Caltech

NANOGrav tutorials

or

How to look for nanohertz gravitational waves

Aaron Johnson
aaron.johnson@nanograv.org

NANOGrav
Physics Frontiers Center

Gravitational Wave
Astronomy Northwest
Student Workshop
6/26/2023

1. Pulsar Timing Array

A window to the nanohertz gravitational-wave sky

Gravitational wave landscape

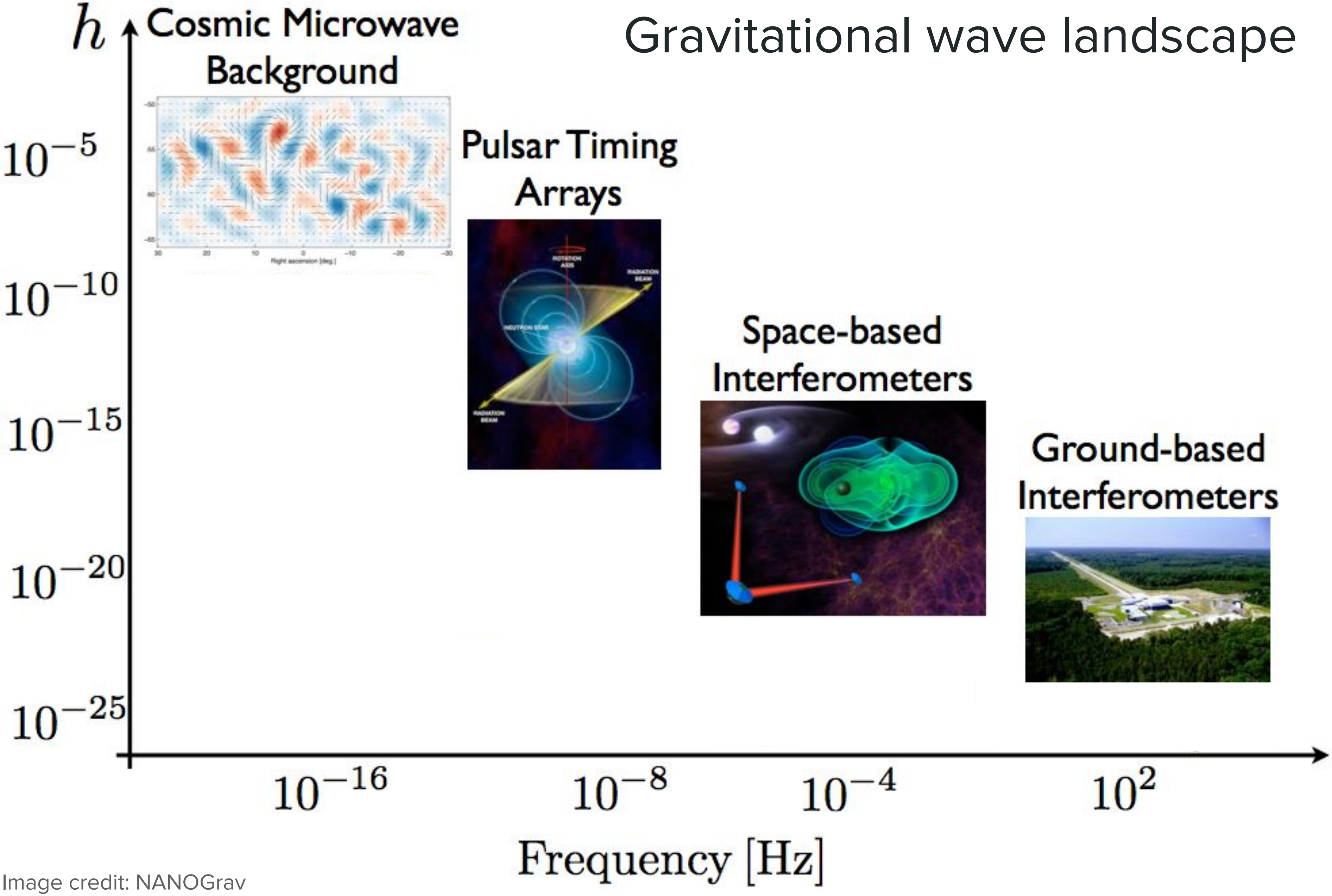
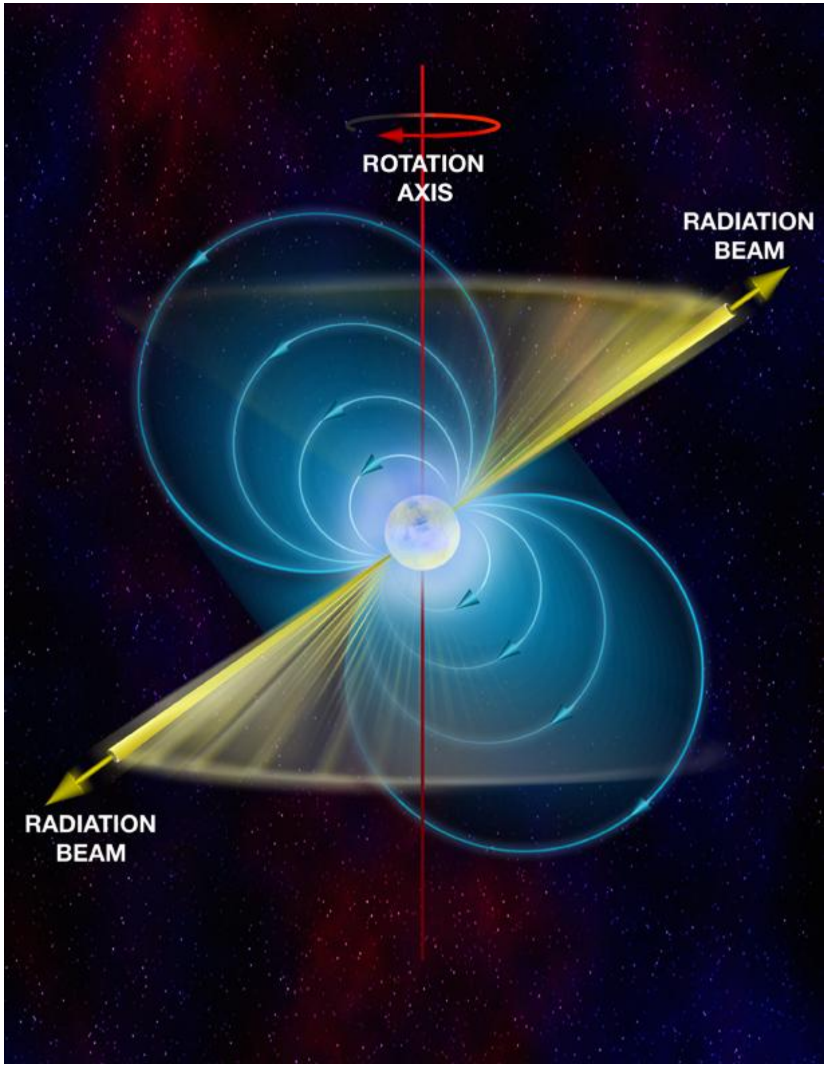
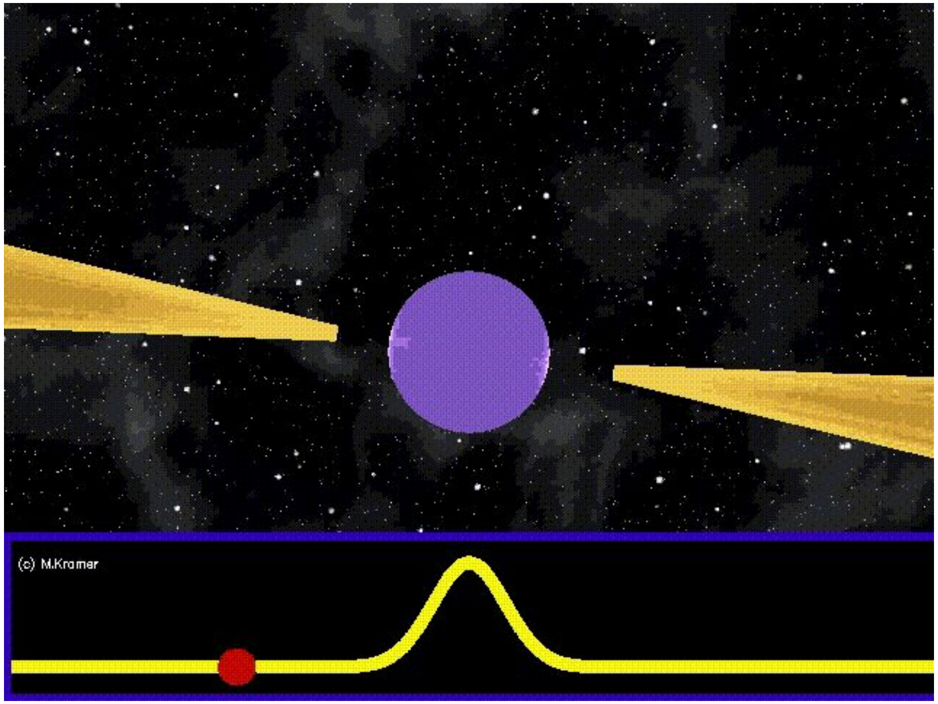


Image credit: NANOGrav

Pulsars





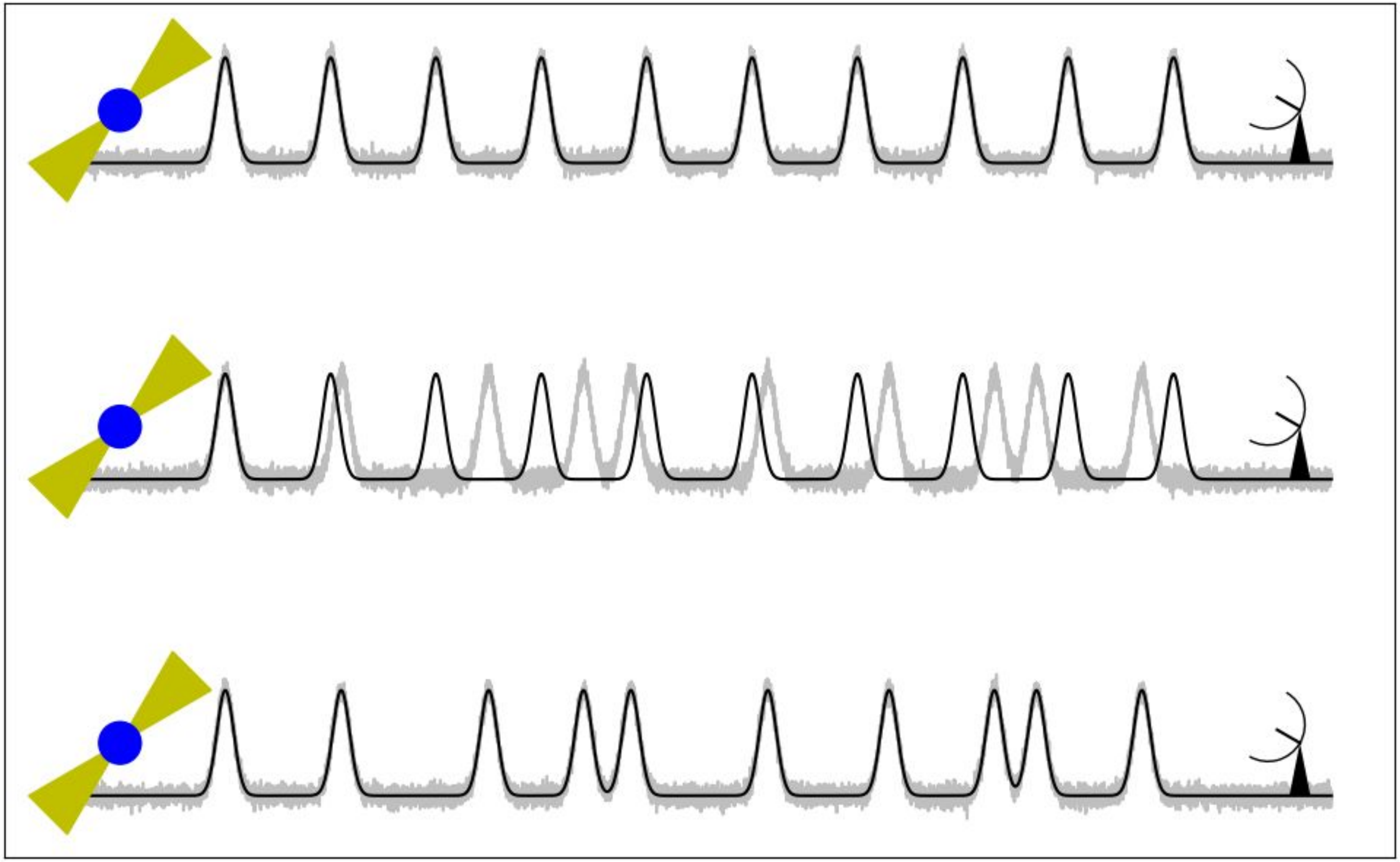
Extreme precision!

Spin period of PSR J0437-4715:

0.00575745193671259±0.000000000000000002s

Image credit: (left) Bill Saxton, NRAO/AUI/NSF; (right) M. Kramer

Pulsar Timing



Pulsar Timing

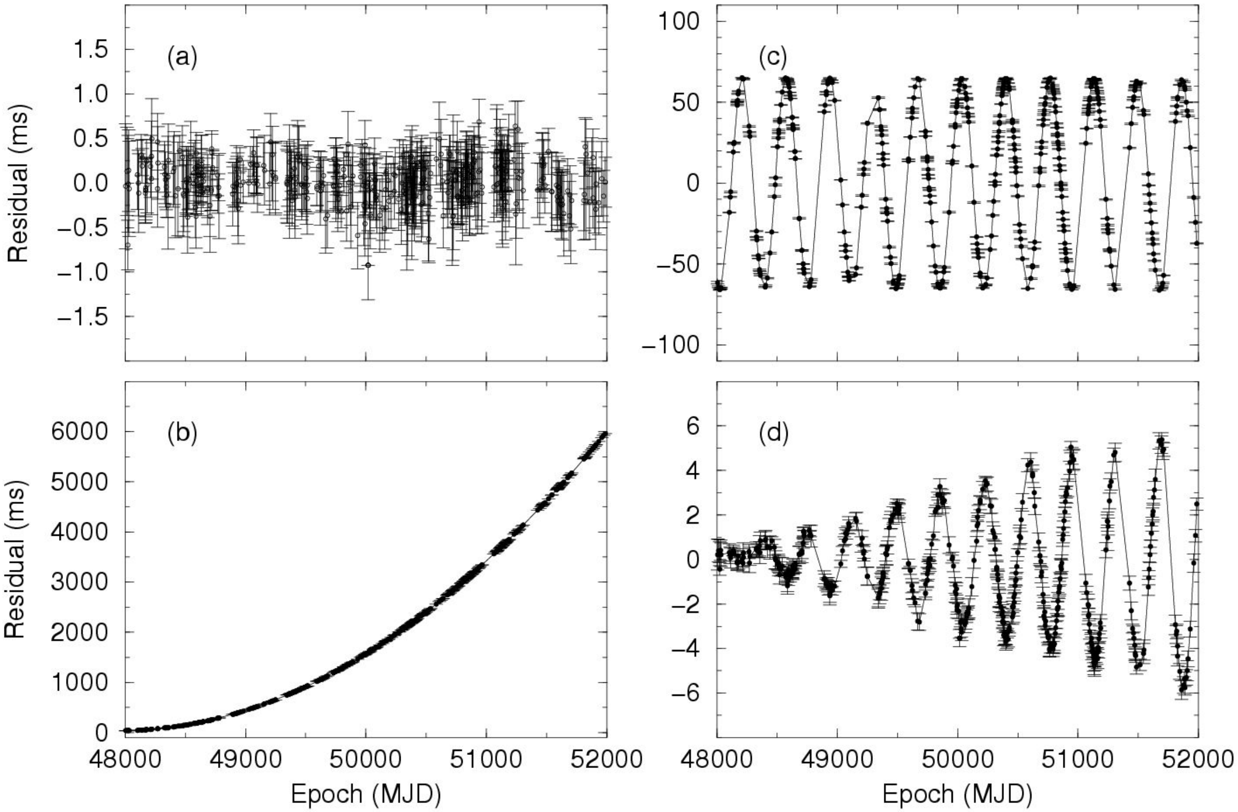


Image credit: Handbook of Pulsar Astronomy, Lorimer and Kramer

Timing model
(a) good
(b) wrong frequency derivative
(c) wrong sky position
(d) wrong proper motion

Other effects:

Earth rotates and orbits, interstellar dispersion, NS system has a proper motion, pulsar spin-down, etc.

- \+ Gravitational waves

Pulsar Timing Arrays

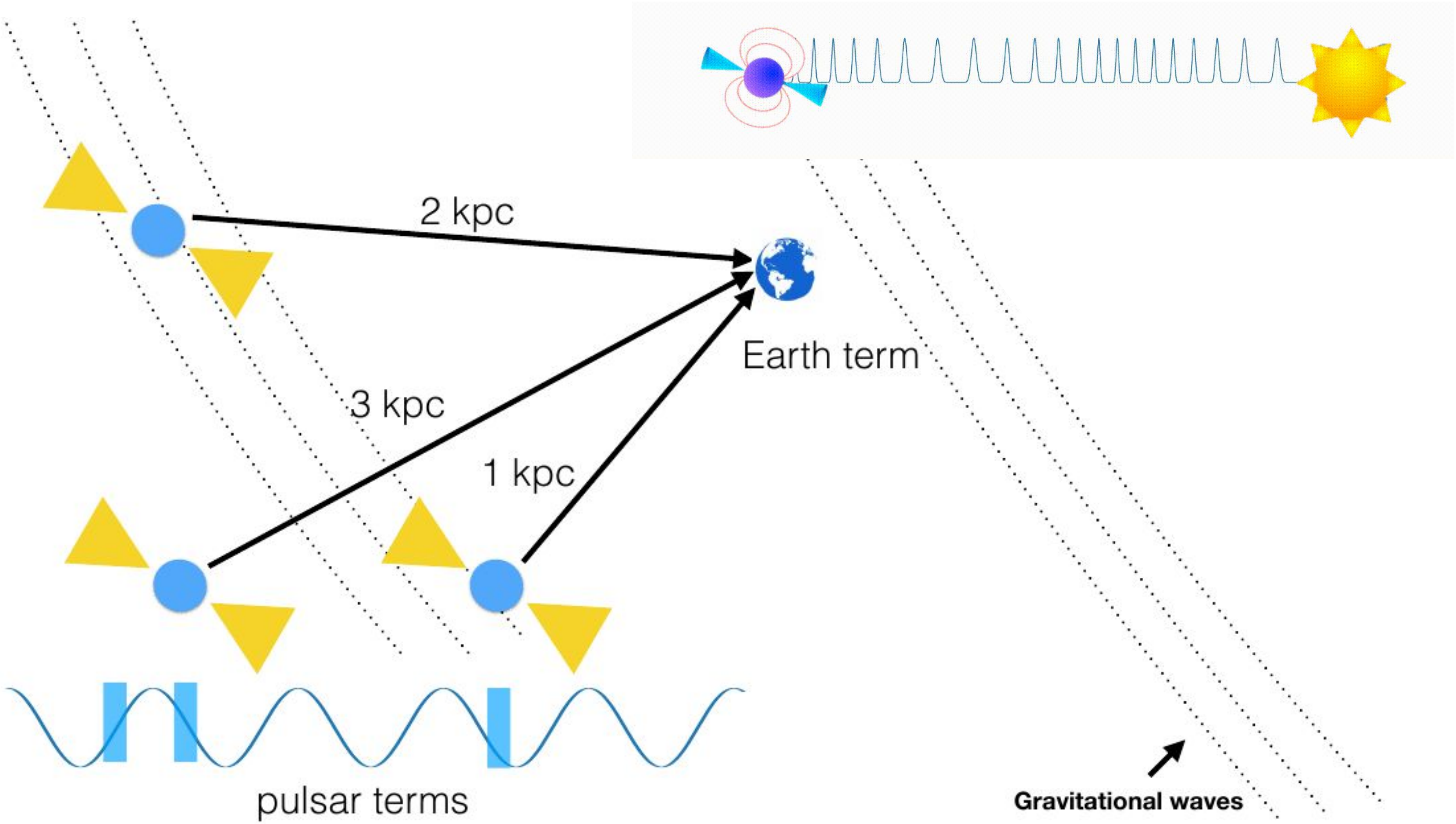


Image credit: Burke-Spolaor, Taylor et al. (2018) arXiv:1811.08826

- North American Nanohertz Observatory for Gravitational Waves
- > 150 members (~65 faculty, ~30 postdocs, and ~45 graduate students)
- > 55 institutions in the US and Canada
- \+ about 100 undergraduate students annually
  through the STARS (Student Teams of Astrophysics Researchers) program

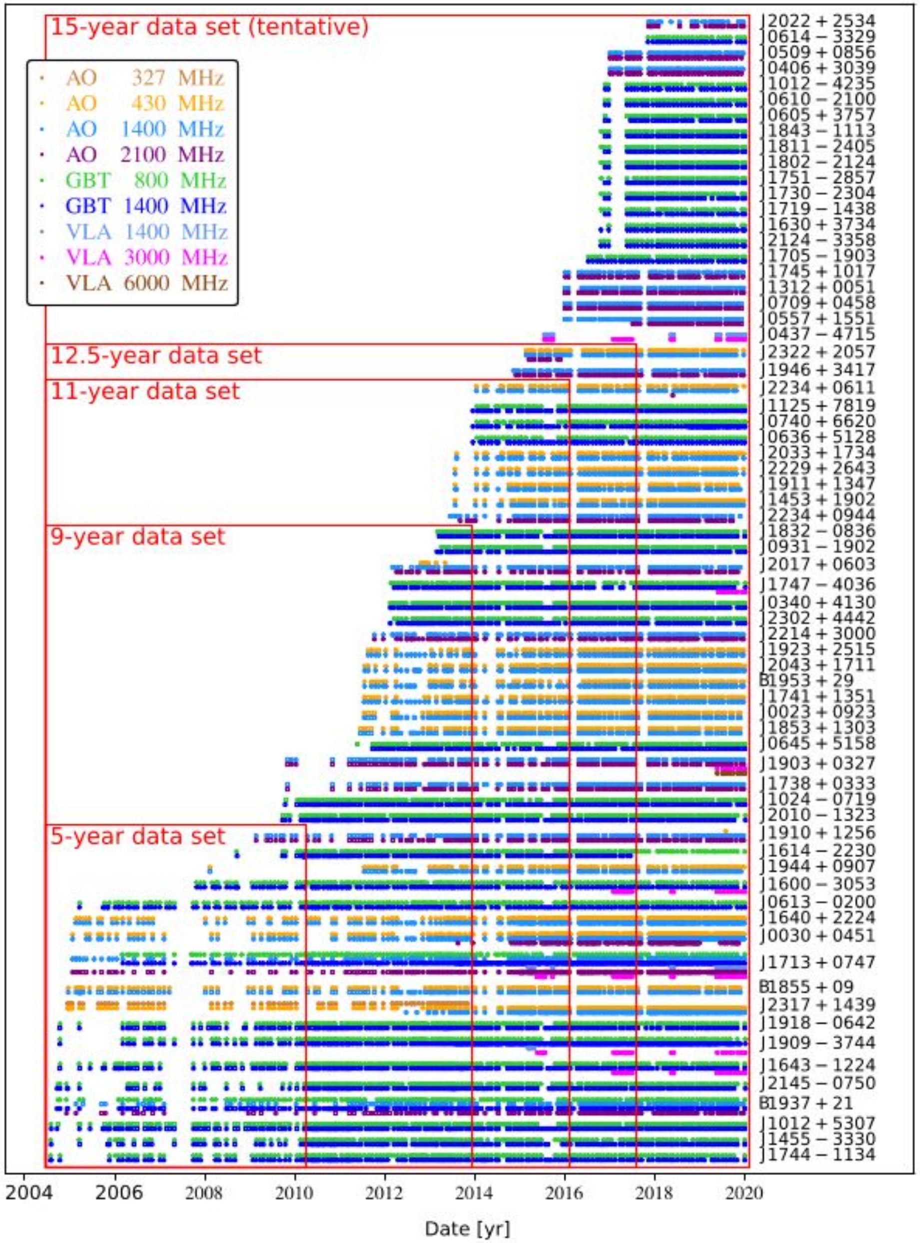


Image credit: (left) David Nice; (right) NANOGrav

Green Bank Telescope

Arecibo Observatory

Potential nHz GW signals

- Deterministic signals
  - Continuous GWs from individual supermassive black hole binaries (SMBHBs)
  - Nonlinear GW memory
  - Generic GW transients (aka bursts)

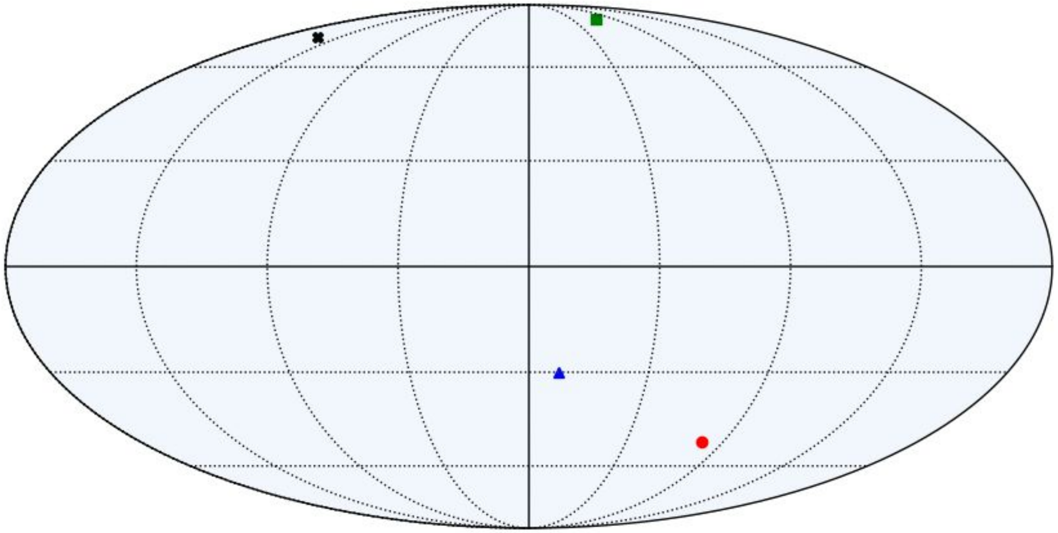
- **Stochastic GW backgrounds** ← subject of this tutorial
  - SMBHBs
  - Cosmic strings
  - Phase transitions

Isotropic stochastic GW background

Stochastic background = superposition of weak signals

Detection progression:

1) All our detectors (pulsars) show the same "noise"
2) Characteristic quadrupolar correlations between pulsars (Hellings & Downs)



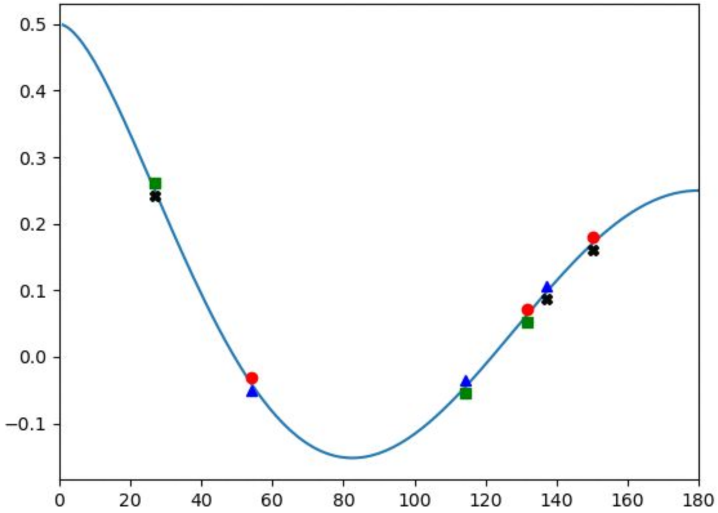
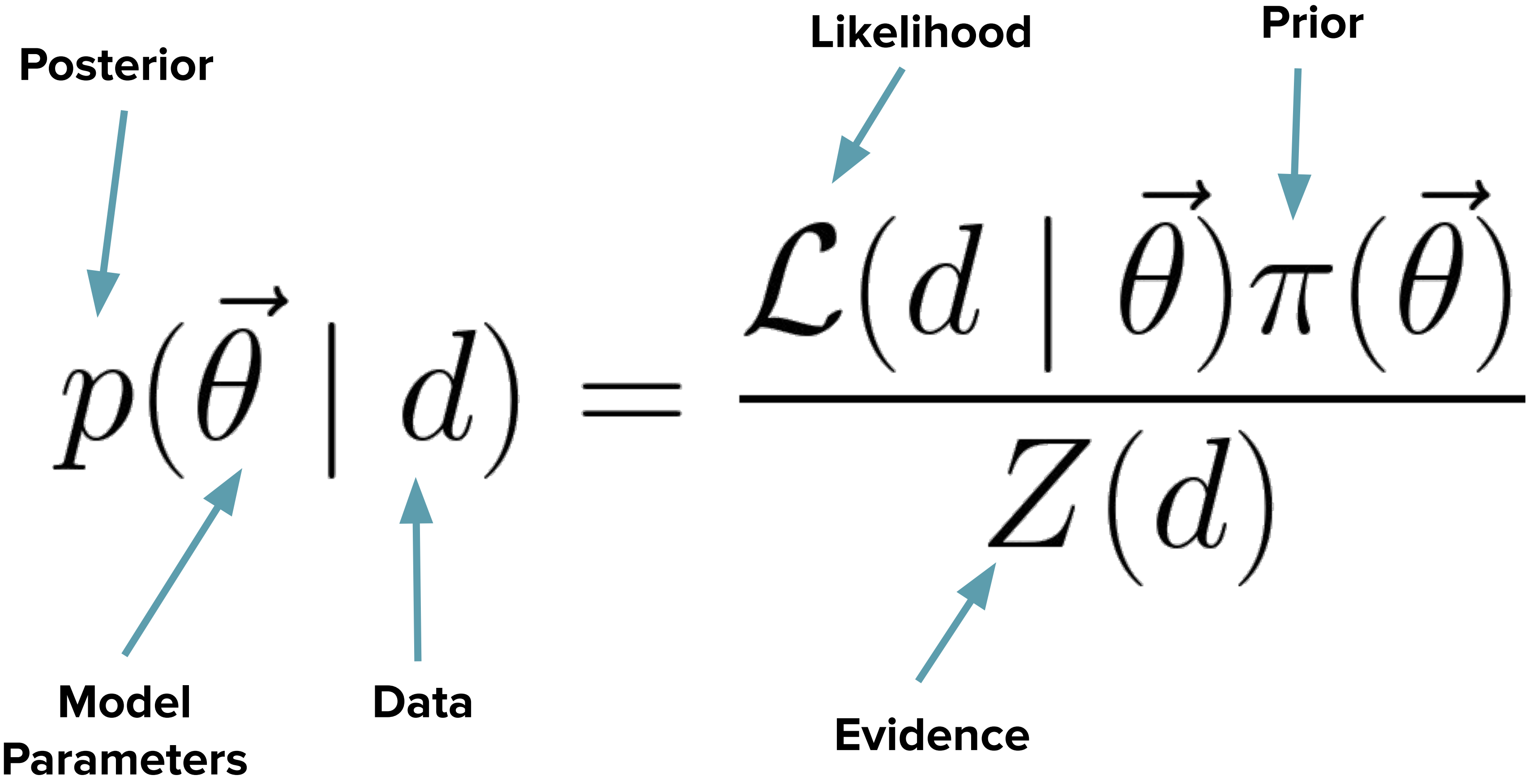


Image credit: Neil Cornish

3. Bayesian methods

Intro to Bayesian PTA model



Posterior

Likelihood

Prior

$$p(\vec{\theta} \mid d) = \frac{\mathcal{L}(d \mid \vec{\theta})\pi(\vec{\theta})}{Z(d)}$$

Model Parameters

Data

Evidence

PTA likelihood

$$\vec{t} = \vec{t}_{\mathrm{det}} + \vec{t}_{\mathrm{stoch}}$$

PTA likelihood

$$\vec{t} = \vec{t}_{\rm det} + \vec{t}_{\rm stoch}$$

$$\vec{t}_{\rm det} \approx \vec{t}_{\rm tm} + M\vec{\epsilon}$$

PTA likelihood

$$\vec{t} = \vec{t}_{\mathrm{det}} + \vec{t}_{\mathrm{stoch}}$$

$$\vec{t}_{\mathrm{det}} \approx \vec{t}_{\mathrm{tm}} + M\vec{\epsilon}$$

$$\vec{t}_{\mathrm{stoch}} = F\vec{a} + \vec{n}$$

PTA likelihood

$$\vec{t} = \vec{t}_{\mathrm{det}} + \vec{t}_{\mathrm{stoch}}$$

$$\vec{t}_{\mathrm{det}} \approx \vec{t}_{\mathrm{tm}} + M\vec{\epsilon}$$

$$\vec{t}_{\mathrm{stoch}} = F\vec{a} + \vec{n}$$

$$\underbrace{\delta\vec{t}}_{\textbf{Residuals}} = \vec{t} - \vec{t}_{\mathrm{tm}} \approx \underbrace{M\vec{\epsilon}}_{\textbf{Timing Model Uncertainties}} + \underbrace{F\vec{a}}_{\textbf{Red Noise}} + \underbrace{\vec{n}}_{\textbf{Gaussian White Noise}}$$

Quick note on Fa

$$F\vec{a} = \sum_{j=1}^{N} \left[X_j \sin\left(2\pi f_j t\right) + Y_j \cos\left(2\pi f_j t\right)\right]$$

$$\mathbf{F} = \begin{pmatrix} \sin\left(2\pi t_1/T\right) & \cos\left(2\pi t_1/T\right) & \cdots & \sin\left(2\pi N_f t_1/T\right) & \cos\left(2\pi N_f t_1/T\right) \\ \sin\left(2\pi t_2/T\right) & \cos\left(2\pi t_2/T\right) & \cdots & \sin\left(2\pi N_f t_2/T\right) & \cos\left(2\pi N_f t_2/T\right) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sin\left(2\pi t_N/T\right) & \cos\left(2\pi t_N/T\right) & \cdots & \sin\left(2\pi N_f t_N/T\right) & \cos\left(2\pi N_f t_N/T\right) \end{pmatrix}$$

$$\vec{a}^T = (X_1, Y_1, X_2, Y_2, \ldots, X_N, Y_N)$$

PTA model

$$\vec{r} = \delta\vec{t} - M\vec{\epsilon} - F\vec{a}$$

$$p(\delta\vec{t} \mid \vec{a}, \vec{\epsilon}) = \frac{1}{\sqrt{\det(2\pi \boldsymbol{N})}} \exp\left(-\frac{1}{2}\vec{r}^{T}\boldsymbol{N}^{-1}\vec{r}\right)$$

Marginalization

Params

$$\mathcal{L}(\delta\vec{t} \mid \vec{\eta}) = \frac{1}{\sqrt{\det(2\pi C)}} \exp\left(-\frac{1}{2}\delta\vec{t}^{T}C^{-1}\delta\vec{t}\right)$$

Check out: “The Nanohertz Gravitational Wave Astronomer” by Steve Taylor (https://arxiv.org/abs/2105.13270)

Noise parameters $(\vec{\eta})$

- White noise
    - EFAC (scale factor)
    - EQUAD (quadrature)
    - ECORR (correlated)
- Red noise (power law)
    - Amplitude
    - Spectral Index

- Timing Model Uncertainties
    - Marginalized
- Power law total params:
    - 3 WN per backend
    - 2 IRN per pulsar
    - 2 CRN/HD

600 total params for 12.5 year data

$$\rho(f) = \frac{A^2}{12\pi^2}\frac{1}{T}\left(\frac{f}{1\mathrm{yr}^{-1}}\right)^{-\gamma}\mathrm{yr}^2$$

Even with white noise parameters **fixed**, PTAs have > 91 parameters.

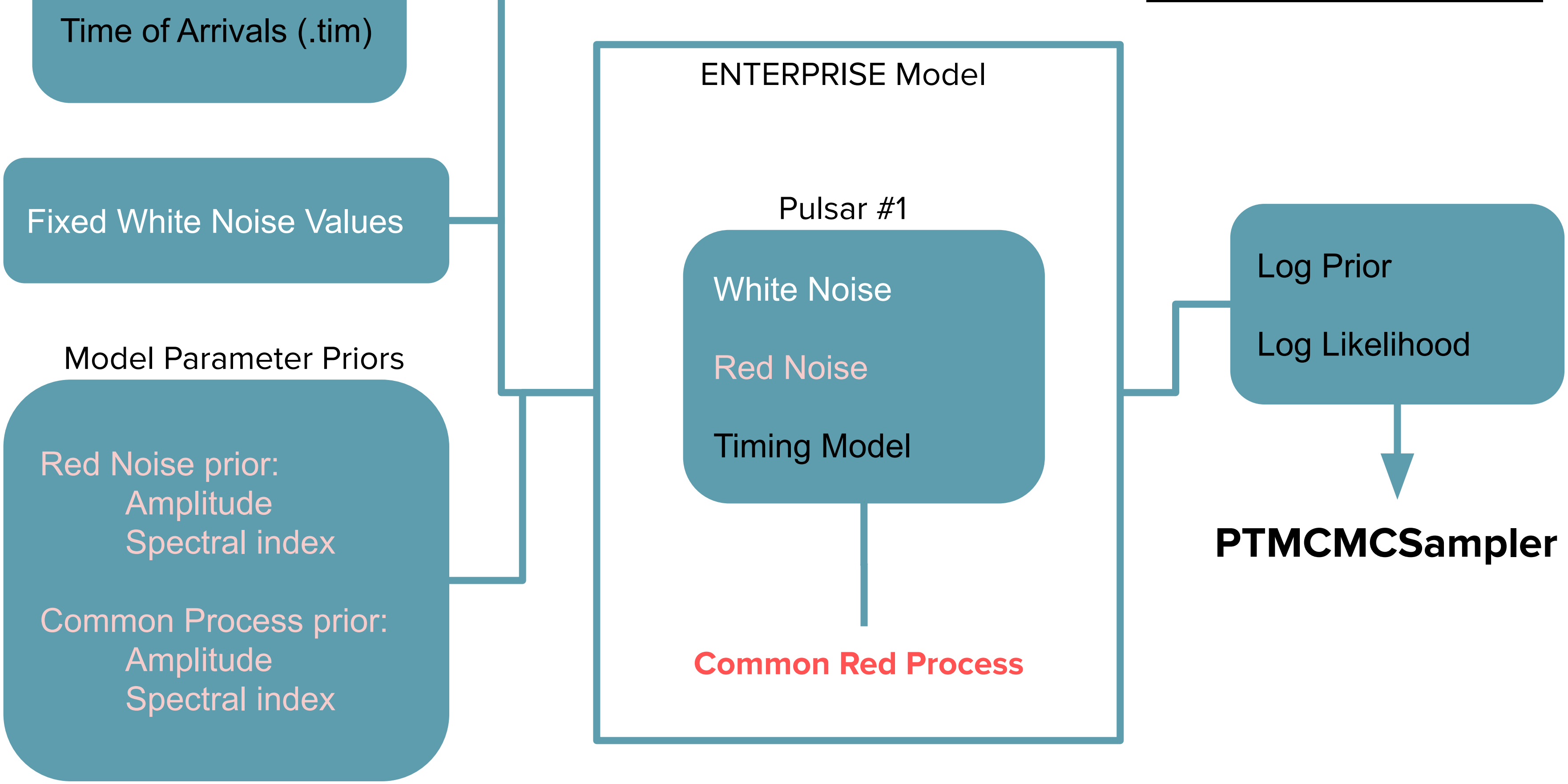
ENTERPRISE (Enhanced Numerical Toolbox Enabling a Robust PulsaR Inference SuitE)

https://github.com/nanograv/enterprise

Data

Timing Solution (.par)

Time of Arrivals (.tim)



Fixed White Noise Values

Model Parameter Priors

Red Noise prior:
Amplitude
Spectral index

Common Process prior:
Amplitude
Spectral index

ENTERPRISE Model

Pulsar #1

White Noise

Red Noise

Timing Model

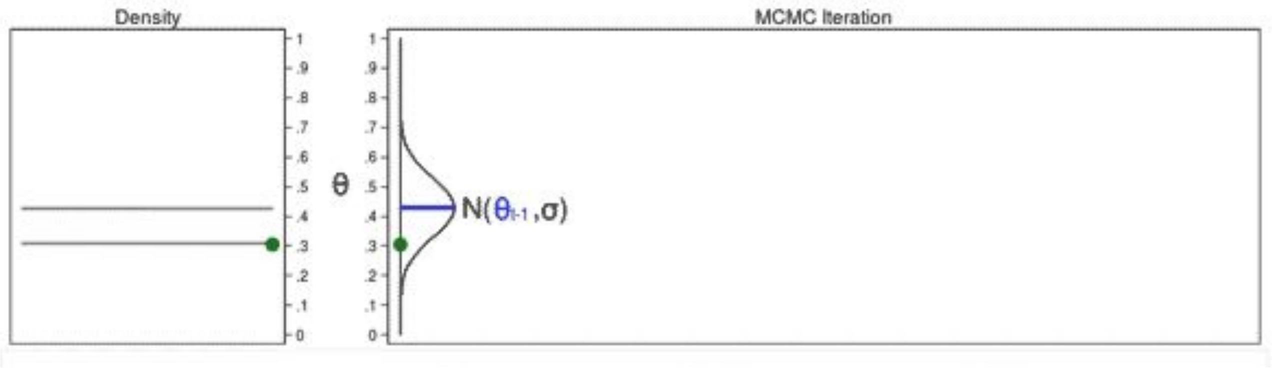
Common Red Process

Log Prior

Log Likelihood

PTMCMCSampler

MCMC: Markov chain Monte Carlo



Goal: Approximate a complicated posterior!

Metropolis-Hastings algorithm

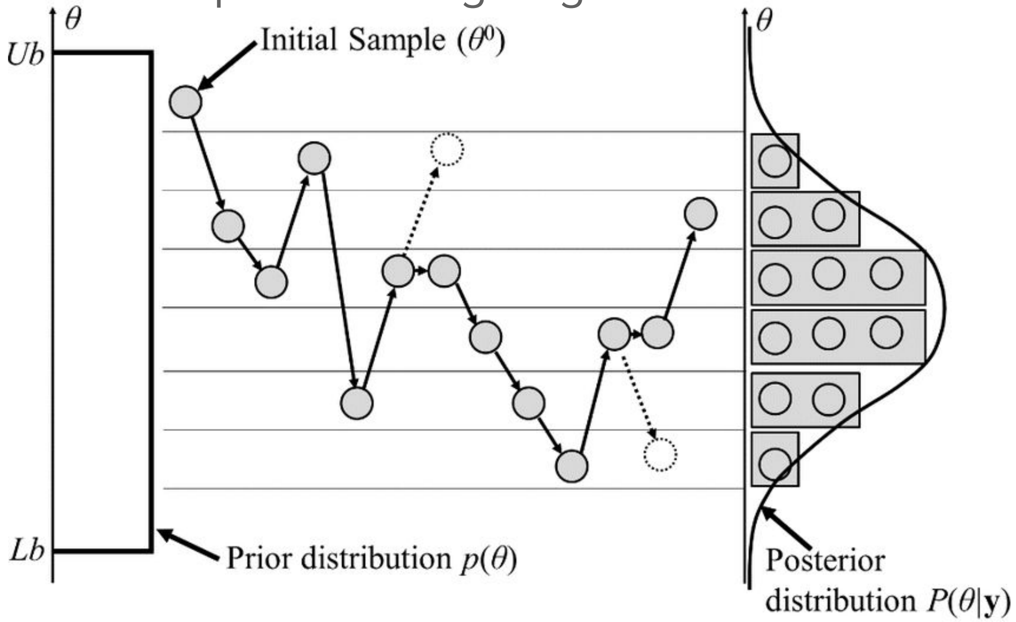


Image credit: Jaewook, Woosuk, Joo-Ho. (2015). Energies. 8. 5538-5554. 10.3390/en8065538.

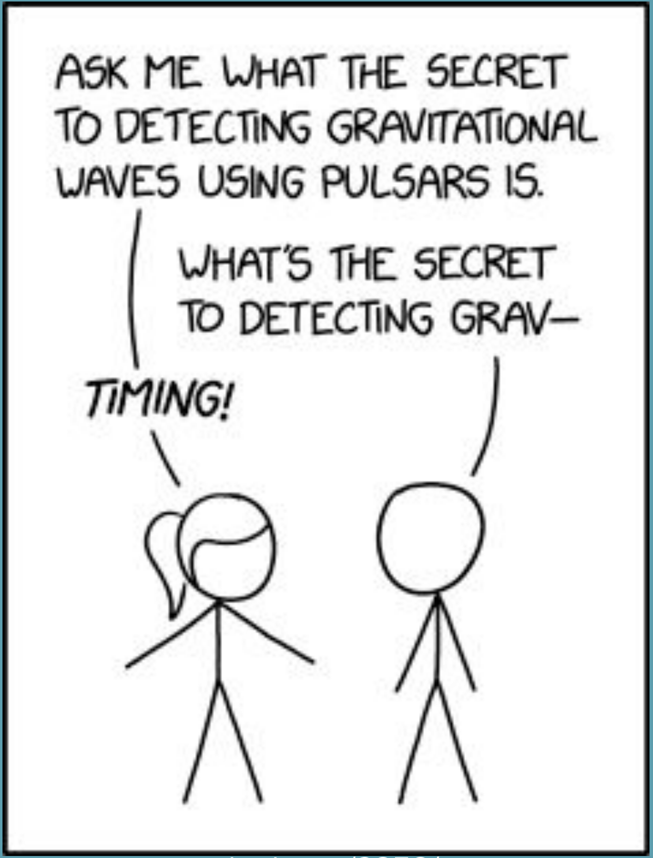
Summary

- Pulsars make great clocks to use as a galaxy scale GW detector
- Using Bayesian model to search PTA data for SGWB
- Software to easily do this is called ENTERPRISE
- Feed in .par, .tim files + parameter priors
- Sample with PTMCMCSampler to get posteriors

- Let's try it…

Questions?

https://tinyurl.com/mwp6vhnx



xkcd.com/2358/

https://tinyurl.com/dz8c2s43

Colab notebooks:

Single-pulsar Bayesian analysis of J1909-3744 (left)

Creating and analyzing a single-pulsar simulated dataset (right)

https://github.com/AaronDJohnson/12p5yr_stochastic_analysis