Incorporating a Stepping-Stone Algorithm into BayesWave

Seth Moriarty
(Dated: May 2022)

BayesWave is used to analyze gravitational wave data that is received by LIGO. It uses Bayesian statistics to simulate signals, and to compare potential models. Currently, thermodynamic integration is used to calculate evidences. An alternative and more efficient method of calculating the evidences is by using a stepping-stone algorithm. We will implement a stepping-stone algorithm in BayesWave, testing its stability and benefits compared to thermodynamic integration.

I. INTRODUCTION/BACKGROUND

- Gravitational waves are ripples in space-time caused by high-energy events in outer space, such as supernovae and the collisions of black holes. LIGO [1], which stands for Laser Interferometer Gravitational-wave Observatory, is a large ground-based interferometer used to detect those ripples. It first detected gravitational waves in 2015, and has been carrying out observational runs since. Constructed of two long orthogonal "arms", LIGO can sense the slight fluctuations in length produced by distant events that produced gravitational waves. Once LIGO has collected data from an event, that data needs to be processed and analyzed computationally.

- Bayesian statistics is a form of statistics in which earlier probability distributions (priors) can be updated to account for new data to produce new distributions (posteriors). This is done using Bayes' Theorem, which is as follows:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \tag{1}$$

  Some $P(X|Y)$ represents the probability of some event X given that Y is true, and $P(X)$ is the probability of some event X independent of other events. Here, A is the posterior, and the prior is B. When you integrate such a function over the entire parameter space, a full model of data called a likelihood is produced. Likelihoods can be compared using the Bayes' factor, a ratio of likelihoods, like so:

$$\frac{P(A|B_1)}{P(A|B_2)} \tag{2}$$

  This can be found with methods such as thermodynamic integration or a stepping-stone algorithm. Being able to compare models using the Bayes' factor is an important step in analyzing data from LIGO, and thus an important component within BayesWave.

- BayesWave [2] is a C-based library which analyzes LIGO data. It is able to model gravitational wave signals and reduce noise. It works using Markov Chain Monte Carlo simulation [3], meaning that it produces posterior samples using a series of events, each of which depends only on the results of the event immediately prior. BayesWave relies on Bayesian statistics for this process.

  Currently, Bayeswave uses thermodynamic integration [4] to calculate the evidences for gravitational models. The evidence is the marginal likelihood of the produced models, marginalized over the entire parameter space. Finding these involves drawing samples from the established data and computing likelihoods for a series of temperature values. (Temperatures scale between 0 and 1, where a 0 indicates a likelihood exactly equal to the prior, a 1 indicates the entire posterior distribution, and a value between 0 and 1 is some combination of the prior and posterior.)

- The stepping-stone algorithm [5] is an alternative to thermodynamic integration, and requires fewer computational steps than thermodynamic integration without compromising the accuracy. It follows much of the same process as thermodynamic integration, but involves a streamlined method for choosing samples and thus fewer computational steps.

  This makes it a good addition to BayesWave's gravitational wave modeling process. The stepping-stone algorithm will increase the speed at which BayesWave is able to produce evidence calculations, which means we will be able to identify noise, glitches, and true signals faster. The next LIGO observing run is expected to detect about one new event per day, so this speed is much needed.

II. OBJECTIVES

My goal this summer is to incorporate a new stepping-stone algorithm into the existing BayesWave system. To do this, I will first be working through the provided code to fully understand it and ensure that it is functional. Once it is running, I will annotate the code and streamline it where I can. Once the code is up and running, it will be connected to the main branch of BayesWave code and I will work to connect it to previously established simulation processes.

The result of this work should be new results for evidence calculation models, which we can compare with other calculations achieved through older methods.

III. APPROACH

The first crucial step to this process will be familiarizing myself with the stepping-stone algorithm code and insuring that I understand each piece of it clearly. This is crucial so that as I am altering pieces of code, I understand what I am working with and where changes need to occur for the code to run smoothly. This step will take some time, as I will likely need to look into other code and explanations of the stepping-stone algorithm itself to fully understand it.

Once my understanding is solidified and the code is fully functioning, I will annotate the code to make it easier to understand to people looking at it with fresh eyes. This step will take less time, as it is just an application of the knowledge I will gain in the first step.

Following that, I will need to work with others to connect the stepping-stone algorithm to the BayesWave main branch, so that it can be used in conjunction with BayesWave modeling. It will likely take time to make the algorithm run smoothly with the rest of the simulation process.

I will be doing this with the oversight of Sophie Hourihane and Katerina Chatziioannou, and will be working in the coding language C.

[1] LIGO- A Gravitational-Wave Interferometer.

[2] N. J. Cornish, T. B. Littenberg, B. Bécsy, K. Chatziioannou, J. A. Clark, S. Ghonge, and M. Millhouse, BayesWave analysis pipeline in the era of gravitational wave observations, Phys. Rev. D **103**, 044006 (2021), arXiv:2011.09494 [gr-qc].

[3] S. Carstens, Introduction to Markov Chain Monte Carlo (MCMC) Sampling.

[4] J. Annis, Thermodynamic Integration and Steppingstone Sampling Methods for Estimating Bayes Factors: A Tutorial, Journal of mathematical psychology **89** (2019).

[5] P. Maturana-Russel, R. Meyer, J. Veitch, and N. Christensen, Stepping-stone sampling algorithm for calculating the evidence of gravitational wave models, Phys. Rev. D **99**, 084006 (2019), arXiv:1810.04488 [physics.data-an].