Interim Report 1: Testing Universal Relations Under Non-Parametric Equation-of-State Models

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Neutron stars have long been a point of interest in astronomy due their extreme qualities. Of these, the core of these super-dense star remains is of especially large interest. Due to the extremely high densities that are present inside of these objects, the way matter moves and acts is unknown to us. At the Laser Interferometer Gravitational-Wave Observatory (LIGO), this knowledge is very important to us as it would give data we collect on gravitational waves much more power in terms of deducing properties of the neutron stars that cause them. We hope to be able to, through the course of this project, see if possible relations between between gravitational wave data we collect and neutron star properties can be found without necessarily knowing how matter acts in these super-dense states with what we call "Universal Relationships". We aim to test these relationships and see if they truly hold irregardless of the Equation-of-State (EoS) that controls how matter moves in neutron stars rigorously so as to ascertain their validity and reliability. With this, we will be better able to collect and restrict neutron star measurements if these relations prove to be fruitful in their utility, and gain insight into our possible incorrect assumptions about how matter interacts in neutron stars if these relationships prove to be less coherent than previously thought.

I. OUTLINE

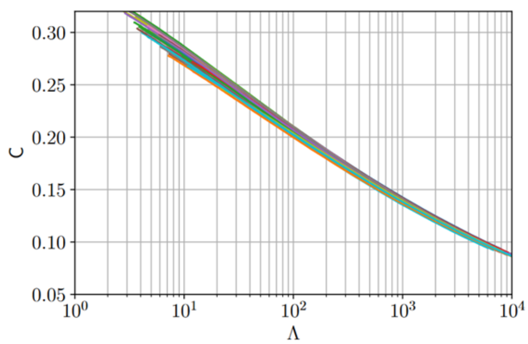
This project is one step in the ongoing understanding of neutron stars in hopes to better make use of data gathered through LIGO observing runs. Due to their very high masses and densities compared to other similar objects like white dwarfs and the general relativity that also plays a part in these celestial objects, current models for relating pressure and density from sources like degeneracy are no longer the only thing at play. With this field having so many unknowns, we find it beneficial to see if we can find any properties of neutron stars that are agnostic to the EoS inside neutron stars in so called "universal relations".

Despite all the complexities present in the makings of neutron stars, literature has been published that has argued for the existence of certain relations between properties of neutron stars that should hold regardless of what EoS makes them up as detailed in Ref. [1]. With this, a large area of research had opened up in seeing if these universal relationships are ,in fact, universal. Of these, an especially important publication was made in the form of Ref. [2]. This endeavor hoped to test these relations under many different possible candidates for the EoS that might make up neutron stars. Specifically, this paper used phenomenological EoS, in which we have a weakly physically motivated form. But we have reason to distrust this data due to the largely model-driven form for the EoS with a set of few parameters that target all possible EoSs for analysis.

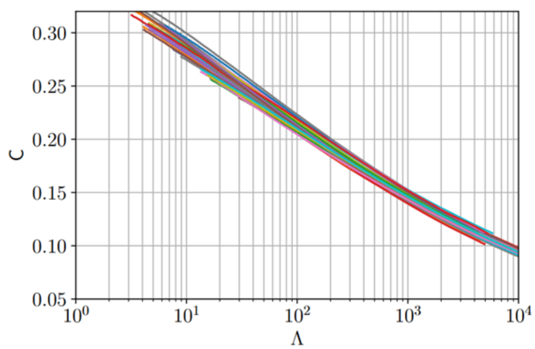
While this has obtained us very beneficial insight, we fear that using this model for the EoS leaves the data over-constrained and does not properly test how universal these relations actually are. With having a functional form without the experimental data to back it up, we run the risk of missing possible EoS models that could accurately describe our matter. In fact, Ref. [3] details exactly how these "parametric" EoS are much more limited in the possible models they can create compared to "non-parametric" EoS, that is, EoS models that were generated algorithmically through a computer that follow no set form. Fig. 1 shows one of the universal relations we are investigating modeled in both parametric and non-parametric models. As one can see through observation, the variance of the relation is much higher in our non-parametric models than our older parametric ones. One can imagine that might've missed the true variance in these relationships if we had simply stuck with our parametric models.

Due to this, we hope to test these relations once again, this time, under many the different algorithmically generated EoS in hopes to more rigorously test them. My project consists of this exact goal, testing these universal relationships under non-parametric EoS to test their validity.

Our project consists of two main phases. First, due to the overwhelming amount of possible EoSs our non-parametric model has created, we need a way to find models that can actually create all the neutron stars we have observed thus far. I must then be able to create a way to quickly search through all of the EoS models, and only grab those that are compatible with our real world as well as taking the neutron star models they create that are likely to ever actually be observed. While the EoS itself may be viable, there's no saying whether or not all the neutron star models it creates are, so this is an important step for analyzing the data if we wish to use in the context of measuring real LIGO data and



(a) Parametric Model



(b) Non-Parametric Model

Figure 1: Graphs showing Compactness (C) vs Deformability (Λ) of neutron stars for 50 different EoS models

in analysis of their viability. These models are important, as they give us many different properties of their neutron stars that we can then plot against each other to develop our universal relations.

Second, we then need to statistically analyze these universal relations to see how variant or invariant they are under many different EoS. This portion of the research is much more involved than the first, due to how many parameters we need to pick ourselves. The general outline we have for this portion of our research consists of making a prediction for these universal relations and seeing how much our models vary from these predictions. We must decide a prediction model to use and what parameters best fit our data. After that, we must also decide on a variance threshold for our relations to be considered valid or not. Luckily, all of these should simply be parameters we are able to change very flexibly.

II. CURRENT PROGRESS

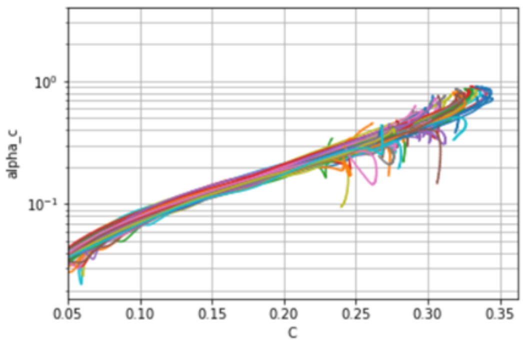
At the time of writing this report, great progress has been made in terms of developing algorithms for retrieving/cleaning data and constructing their universal relations So far, I've built a method to randomly select certain EoS models with weights given by astrophysical data above a certain amount. This is done by taking a file which documents any EoS models that have some sort of physical feasibility along with many important properties, most important of which is their weights given by physical data. The code takes in the amount of models the user wish to inspect, along with a minimum weight the user would like for these models and then randomly selects from the list to pick out. This code then allows the user to pick certain properties of neutron stars modeled by these EoS and use them for any analysis they may wish. For us, we have used this code to take parameters such as deformability, compactness, mass, etc. and plot them for our universal relationships.

Of course, much of the above information will be associated with neutron star models that either simply can't exist (as far as we know) or we are unlikely to ever see in some sort of observation. So, in order to make sure that the universal relations we get back are "clean" and much easier to analyze, we then get rid of data that corresponds to neutron stars below 0.8 solar mass or neutron stars that are "unstable", essentially meaning that we won't feasibly find a neutron star in that state. Luckily for our analysis, these stipulations end up aligning with the "ends" of our data and can be taken off quite easily. Fig. 2 shows another universal relation candidate that we have been able to plot, showing some of the data before and after such cleanups.

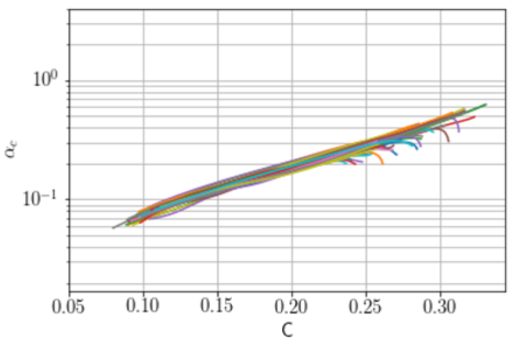
After this, we then need to analyze these models to see if the amount of variance between them is small enough to warrant the relations to be considered truly universal. This is where our current effort lies, and will likely be what the majority of our effort will go to.

III. CHALLENGES AND EXPECTATIONS

So far, the biggest challenges have been learning more modern techniques for data retrieval and manipulation. Much of the data I have been using for analysis has been in the form of comma-separated values, which must be read into Python as dataframes through the use of Pandas. Pandas is a library with which I had no experience before starting my research here, so learning methods to best work with and manipulate this data was a bit



(a) before cleanup



(b) after cleanup

Figure 2: Graphs showing $\alpha(\frac{Presure}{Density})$ in the core vs Compactness of neutron stars for 50 different EoS models

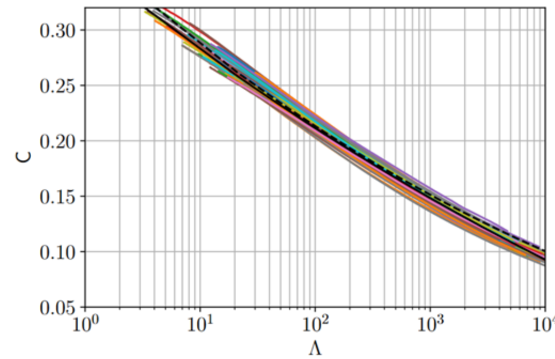


Figure 3: Compactness vs Deformability with 2 trial predictions overlaid on top

tricky at first. As well as this, I also had to learn many more modern techniques for Python to best make my code as general as possible for future use. Writing algorithms that can efficiently retrieve data while also being general enough to be easily modifiable for future use is something I haven't had experience with prior, so getting used to methods and techniques in this area was a challenge. However, at the point I am writing this report, these challenges have been overcome and I can confidently say that my code works very well to serve its purpose of data retrieval and cleaning.

The challenges I anticipate moving forward will likely be in the next step of our methodology: analyzing the data. Statistical methods for quantifying the quality of each of our universal relations is yet another field I do not have much experience with, and so I foresee a challenge very similar to my previous in which I have to learn many new methods and techniques.

Fig. 3 shows a mock graph of one our universal relations of great interest with some predictions used in Ref. [1] overlaid on top. These predictions will not be the one we use for our analysis as they are based on real data and not our models, but this kind of prediction is what we are aiming for. Our goal is to recreate something like this, but instead of having 50 EoS analyzed, we hope to investigate orders of magnitude more at one time with a specific prediction we create to better suit our data.

Although, what I am most anxious about is in integrating all of this with my previous made functions. Part one of my research has already required me to make one main function and 2 helper functions with a total of 8 arguments in order to keep the code as general and as flexible as possible in simply what data we want to retrieve and from where. So then when I am adding in new features to my code in order to also analyze the overall variance within these relations, I do fear keeping on top of all the moving parts to the tools I am making. This is the largest network of interconnected tools I've had to create for a research project, so I expect there to be many problems with getting everything to work well together, especially when optimizing specific tools while ensuring they still work with eachother.

[1] N. Y. Kent Yagi, Approximate Universal Relations for Neutron Stars and Quark Stars 10.48550/arXiv.1608.02582 (2016).

[2] C.-J. H. K. Y. Zack Carson, Keterina Chatziioannou and N. Yunes, Equation-of-state insensitive relations after gw170817 10.1103/PhysRevD.99.083016 (2022).

[3] R. E. Isaac Legred, Katerina Chatziioannou and P. Landry, Implicit correlations within phenomenological parametric models of the neutron star equation of state 10.1103/PhysRevD.105.043016 (2022).