtonian amplitude and 2PN order Fourier phase. This study[24] also provides valuable insights into the characteristics of eccentric gravitational waveforms and their possible uses in examining the cosmos.

The research will analyze existing data and compare eccentric and non-eccentric waveforms to achieve these goals. This will provide insights into the effects of eccentricity on the waveform, which will then be used to conduct more in-depth studies on this parameter.

According to the study conducted by Philip Carl Peters[25], the time average of the rate of change of eccentricity of the orbit is given by:

$$\langle\frac{de}{dt}\rangle = -\frac{304eG^3m_1m_2(m_1+m_2)}{15c^5a^4(1-e^2)^{5/2}}(1+\frac{121e^2}{304}). \tag{10}$$

Using this equation, I can try to find out how the eccentricity of eccentric BBH orbit changes when they merge.

In the coming research period, I will study eccentric waveforms in the following aspects:

1. How does the eccentricity evolve with time as the binary system approaches merger?
2. What do eccentric waveforms look like in the time and frequency domain?
3. Develop a framework to calculate the eccentricity value when constructing parameter estimation.
4. How can eccentric waveforms be searched with parameter estimation?

5. Do the eccentric waveforms pass a set of Sanity checks in which they look reasonable and have proper limiting behavior?

6. Comparison between eccentric and non-eccentric waveforms.

7. Is it possible to find the eccentric waveform using a quasi-circular waveform in our template bank?

8. Find out the minimum eccentricity with which the eccentric waveform cannot be distinguished from a regular waveform.

9. By investigating their differences, try to distinguish eccentric waveforms and processing waveforms (non-aligned spin waveforms).

Since current research regarding eccentric gravitational waveforms only considers small eccentricity, studying eccentricity in gravitational waveforms, regardless of its magnitude, is crucial as it can provide a more comprehensive understanding of the behavior of compact binary systems in a broader range of eccentricities. This research can shed light on the physics behind the inspiral and merger of binaries with higher eccentricities, which can lead to the detection of more GW signals from such systems. Moreover, it can help improve our existing models for the dynamics of compact binaries, which can lead to more accurate parameter estimation and GW detection. Additionally, studying high eccentricities can help us understand the astrophysical processes responsible for producing such systems and their implications for cosmology and astrophysics. Therefore, this research can have significant implications for understanding the universe and the properties of compact objects such as neutron stars and black holes.

III. METHODS

To conduct this study, simulated eccentric waveforms with different magnitudes of eccentricity will be generated. These waveforms will be compared to standard quasi-circular waveforms using matched-filtering and sanity checks to identify apparent differences. The study of eccentric waveforms involves using waveform overlap with standard waveforms. By comparing them, we can quantify the waveforms using parameter estimation, ensuring that eccentricity is considered when generating or detecting GW signals. Any waveforms with apparent differences from regular waveforms will be saved for further usage, including the construction of an eccentric waveform template bank.

The next step involves performing an injection study to determine the minimum eccentricity with which the eccentric waveform can be distinguished from a regular waveform. This involves injecting a simulated eccentric waveform into a random signal and attempting to use a non-eccentric waveform template to recover it.

The injection study will allow researchers to understand better the properties of eccentric waveforms and how they differ from regular waveforms. This information can then be used to improve the detection of GW from eccentric sources, such as binary systems with large eccentricities.

I will study eccentric waveforms by the following steps in short:

1. Generate eccentric and non-eccentric waveforms

2. Compare the waveforms

3. Quantify the waveform overlaps

4. Compute and plot the match in both time and frequency domain

5. Repeat Steps 1 4 for similar eccentricity

6. Repeat Steps 1 4 for non-aligned spin waveforms

IV. TIMELINE

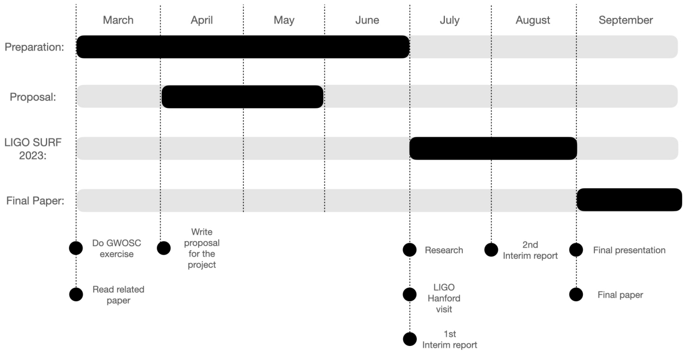

Figure 3. Timeline of this proposed project

This research project will last two months, from 21^{st} June to 25^{th} August in Caltech in the United States of America. Before the start of the project, preparation is necessary. From late February to mid-April, signal processing skills and background knowledge is learned. This proposal will be submitted in May. The project will then start from 21^{st} June. Interim reports will be written in weeks 3 and 7 of the 10 week project period. While constructing the project, there will be a detector visit in LIGO Hanford in the mid of July. In the tenth week, a comprehensive presentation about the result of this project will be performed. After that, a final report will be written in Fall (After August). A timeline chart is shown in Figure 1.

V. SUMMARY

To prepare for the project at Caltech, it is vital to delve deeper into the subject matter in the upcoming months. This includes reading more scientific papers about GW, particularly those related to eccentricity. Through this, one can gain a more profound understanding of the phenomenon and the parameters that govern it. In addition, consolidating signal-processing knowledge and programming skills is crucial, enabling the individual to analyze and interpret the collected data accurately. Furthermore, generating eccentric waveforms using parameter estimation techniques can provide valuable insights into the characteristics of the waves. Comparing these new signals with those obtained from previous observing runs can help identify trends, patterns, and anomalies, paving the way for discoveries and breakthroughs.

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