Testing General Relativity with Gravitational Wave Signals

Noah Wolfe, Jacob Golomb, Colm Talbot

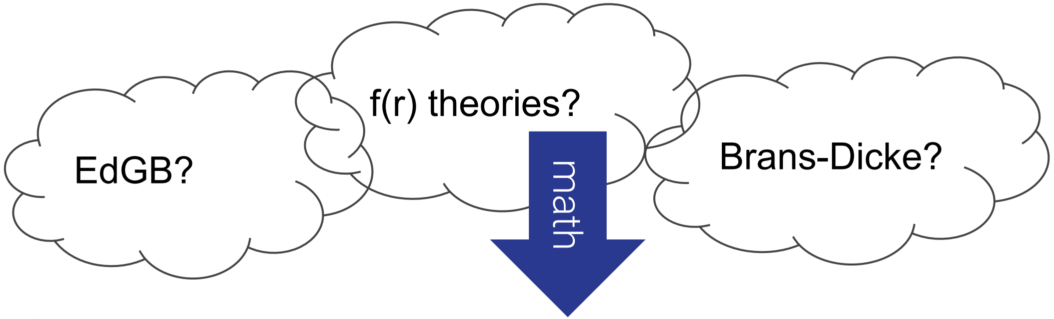
GR is wrong!

...in the signals I injected assuming GR is wrong

Alternatives and Post-Newtonian Formalism

- General Relativity (GR) is successful in predicting GW properties
- Many alternative theories
- Use **generic post-Newtonian (PN) expansion** of GR metric

Alternatives and Post-Newtonian Formalism

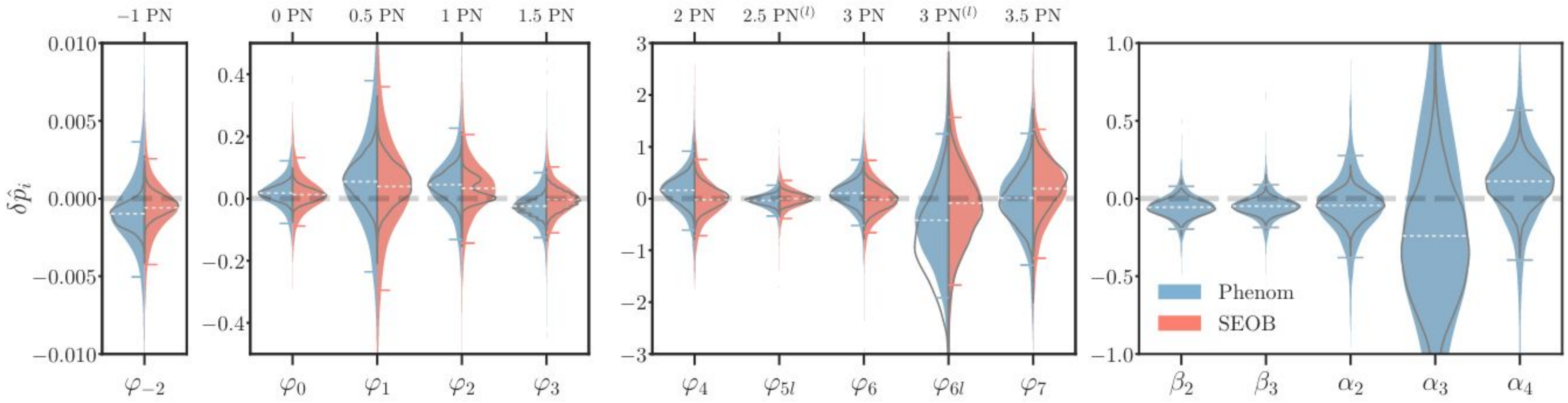


$$g_{00} = -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1 + \zeta_3)\Phi_3$$
$$+ 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U - \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}((v^2/c^2)^3)$$

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\psi(f)} \quad \psi(f) = -\frac{\pi}{4} + \frac{3}{128\eta v^5}\sum_i^7 \boxed{\varphi_i} v^i$$

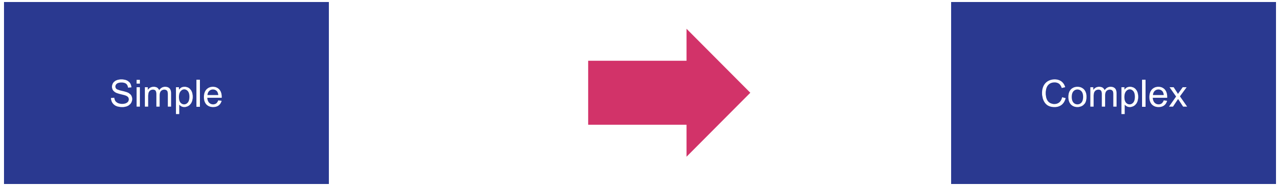
Approximants like TaylorF2

Strong-Field Tests of General Relativity



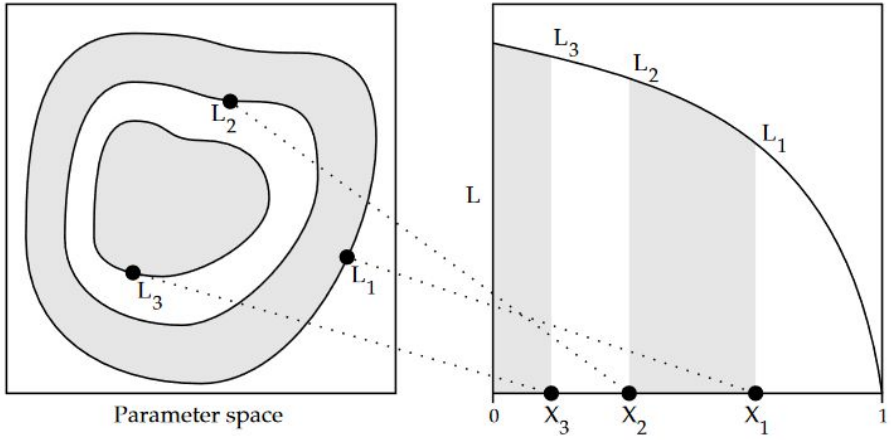
Abbott et al. (2021)

Hybrid Sampling



Nested Sampling

- Fast to initialize
- We know it works!
- Fixed number of samples
- Can't start from other points



Skilling (2006)

Monte Carlo Markov Chains

- Random walker explores parameter space to reconstruct posterior
- Possible to continue ~infinitely
- Can initialize at any point in the space*
  - Fast if **well-initialized**

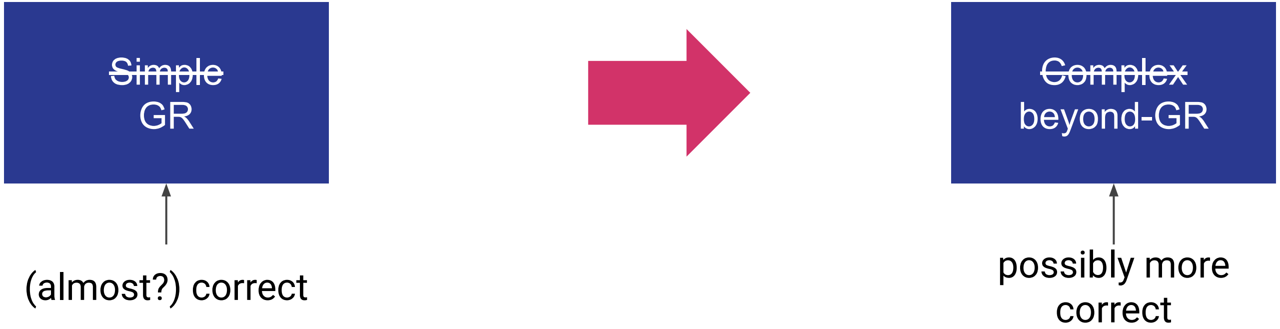
Ensemble Monte Carlo Markov Chains

- ~~Random walker explores parameter space to reconstruct posterior~~
- Random ***walkers*** explore parameter space to reconstruct posterior
- Possible to continue ~infinitely
- Can initialize at any point in the space*
  - Fast if **well-initialized**

Parallel-Tempered *Ensemble* Monte Carlo Markov Chains

- ~~Random walker explores parameter space to reconstruct posterior~~
- ~~Random ***walkers*** explore parameter space to reconstruct posterior~~
- Random ***walkers***, at different "temperatures", explore parameter space to reconstruct posterior
- Possible to continue ~infinitely
- Can initialize at any point in the space*
    - Fast if **well-initialized**

Back to GR!


~~Simple~~
GR
(almost?) correct
~~Complex~~
beyond-GR
possibly more correct

Back to GR!

~~Simple~~
GR

→

~~Complex~~
beyond-GR

(almost?) correct

possibly more correct

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\psi(f)}$$

$$\psi(f) = -\frac{\pi}{4} + \frac{3}{128\eta v^5} \sum_i^7 \varphi_i v^i$$

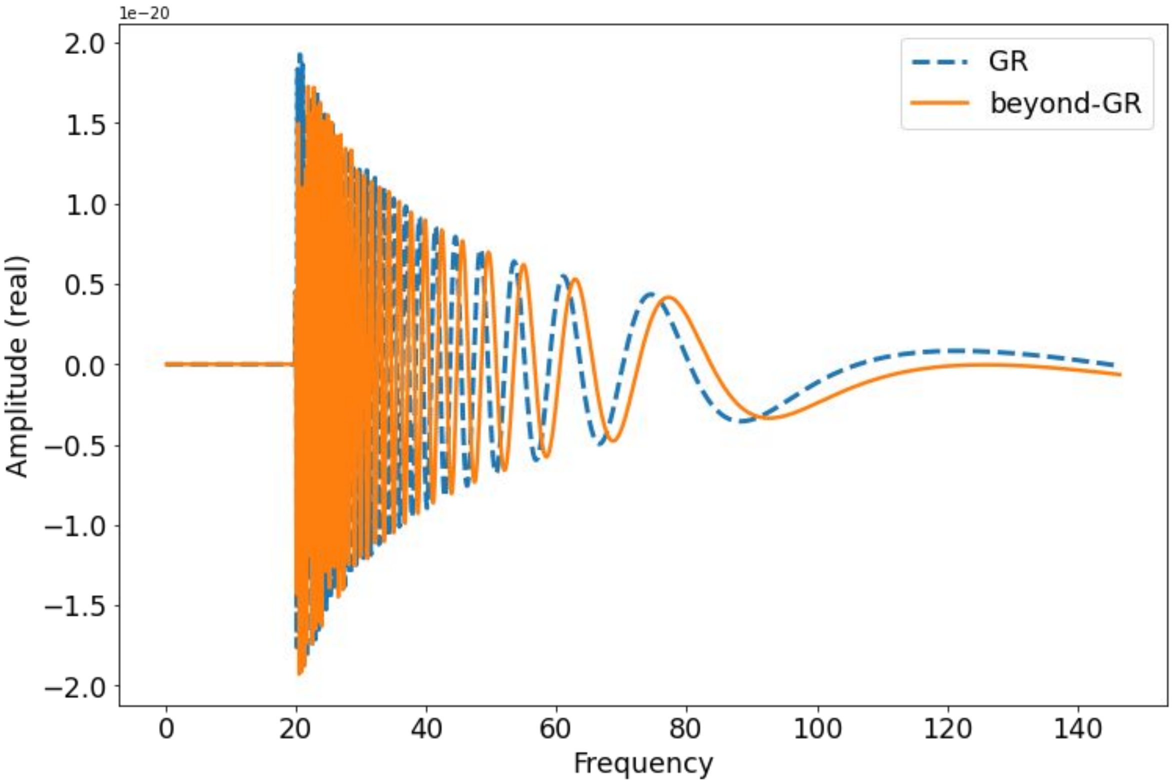
$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\psi(f)}$$

$$\psi(f) = -\frac{\pi}{4} + \frac{3}{128\eta v^5} \sum_i^7 (\varphi_i + \boxed{\delta\varphi_i}) v^i$$

$$v = \left(\pi f \frac{GM}{c^3}\right)^{1/3}$$

Waveform Generation

- d_L = 200 Mpc
- q = 1
- $\delta\varphi_1 = 10^{-5}$
- No spin
- Chirp masses in {8, 15, 30}

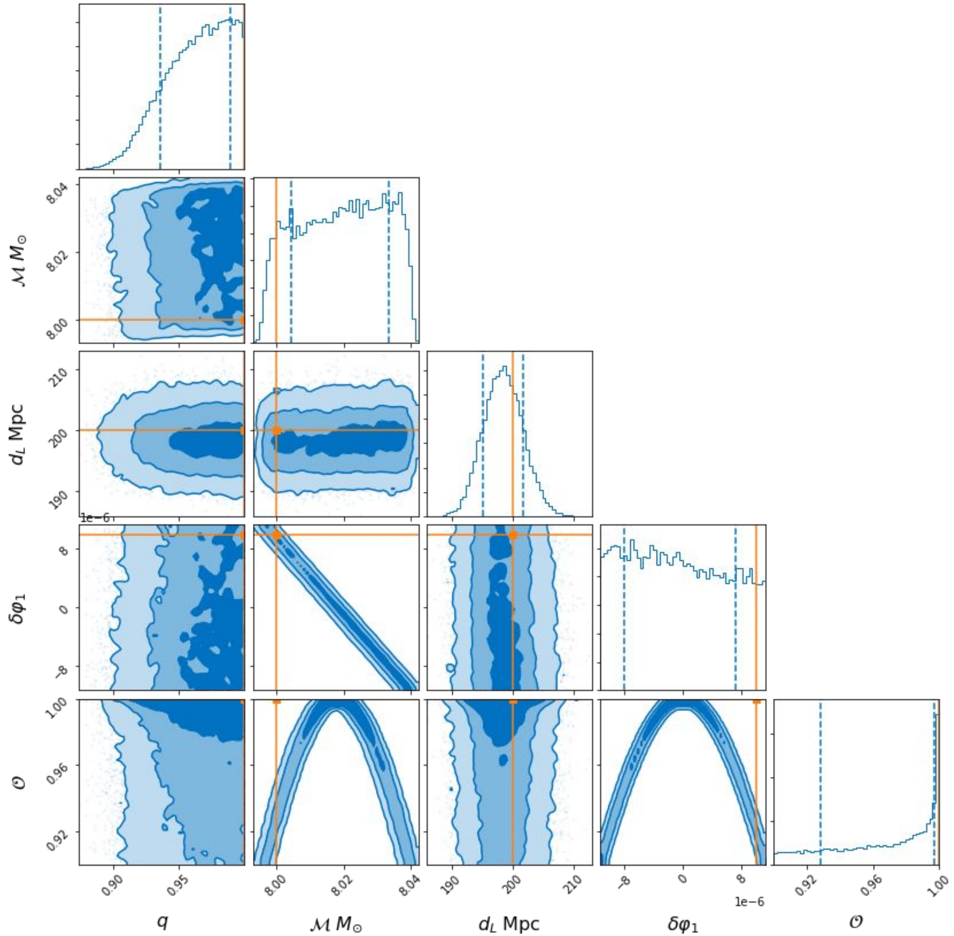


Initial Results

- Limited set of parameters
- Check that hybrid sampling returns the same as dynesty
- Investigate chirp mass, signal-to-noise ratio dependence
- Prior limit of $\mathcal{O} \geq 0.9$

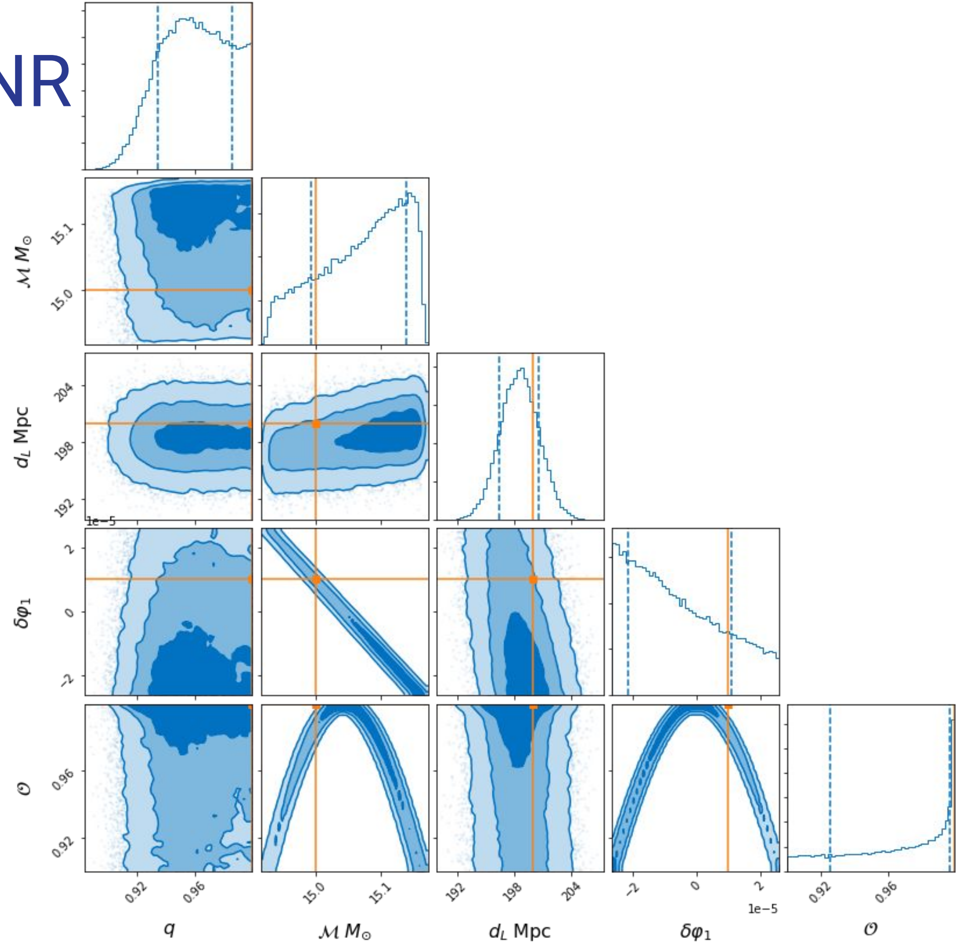
“Low” SNR

Chirp Mass = 8
SNR ~ 50



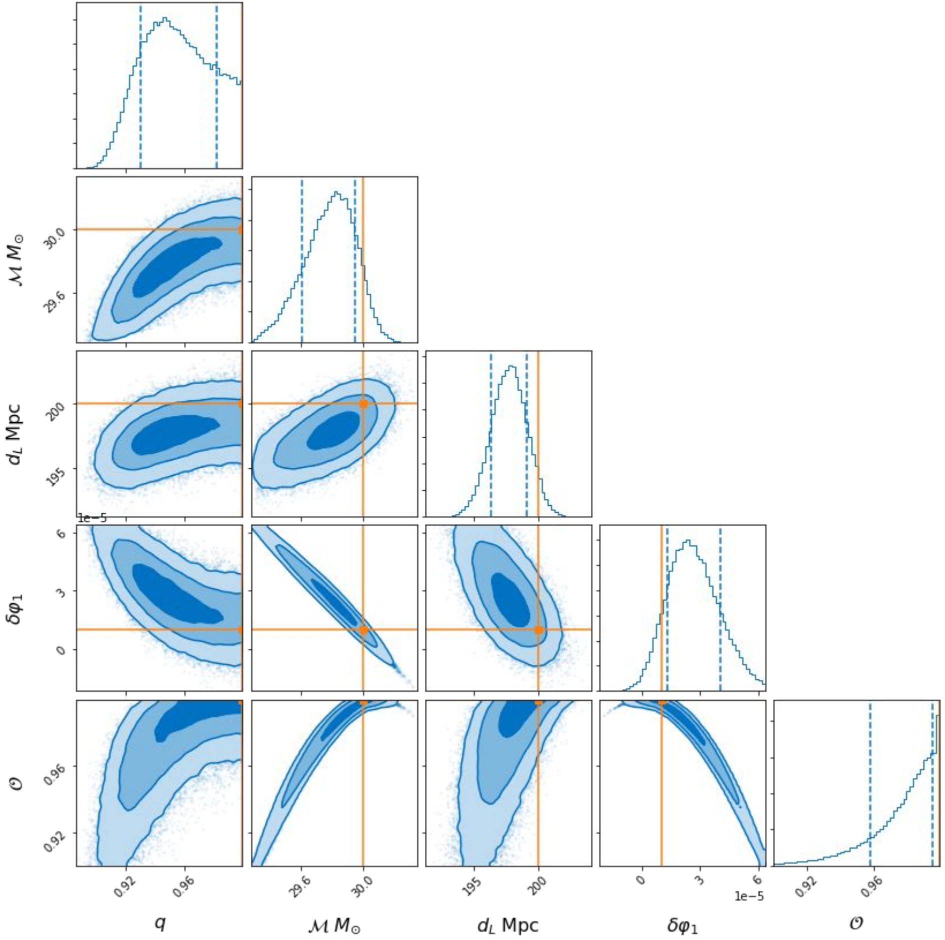
“Medium” SNR

Chirp Mass = 15
SNR ~ 80



High SNR

Chirp Mass = 30
SNR ~ 140



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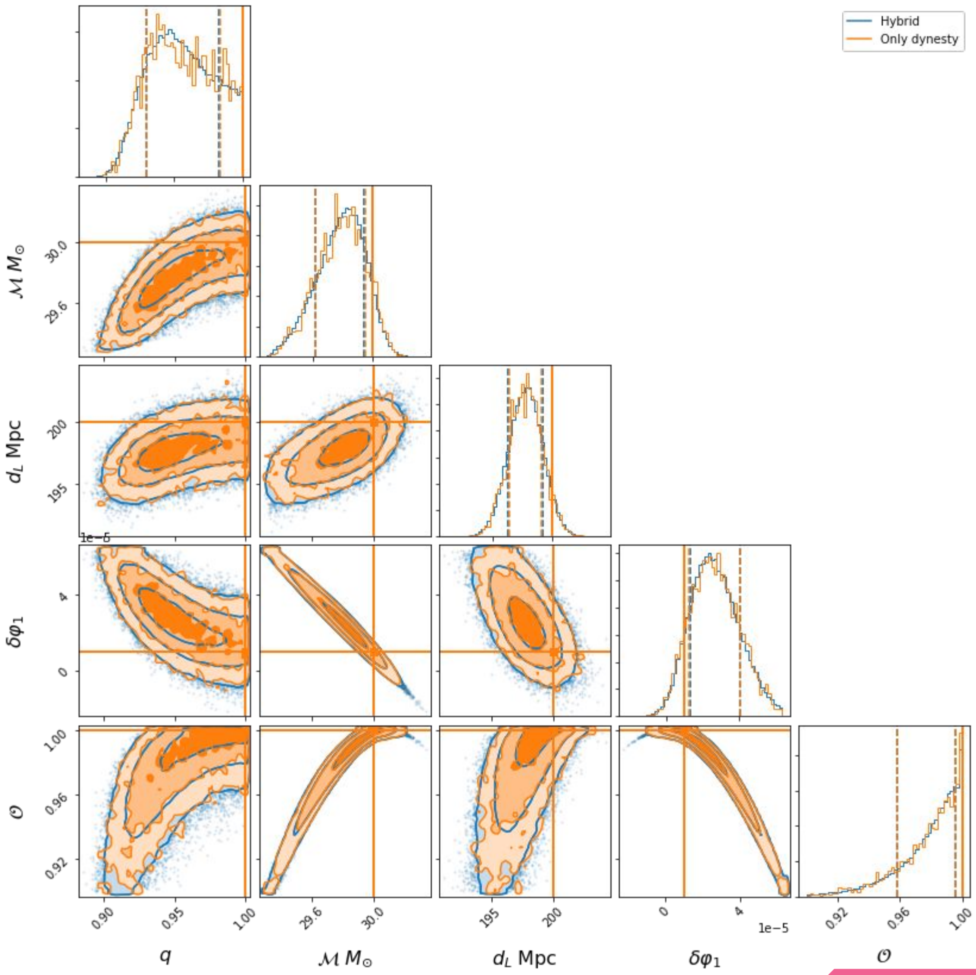


Summary

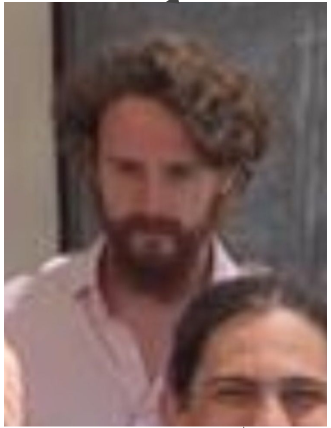
- ✔️Generate beyond-GR phase variations
- ✔️Test hybrid parameter estimation scheme
- ✔️Implement modified source model with beyond-GR waveform
- ✔️Compare hybrid sampling & dynasty on beyond-GR waveform
- Hybrid sampling with all GR parameters
- Hybrid sampling with real data
- Vary multiple deviation coefficients simultaneously
- 🤔Prove GR wrong

Acknowledgements

Paid me

Caltech

Paid me
(in knowledge)



Me (about to be taught
nested sampling)

LIGO-T2100339

Backup Slides

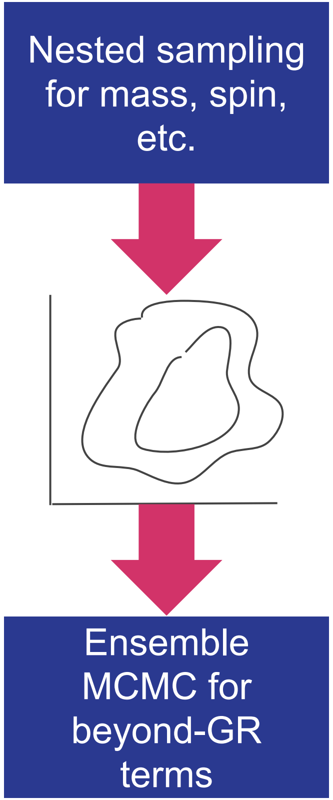
- Overlap function

$$\mathcal{O} = \frac{\left\langle \tilde{h}_1(f), \tilde{h}_2(f) \right\rangle}{\sqrt{\left\langle \tilde{h}_1(f), \tilde{h}_1(f) \right\rangle \left\langle \tilde{h}_2(f), \tilde{h}_2(f) \right\rangle}}$$

$$\left\langle \tilde{h}_1(f), \tilde{h}_2(f) \right\rangle = 4\Delta f \sum_i^N \frac{\tilde{h}_{1,i}\tilde{h}^*_{2,i}}{s_i}$$

Hybrid Sampling Scheme

- Nested sampling
  - Directly calculate the Bayesian evidence, then posteriors
  - Traveling up the likelihood surface, accumulating prior mass
- Ensemble Monte Carlo Markov Chain
  - Multiple walkers, in-concert, exploring the parameter space
- Possibility to ‘temper’ both methods
  - Increase the ‘temperature’ of the likelihood to more easily explore all modes

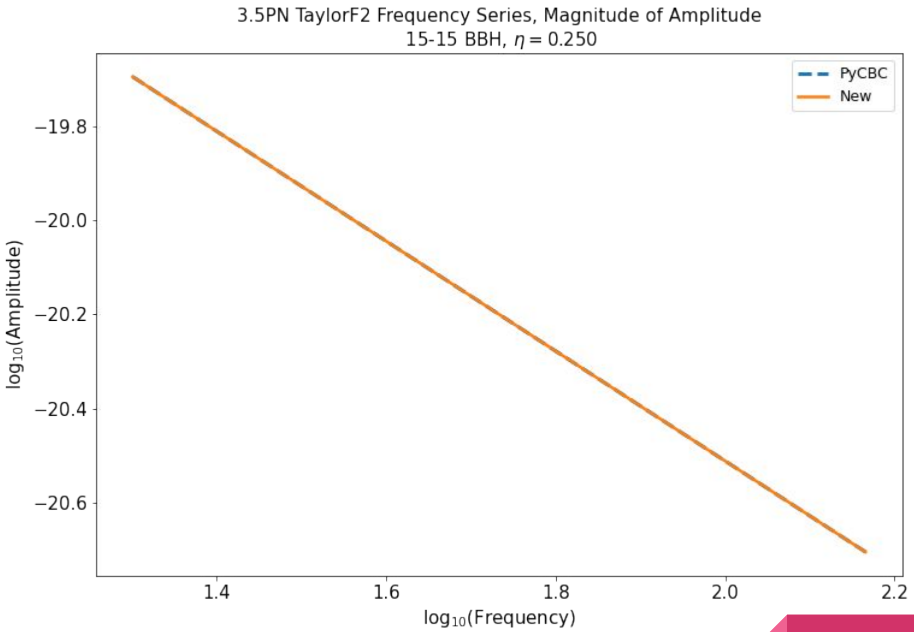


TaylorF2 Waveform Generation

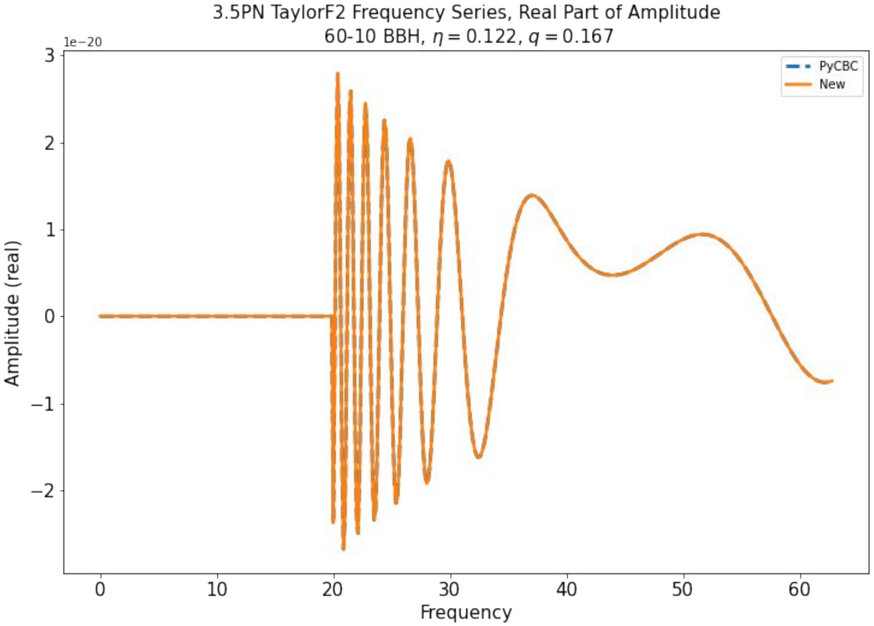
$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\psi(f)}$$

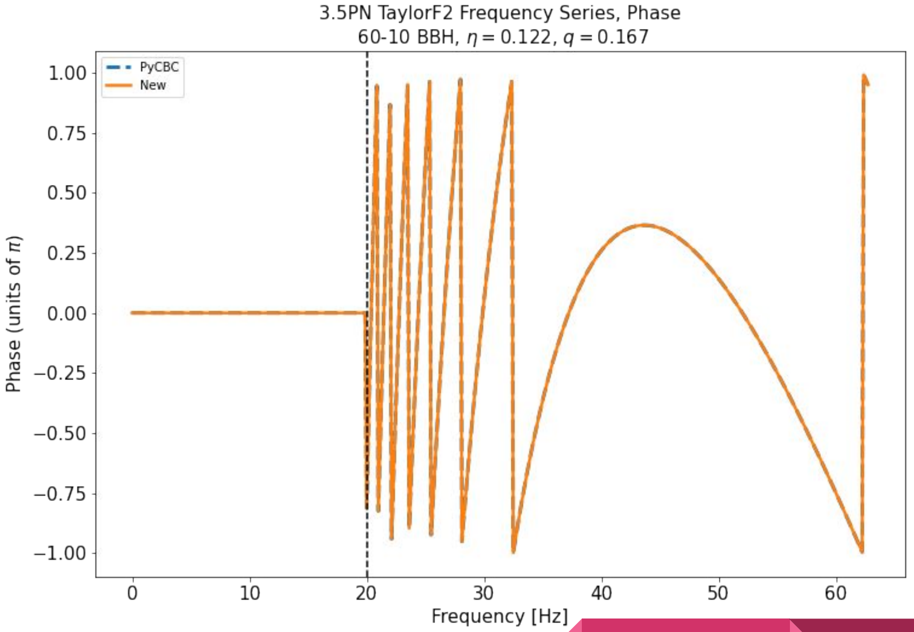
$$\psi(f) = -\frac{\pi}{4} + \frac{3}{128\eta v^5} \sum_i^7 \varphi_i v^i$$

$$v = (\pi M f G c^{-3})^{1/3}$$

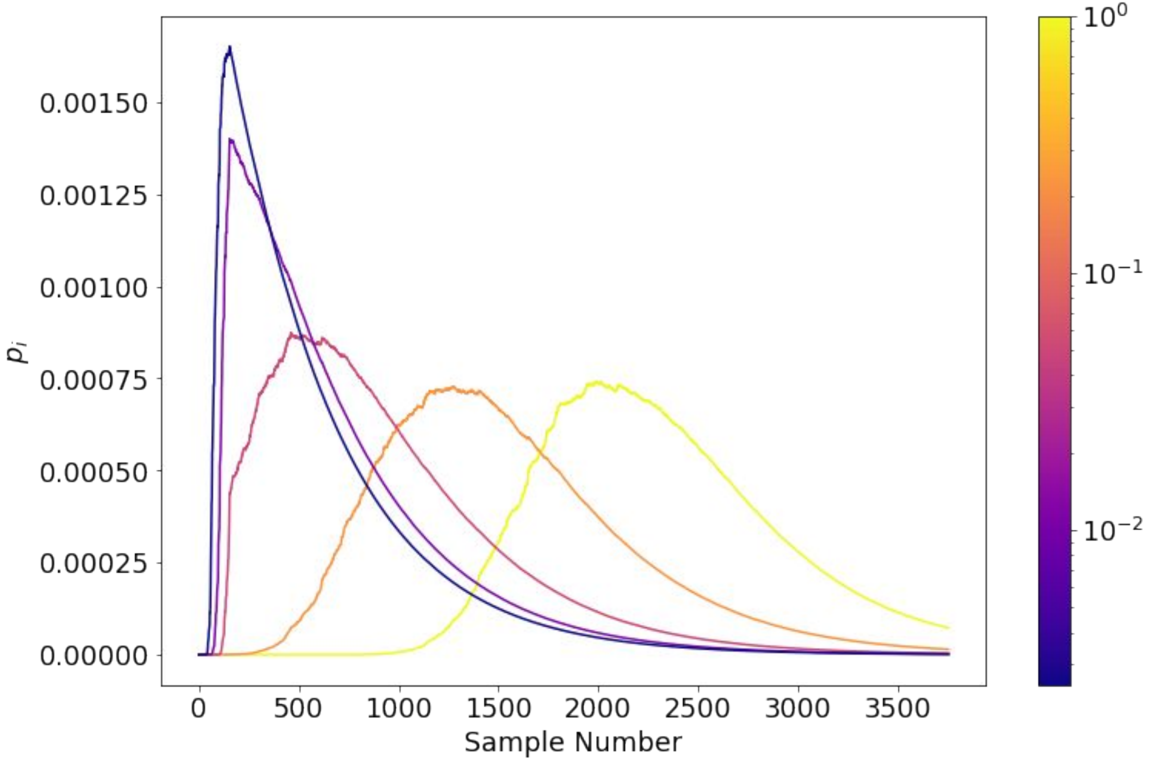


Phase Evolution Agreement





Tempered Posterior Weights

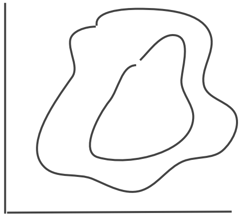


$$p_{i,\beta_T} = \left(\frac{L_i w_i}{Z}\right)^{\beta_T}$$

Hybrid Sampling

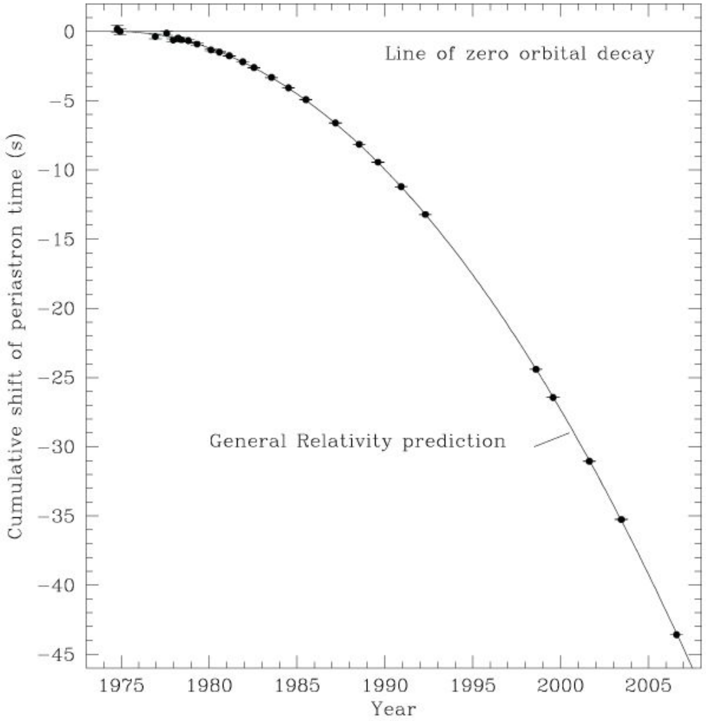
<u>Finish this?</u>

Simple
Dynesty

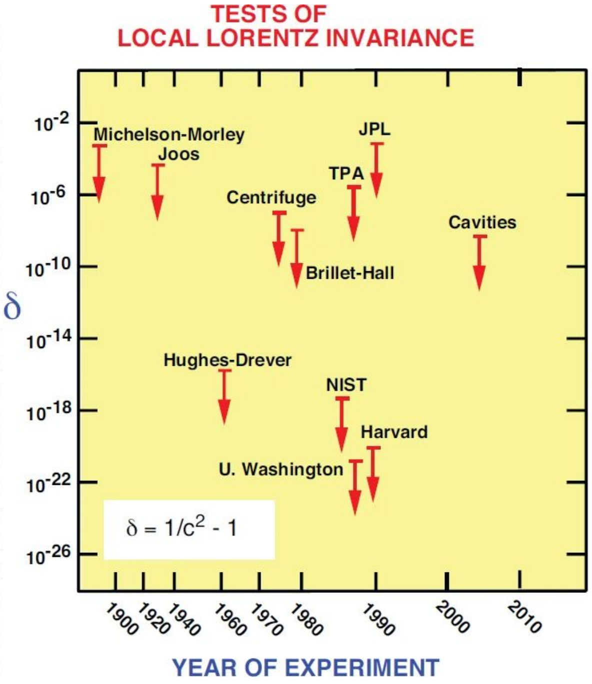


Complex

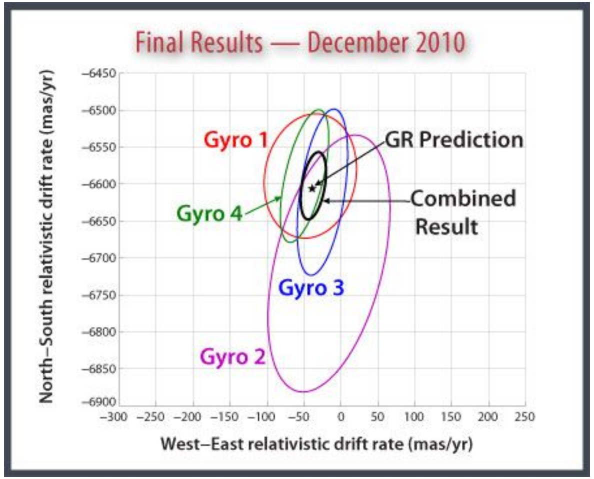
Weak-Field Tests of General Relativity



Weisberg et al. (2010)



Will (2014)



Everitt et al. (2011)

Our Goal

- Previous test with GWTC-2
  - Only **varied one beyond-GR parameter** at a time
  - Required full parameter estimation on **15 GR parameters + 10 beyond-GR parameters**
- Hybrid nested sampling and ensemble monte carlo markov chain scheme
  - More efficient parameter estimation

TaylorF2 Waveform Generation

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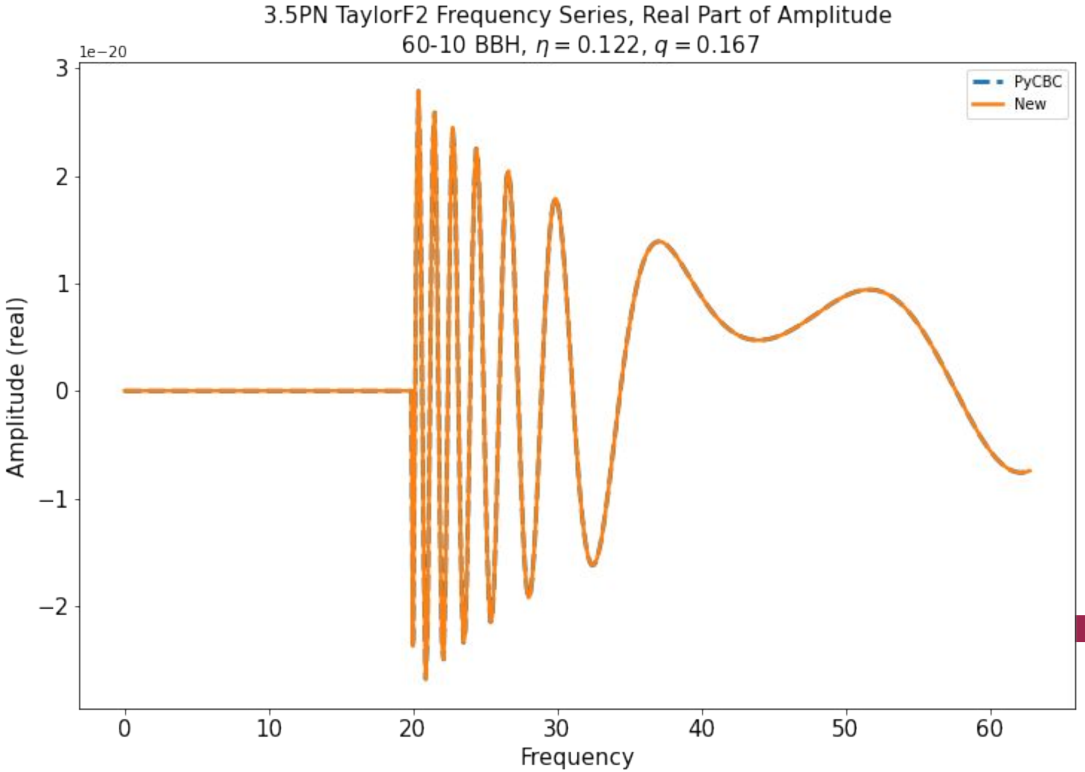
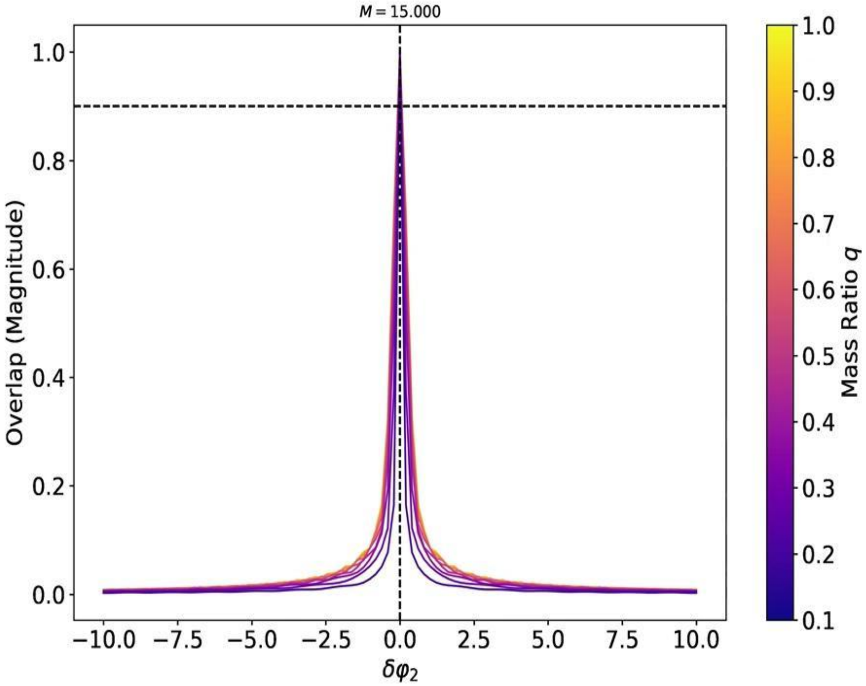


Overlap Phase Variations



$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\psi(f)}$$

$$\psi(f) = -\frac{\pi}{4} + \frac{3}{128\eta v^5} \sum_i^7 (\varphi_i + \delta\varphi_i) v^i$$

Left: variation of the i = 2 term

Toy Model

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-((x-\mu)/(\sigma\sqrt{2}))^2}$$

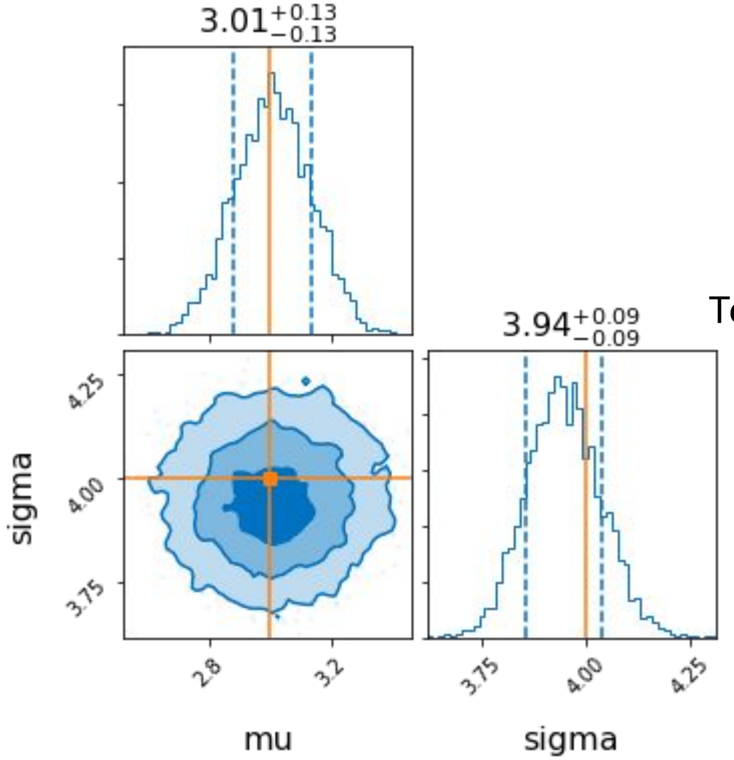
$$P(x) = \frac{\beta}{2\alpha\Gamma(1/\beta)} e^{-(|x-\mu|/\alpha)^\beta}$$

$$\alpha = \sigma\sqrt{2}$$

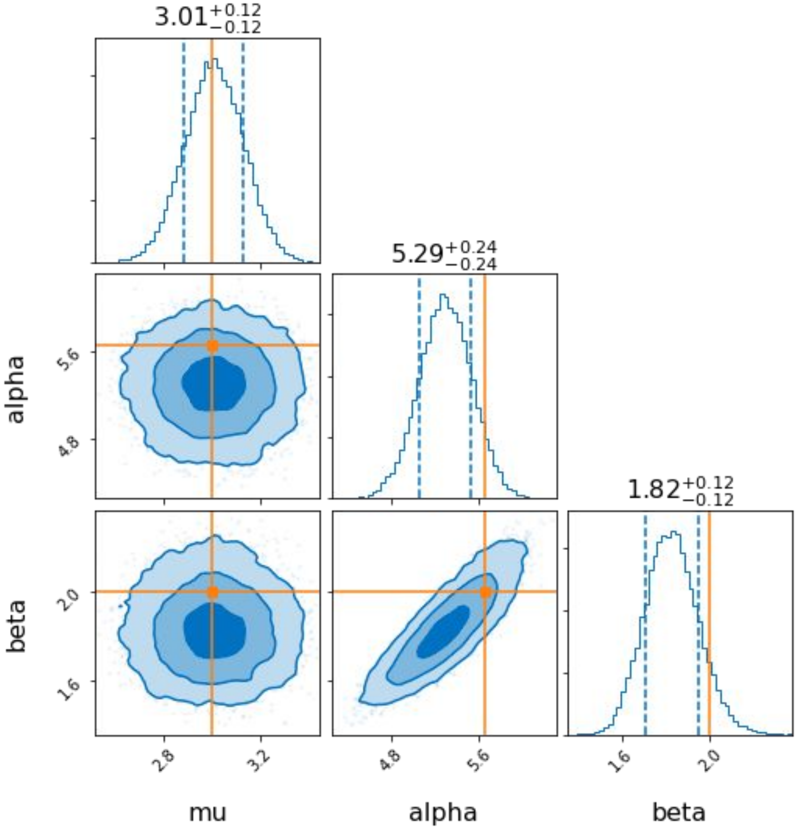
Sample in μ, σ
(dynesty)

Sample in μ, α, β
(ptemcee)

Toy Model Results



Tempered Posterior



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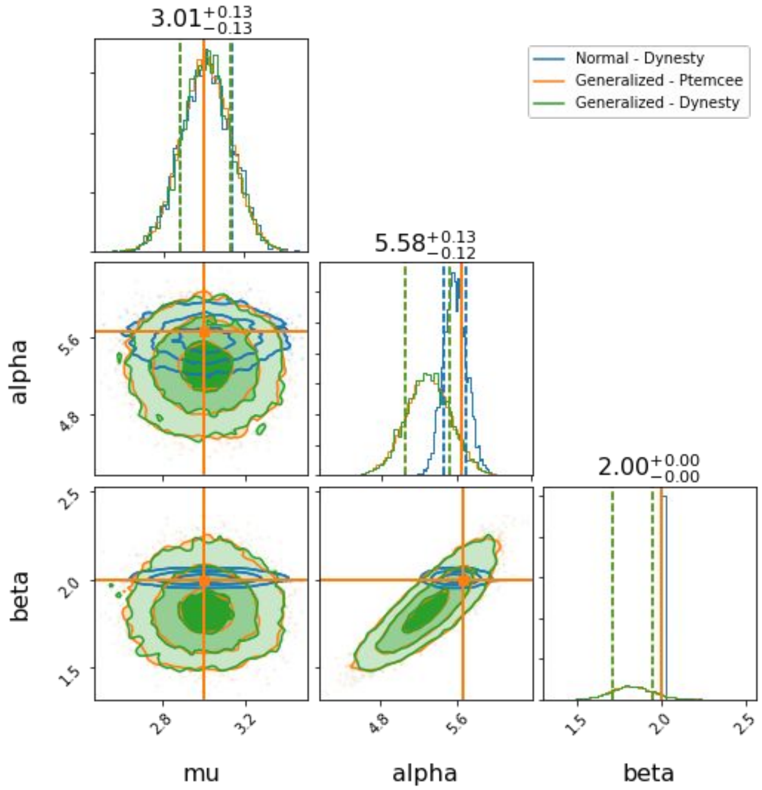
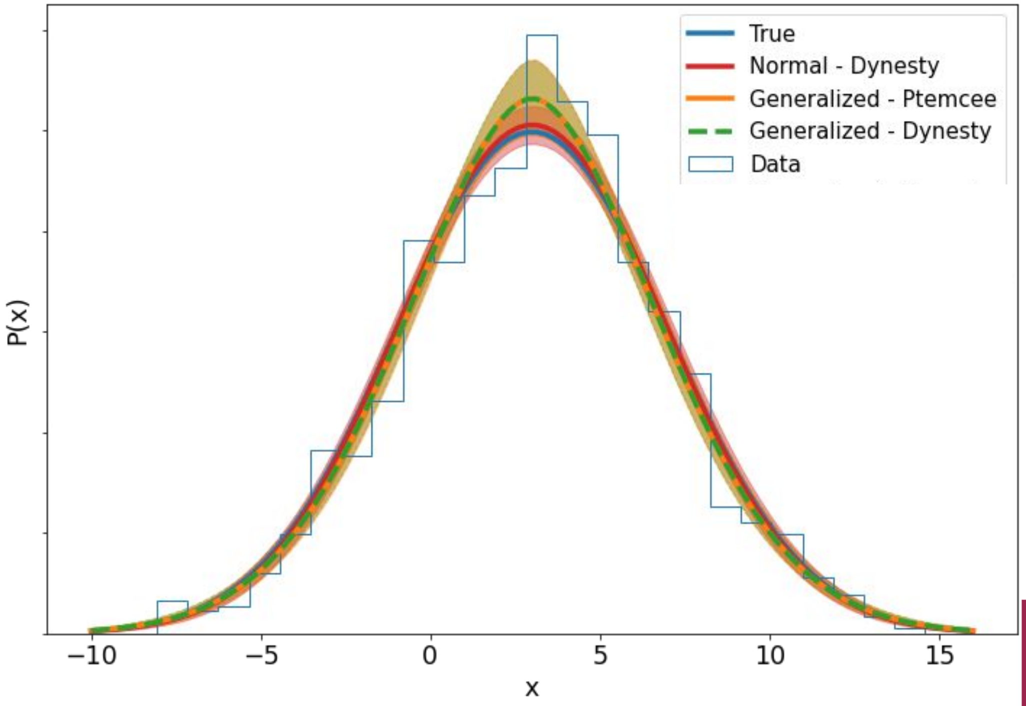
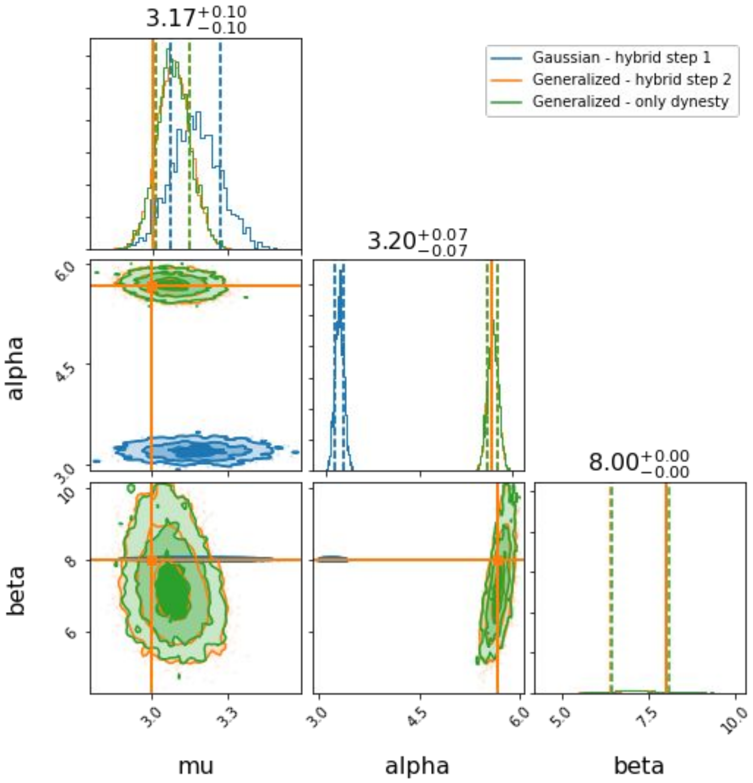
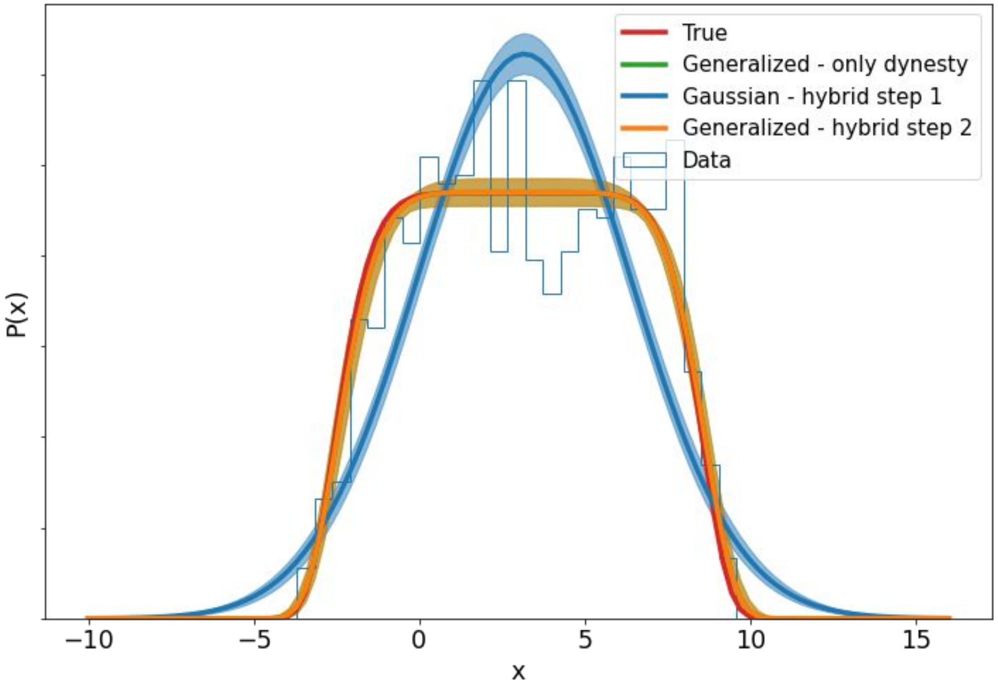




Model Misspecification





Overlap Calculation

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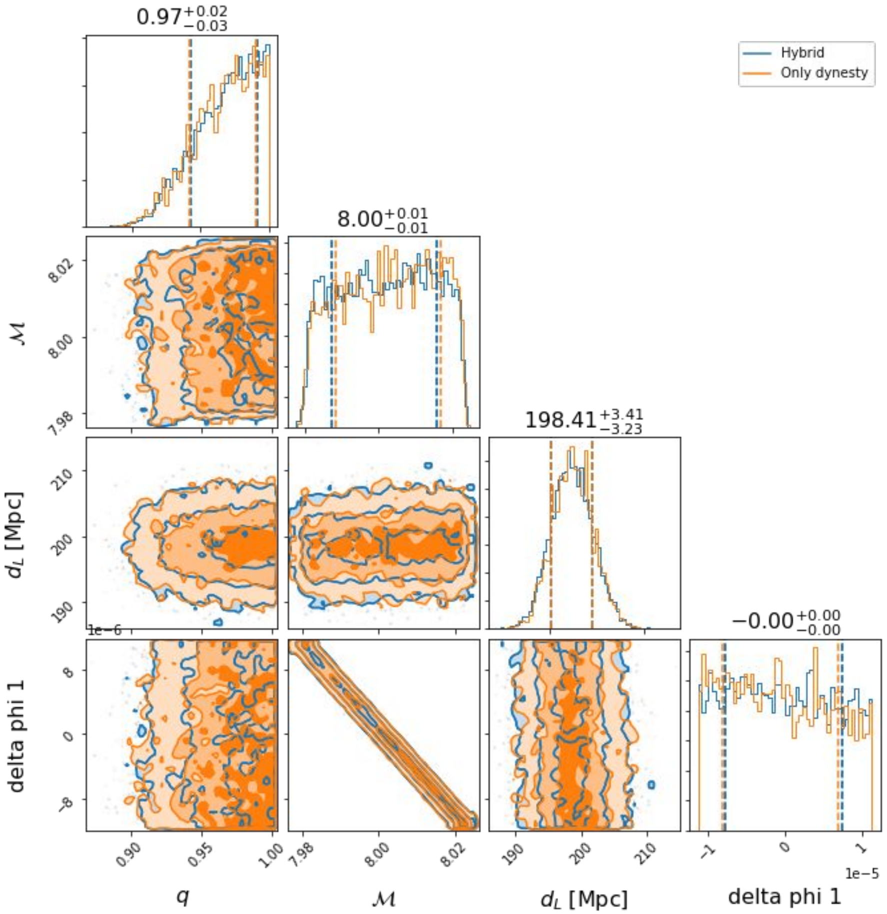
CUT
“Low” SNR

SNR ~ 50



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