Recovering Higher Order Quasinormal Modes and Overtones from Compact Binary Coalescences

Rachel Mechum*
University of Florida

Mentors: Richard Udall† and Alan Weinstein‡
California Institute of Technology
LIGO Laboratory, Caltech
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Disturbances in the curvature of spacetime from the coalescence of binary black holes can be probed by the gravitational radiation emitted by these sources and recorded by Advanced LIGO and Virgo. The merger of such objects allows us to test Einstein's theory of general relativity in the regime of strong and highly dynamical gravity - specifically, the newly-formed single black hole rings down in a series of quasinormal modes, whose frequencies and damping rates are fully predicted by general relativity. Deriving characteristics from this powerful signal is one of the ways we are able to familiarize ourselves with such distant and exotic objects. Deviations from predictions may provide phenomena beyond general relativity that we are not yet familiar with. We propose to study ways in which this may be revealed in gravitational wave data.

Keywords: gravitational waves, general relativity, LIGO, quasinormal modes, compact binary mergers

I. MOTIVATION

Gravitational wave (GW) signals from compact binary coalescences (CBC) provide crucial information to understand what remains of the remnant black hole (BH) and allow us to test general relativity (GR) in the regime of strong and highly dynamical gravity.

Binary neutron stars (BNS), black hole-neutron star (BHNS), and binary black holes (BBH) are the three main classes of detectable CBCs from our current ground-based detectors. Future detectors such as LISA will allow us to study more types of CBCs. More specifically, we will look at BBH merger events. The data from BBH mergers come from real events, but are simulated to better understand our current models and refine our analysis techniques. During a BBH coalescence, there are 3 stages: the inspiral, merger, and ringdown (IMR). The remnant of merged BHs is a single perturbed BH with a GW waveform characterized as a set of complex frequencies and damping times known as quasinormal modes (QNMs), which are unambiguously predicted by GR. The gravitational radiation from this remnant is called the ringdown phase [1].

BH ringdown is an effective probe GR in the strong field, notably the "no-hair theorem" (NHT). Detections of deviations from GR in the form of violations of NHT can point to physcis beyong GR. We model the ringdown to be a linear superposition of damped sinusoids,

$$\sum_{lmn} A_{lmn} e^{-t/\tau_{lmn}} sin(2\pi f_{lmn} t), \tag{1}$$

where l and m are represented as the angular modes, n is the overtone, A is the amplitude of the waveform, τ is the damping time, and f is the frequency (see FIG. 1). Compactly, the ringdown is a set of complex frequencies, ω,

$$\omega = 2\pi f + i/\tau, \tag{2}$$

determined by the nature of the remnant black hole [2].

The dominant quasinormal mode in GR is recognized as (220), where $l = 2$, $m = 2$, and $n = 0$. This mode displays the highest frequency and is the least damped, which we label as the fundamental (22) mode. Higher order modes (HOMs) of QNMs are the modes with smaller amplitudes than the dominant (22) mode: (330), ... , (440).

Higher order modes (HOMs) that have a radial mode $n > 0$ are referred to as overtones. Overtones are the QNMs with faster decay rates than $n = 0$, but also the highest amplitudes near the waveform peak [3]. In previous data analysis, the inclusion of overtones was omitted which led to loss of signal content. That is to say, the inclusion of overtones is important to extract the parameters of the signal more accurately [4] and further the field of black hole spectroscopy.

The GW ringdown frequencies and damping rate reveal the final mass and spin of the merged BH. The frequencies for a Kerr black hole do not depend upon its dynamical past, but the amplitudes of the ringdown do. This leads to the discussion of the NHT. The NHT states that mass and spin are the only two properties of Kerr BHs in GR. Therefore, they uniquely determine all of the f_{lmns} and τ_{lmns}. We can test the NHT with the data collected during previous and future runs of GW detectors [3].

* rachel.mechum@ufl.edu
† rudall@caltech.edu
‡ ajw@caltech.edu

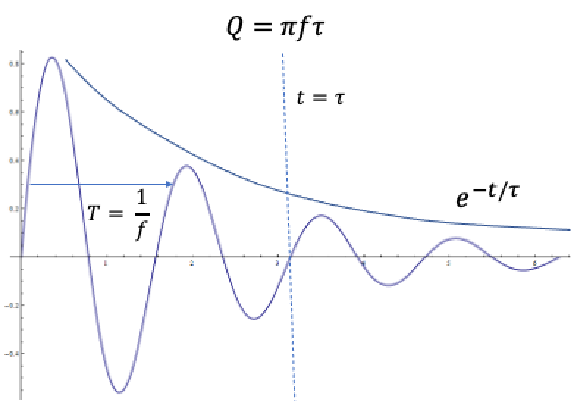

FIG. 1. Example of a singular damped sinusoidal QNM. T is the period, Q is the quality factor, and $e^{-t/\tau}$ is the exponential decay. By setting $t = \tau$, we can see the quality factor will be ~ 2. When adding multiple QNMs, the equation will result in a linear superposition of damped sinusoidal oscillations, as seen in Eqn. 1.

II. APPROACH AND METHODS

Recovering higher order QNMs is a powerful way to test Einstein's theory of GR above the dominant mode. This proves to be more difficult once passing the dominant angular mode, ($l = m = 2$). We will use real GW signals from the third LIGO-Virgo observing run (O3), as well as simulated data. The simulated data will be analyzed in the time domain.

The framework for this research is based on work done by Maximiliano Isi and Will Farr, who analyzed ringdowns not in the frequency domain, but in the time domain. However, this approach demands truncating the GW signal at a specific time, which is difficult to handle with the usual LIGO-Virgo analysis techniques. Instead, it calls for special treatment in the time domain, or an equivalent nontrivial procedure in the frequency domain [3, 5–8]. We chose to work with the former and use the *RINGDOWN* software package [5].

III. RESULTS

Using the *RINGDOWN* software package, we are able to produce simulated ringdowns that are consistent with GR. The simulated ringdown is a 'ringup-ringdown' where the ringdown is symmetrical to the ringup. This does not correspond to real astrophysical events that have been detected. We begin by simulating a noiseless, time-domain waveform to see how different angular modes and overtones behave.

For these simulated post-merger events, we are able to specify the angular modes, l and m, as well as the overtone, n. In return, we have access to the frequecies and damping times of different modes and overtones. The primary noiseless waveforms are simulated to analyze the behavior of the ringdown oscillations alone.

A. Complex Frequencies and Damping Times

As previously stated, all QNMs have their own distinct frequencies and damping times that are derived directly from the mass and spin of the remnant BH. With these frequencies and damping times, we can plot them against respective values of χ ranging from 0-1 (see FIGs 3-5).

Analyzing the plots, we can see that as by increasing multipoles, their frequency and Q factor is climbing. We can also note that in each leftmost plot, the frequencies of each overtone are spread at $\chi = 0$ (assuming the Kerr solution breaks down here). The frequencies and damping times diverge towards infinity at $\chi = 1$, but never reach due to the limit of astrophysical BHs.

B. Dominant Mode and Overtones

Starting by plotting the fundamental (220) mode (see FIG. 2), we are able to understand why this is labeled the dominant mode. This mode can be more easily recovered due to how 'loud' it is compared to subdominant modes. Berti et al. [9] showed that the ringdown analysis with only ($l = 2, m = 2, n = 0$) mode can lose 10% of potential LIGO events [1]. Since this fundamental mode behaves just as we expect (by having the highest frequency in the (22) mode and showing to be the least damped), this makes it the best recovery target.

When adding overtones, the frequencies and damping times of the wave decreases (see FIG. 6). Recovering ringdowns with higher overtones is where the task becomes more difficult. For us to efficiently be able to recover the (22) mode and higher overtones, we would need a louder event or a more sensitive detector.

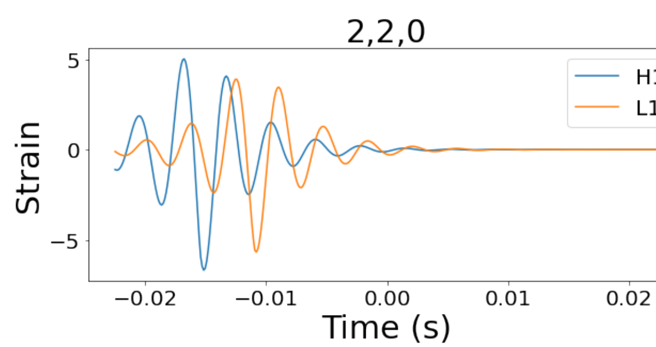

FIG. 2. Simulated ringdown waveform of the dominant fundamental (220) mode. This produced waveform takes multiple parameter arguments such as mass and spin of the remnant black hole. Here, I have used the remnant BH of GW150914's mass and spin as $M = 62M_{\odot}$ and $\chi = 0.67$.

C. Subdominant Modes and Overtones

Simulating the subdominant modes along with their overtones is imperative to understand how to deconstruct real waveforms. When we begin recovering these HOMs in true ringdown data, we will know what to search for and soon be confident in what and where the best events are to spot them.

Beginning with the first subdominant angular mode ($l = 3, m = 3, n = 0$), we note an increase in frequency from the dominant fundamental mode ($l = 2, m = 2, n = 0$). The subdominant mode of ($l = m = 3$) is consistent in the decrease in frequency with increasing overtones (see FIG. 7). We see the same information when simulating the ($l = m = 4$) fundamental mode along with various overtones (see FIG. 8).

D. Next Steps

Beyond simulating the fundamental dominant and fundamental subdominant modes along with their respective overtones to explore the raw ringdown waveform, we don't want to just examine the noiseless scenario.

After analyzing the most basic waveform, we plan to then add Gaussian distributed noise to the simulated ringdown. By adding noise, it will then be possible to explore the recovery of HOMs and their overtones in a more realistic scenario.

Along with noise addition, we will prepare a BBH coalescence IMR waveform. This will allow us to attempt to fully recover specific HOMs along with their respective frequencies and damping times.

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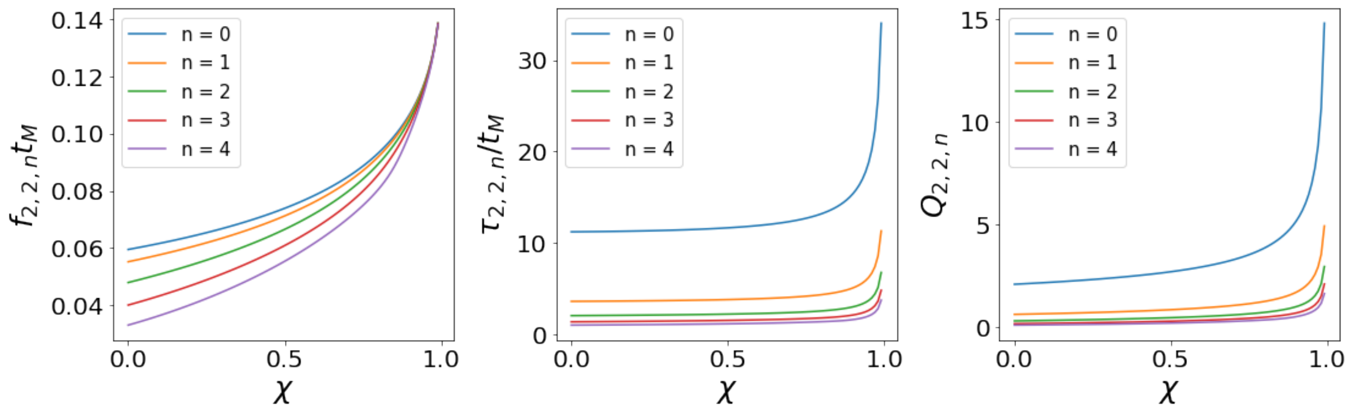

FIG. 3. Frequency f_{22n} (left), damping time τ_{22n} (center) and quality factor $Q_{22n} = \pi f_{22n}\tau_{22n}$ (right) for changing l = m = 2 tones, as a function of dimensionless BH spin χ. Times are measured in units of $t_M \equiv GM/c^3$ for BH mass M.

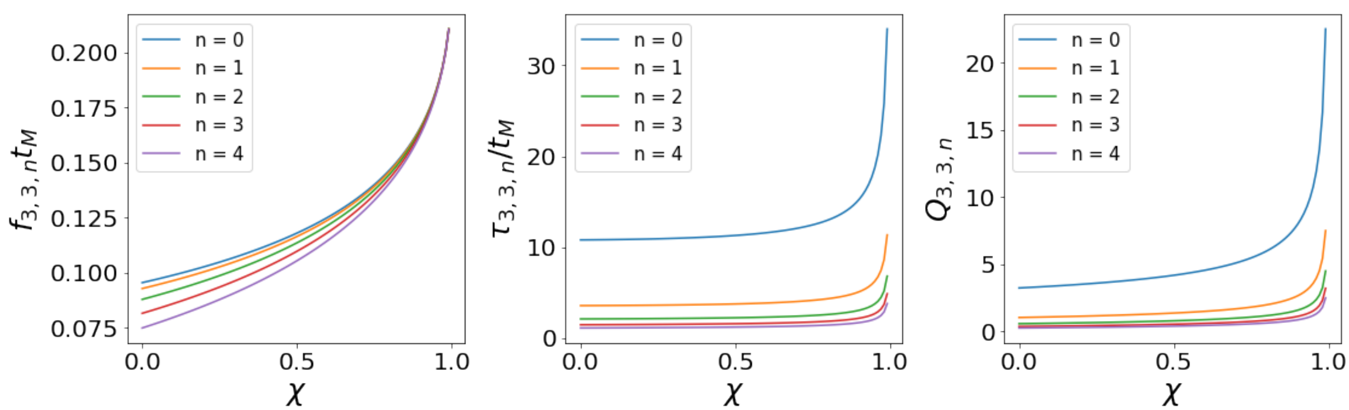

FIG. 4. Frequency f_{33n} (left), damping time τ_{33n} (center) and quality factor $Q_{33n} = \pi f_{33n}\tau_{33n}$ (right) for changing l = m = 3 tones, as a function of dimensionless BH spin χ.

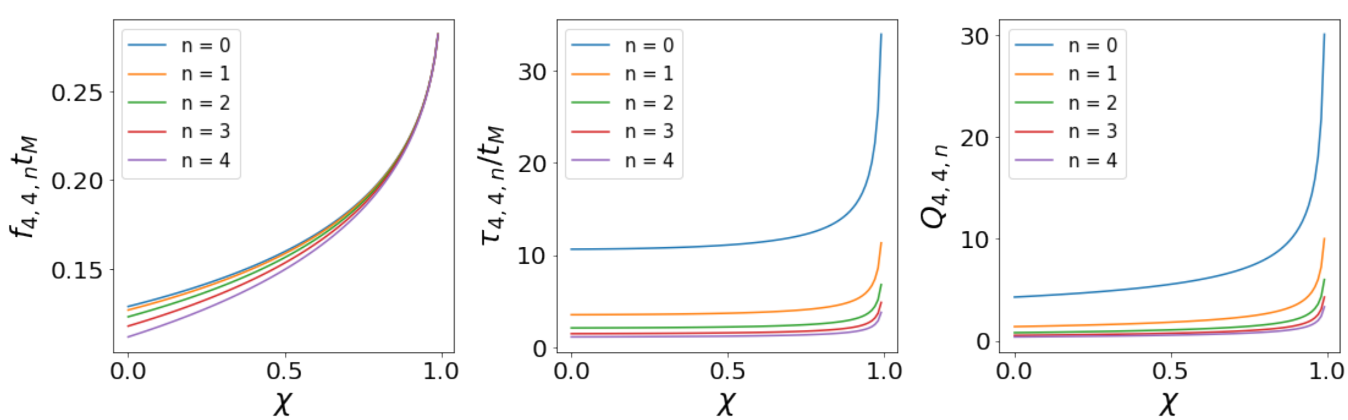

FIG. 5. Frequency f_{44n} (left), damping time τ_{44n} (center) and quality factor $Q_{44n} = \pi f_{44n}\tau_{44n}$ (right) for changing l = m = 4 tones, as a function of dimensionless BH spin χ.

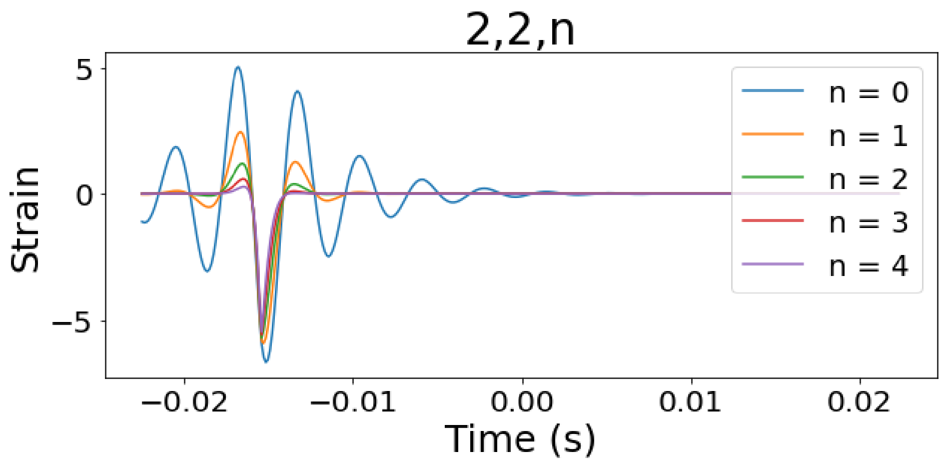

FIG. 6. Simulated ringdown waveform with overtones ranging from 1-4 in addition to the fundamental mode. Here again, using GW150914 data with $M = 62M_{\odot}$ and $\chi = 0.67$. We clearly see the higher overtones that are added, the lower their amplitude becomes. The frequency of the overtones is also diminishing.

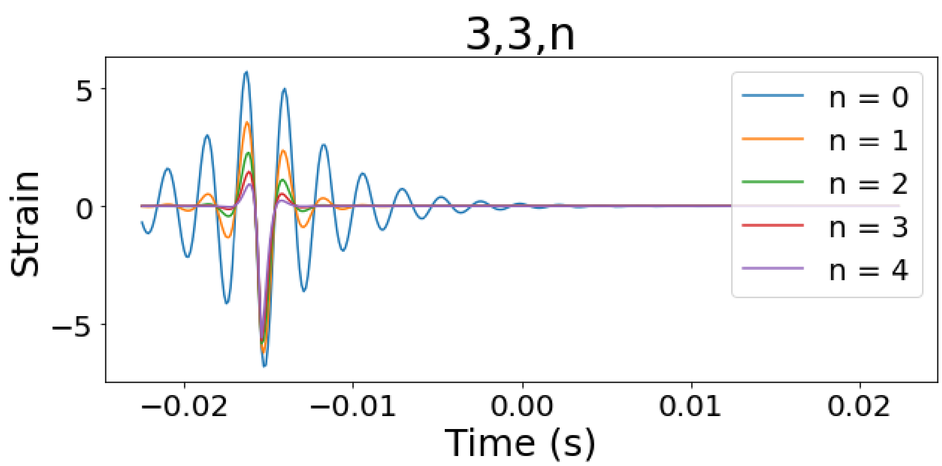

FIG. 7. In the (33) mode and its overtones, we notice an increase in frequency from the dominant fundamental (220) mode (above), but the contracting overtones are still present. (Using the same GW150914 remnant BH parameters as above.)

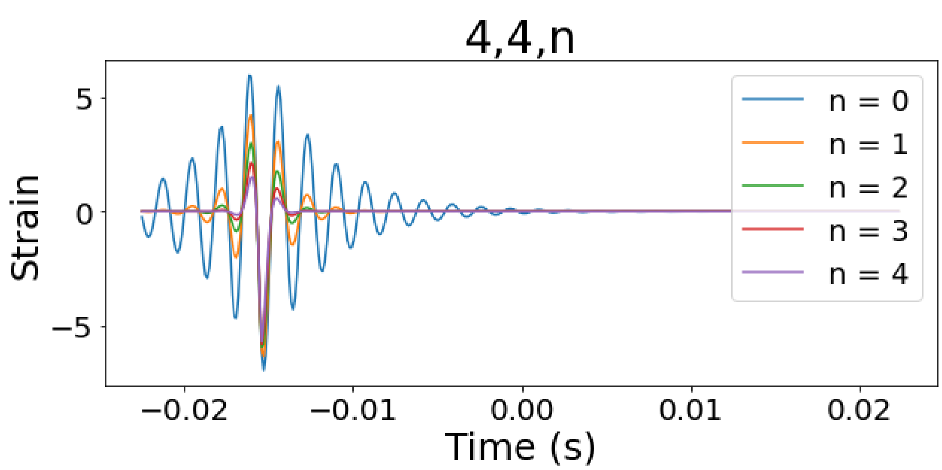

FIG. 8. Simulated ringup-ringdown waveform that dispalys the same explanations as previous plots. Higher frequency in fundamental (44) mode than previous (33) fundamental mode. The amplitudes and frequency of the overtones are decreasing as well.