Studying the Detectability of High Mass Black Hole Binary Mergers with Future Gravitational Wave Detectors

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1. Introduction

1.1. Overview of the Study

This project aims to determine the luminosity distances to which gravitational waves from binary Intermediate Mass Black Hole (IMBH) [1] mergers can be detected in observing run four (O4), and beyond, of the Laser Interferometer Gravitational-wave Observatory (LIGO) network. By using waveform approximant models, like the IMRPhenom model family, to simulate gravitational waves (GW) strain (in either the time or frequency domain) produced from the merger of binary black holes (BBHs) we will study the factors that affect sensitivity; specifically those in the intermediate mass range. Considering these simulated strains —averaged over several physical attributes of the merger and external parameters specific to observation —will allow the prediction of the distance sensitivity to be expected for future observing runs.

These simulations use predetermined ratios of fifteen astrophysical parameters [2], that will be sampled over using Monte Carlo methods. Bayesian inference posteriors can then be created and informed with the sampled information [1] in order to determine the posterior probability distribution of the noise and maximize the detected signal. Since IMBH mergers are expected to be very rare, occurring at a rate density of $0.13^{+0.30}_{-0.11}$ $Gpc^{-3}yr^{-1}$ [2], determining the future sensitivity distance relates directly to increased chance of detecting them, since an increase in astronomical distances correspond to cubic increase in space-time volume (VT). These VTs refer to the co-moving volume, which experience cosmological effects like red shift (on the time domain waveforms/frequency/BBH mass) and time dilation. Strictly speaking this makes the relationship between distance and volume slightly less than cubic. Additional considerations during waveform correction will include detector antenna response from the gravitational wave detector network. The corrected waveforms from the sampled parameters will be subtracted from strain data to compute Gaussian noise likelihoods for each point in parameter space. From the resulting Gaussian likelihood, posterior probability distributions will be created, drawing on results from observing run 3 (O3) to inform the expectation. Based on these results predictions of the signal to noise ratios of various events can be calculated to ultimately determine the sensitive distance to which GWs from intermediate-mass BBH mergers will be detectable in upcoming observing runs.

1.2. Gravitational Wave Background

Binary black holes, predicted and later confirmed with the detection of GW150914, are thought to arise from co-evolved binary star systems or dynamical capture in dense stellar environments [3]. Once formed general relativity (GR) predicts that the BHs will orbit each other, losing energy in the form of gravitational radiation and move closer together, until finally merging into a single object [4]. The gravitational waves produced by BBH mergers are a result of the relativistic orbit that ripples space-time [5], emitted at a frequency equivalent to twice of the orbital frequency [6]. Gravitational wave data are consistent with GR so far, so waveform simulations used throughout this study are based on GR simulations.

Although GWs are produced by all moving matter in the universe, only merging events of neutron stars and black holes are loud enough to be visible to the gravitational wave detector network. Loud in terms of Gravitational wave physics meaning a large strain amplitude in the collected frequency data. Even still, not all mergers are visible to the network; for example the detection threshold in O3 required a signal to noise ratio (SNR) of 12 [7] for an event to be distinguishable from noise in the data, and the recorded data had to be within the LIGO frequency band, between 24Hz to 2048Hz [8], to be detected. These basic conditions dictate what can be detected by the gravitational wave detector network, but there are multiple intrinsic and external variables that can affect whether the data collected from a merger will meet this criteria.

The gravitational wave detector network uses Michelson Interferometers to measure the GWs, in which the mirrors of the detector are free to be moved by passing GWs. The interferometer bounces lasers between these mirrors, allowing the motion to be quantified as strain of the laser cavities, calculated from the phase difference of the lasers [9]. The gravitational waves arriving at the detectors are measured from their distortion of space in this manner, which is collected as strain data characterized by the following relation [10]:

$$h = h_+F_+ + h_\times F_\times. \tag{1}$$

Where h is the strain, plus and cross represent the polarizations,

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[1] Explained in depth in section 1.3, Motivation, for the purposes of this study we will refer to black holes between 65 and 150 $M_\odot$ as intermediate mass. This is in order to consider black holes in the pair instability/pulsational pair instability mass gap.

[2] The fifteen parameters include the individual BBH masses, the three spin components for each BH, plane inclination, azimuthal angle, total angular momentum of the system, luminosity distance, time of merger, right ascension and declination. We will assume circular orbits.

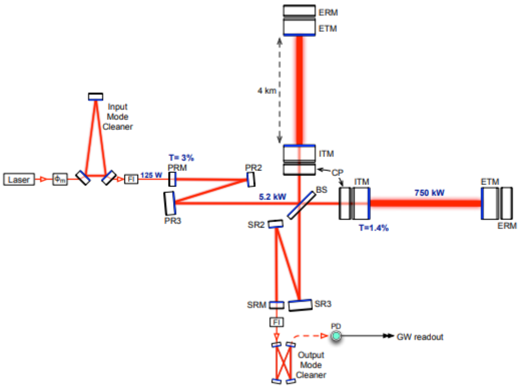

Figure 1: *Advanced LIGO schematics [[11], fig.2]*

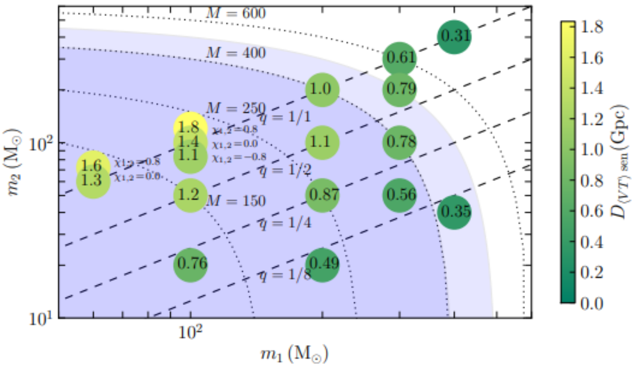

Figure 2: *Distance sensitivity, in Gpc, of IMBH binary mergers events with varying constituent masses [[17], fig.1].*

and F represents the detector antenna response to each polarization. Each polarization's strain is dependent on the intrinsic properties of the merger event, whereas the detector antenna response is dependent on the detector's antenna response to varying sky location/orientation.

This strain, h, is seen by the gravitational wave detector network only if it both is above the SNR detection threshold and within the LIGO frequency band. The SNR of a merger event can be calculated with the following [12]:

$$\langle a(\theta, f)|b(\theta, f)\rangle = 2\int_{f_{low}}^{f_{high}} \frac{a(\theta, f)b(\theta, f)^* + c.c}{S_n(f)} df. \quad (2)$$

Where $a(\theta, f)$ is the strain in the frequency domain, $b(\theta, f)$ is the template, and $S_n(f)$ is the power spectral density of the noise. This optimal SNR relationship is also applicable to simulated strains, which will enable predictions of the BBH mergers visible in observation run 4 and beyond.

1.3. Astrophysical Motivation for Finding Distance Sensitivity of High Mass Binaries

Astrophysicists predict a deficit of black holes whose progenitor stars are between the masses of about 95 $M_\odot$ and 130 $M_\odot$ [13] attributed to the pulsational pair instability mass gap. Stars that begin hydrogen fusion at this mass may undergo a pulsational pair instability supernova near the ends of their lives due to the internal thermal conditions. Stars outside of this mass range, conversely, may transition to their final states more immediately, be that a black holes, neutron stars, white dwarf, or other. The stars in the pair instability mass range finish their hydrogen fusion and begin to form heavier cores, containing helium and other heavy elements up until the typical lead barrier. The pressure, and thermal energy, within these heavy cores will build through each burning stage until the heat reaches a critical temperature, exceeding $10^9 K$ [13], at which point the environment creates electron-positron pairs from photons. While the photons provided thermal pressure to support the star's radius, the pair instability electrons and positrons contribute significantly less outward pressure, which leads to disruption of the star's hydro-static equilibrium. The electron-positron pairs are created with a cascading effect, so large quantities of thermal energy go into creating these particle pairs which do little to support the stars radius, eventually leading to overall contraction of the star due to lack of pressure. A chain effect soon follows; the sudden contraction, and therefore increased temperature, creates a period of explosive element burning, providing more fusion pressure within the star, which in turn causes an increase in radius that may be fast enough and ejects many solar masses worth of material from the star. Pair instability stars may be completely decimated from this collapse, but pulsational pair instability stars may go through this cycle many times over, losing layers of mass each time. After the ejection the pulsational stars will contract again, releasing both light and neutrinos, and encounter another instability; a process that repeats until it reaches a stable mass around 65 $M_\odot$, with a heavy core of 40 $M_\odot$ [13]. The succeeding pulses will eject less material, but have higher energy, and can collide with the initial material becoming extremely luminous [13].

Due to the Pulsational Pair Instability (PPI), and Pair Instability (PI), processes there is an observed gap in the 50 to 135 $M_\odot$ range [14]. However with the discovery of GW190521, a high mass BBH merger, during observing run 3 of the LIGO and Virgo detector network, researchers were able to confirm one of the constituent black holes was in the IMBH range. The GWs originated from a BBH merger with constituent masses of 66 $M_\odot$ and 85 $M_\odot$, leaving behind a remnant of 142 $M_\odot$ [2]. There are contending theories on the production of IMBHs, like hierarchical merging of many smaller black holes [15], or primordial origins allowing for masses in this range [16]. Theory and observation aim to elucidate the formation mechanism for super massive black holes in galactic centers, which IMBH formation could provide a clue to. Thus detecting more IMBHs is crucial to develop our understanding, yet these events are few and far between with such a low production rate of $0.13 Gpc^{-3} yr^{-1}$ [2]. Therefore this project aims to determine the sensitive volume for detection of GWs from BBH mergers containing high mass black holes, with data from current and future ground-based detector networks.

Determining the distance sensitivity of the detector network has been important in past observing runs as well. In observing runs one and two (O1 and O2) the sensitivity distance was calculated for differing constituent masses of IMBH binary mergers, as shown below [17]. The maximum distance calculated by this previous study is around 1.8 Gpc [17] for optimal conditions of a 100 on 100 $M_\odot$ black hole, aligned-spin source. This project expects to see improved sensitivity distances due to better detector sensitivity in run 4, O4, and more advanced modeling.

1.4. Gravitational Wave Tools and Considerations

To accomplish this goal the project will simulate merging events of BBHs with different models with pycbc waveform model families that are partially solved using numerical relativity methods. Numerical relativity is crucial to these waveforms because they reduce the run time of general relativity simulations from as much as a few months, typical of Spectral Einstein Code, to a few seconds [18]. This allows us to run thousands of simulated events in greatly reduced time, and without usage of super computers. Although this method comes with some inherent limitations, like preset mass ratios, there is little error, comparable to the estimated numerical error of generated waveforms, associated with numerical relativity based simulations [18].

There are several waveform models in pycbc, and must all be considered for their potential impact on this study. This project will be utilizing the IMRPhenomXP model to simulate waveforms. After comparing several waveform model families we have determined that each model has correlation significant enough to be negligible. In the coming steps of the study we will produce a complete comparison to demonstrate the validity of this model. IMRPhenomXP is chosen over it's aligned spin counterpart, IMRPhenomXAS, in order to add spin to the component BHs. The Phenom family does an especially good job of modeling the early inspiral, which although not crucial to this study, may be important to considering lower mass systems. In particular the SEOBNR family does not extend it's waveforms to the early inspiral, but this effect is, in practice, inconsequential to this study.

2. Progress and Next Steps

2.1. Simulated Waveforms

This study looks to average over several source factors, and therefore requires an array of conditions for merging BBHs to determine how select parameters will influence merger detectability. The first step then is to begin generating waveforms with various parameters, both intrinsic and external. I have begun simulating waveform strains with randomized initial conditions in order to visualize the many factors that influence strain.

The waveforms I have generated so far utilized the IMRPhenom waveform waveform model families from the python pycbc library, which refrence general relativity to determine the inspiral, merger, and ring-down forms. Each portion of the event i based on the parameters given to the approximant. In figure 2 an example is given, where the spins randomly assigned to the waveforms create a secondary fluctuation in amplitude as the BHs are predicted to disperse of both orbital and spin-angular momentum. These waveforms are also red shifted in amplitude based on the distance using the flat lambda-CDM model.

The plot displayed in figure three illustrates the strain observed in each frequency band of the gravitational wave detector network. These BBHs fall well within the LIGO band, spending more time at lower frequencies, but quickly climbing through frequencies near merger.

These techniques will be applied, using a Monte Carlo sampler to choose parameters, over thousands of simulated GW events in order to characterize distance sensitivity under different parameter values.

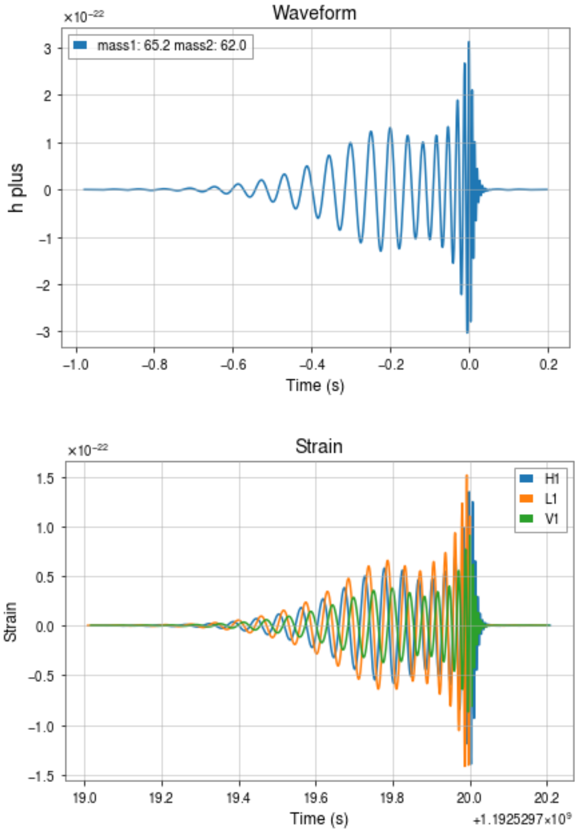

Figure 3: *An example of a simulated merging event using random parameter selection. The waveform was produced with the IMRPhenomXP waveform approximant from pycbc for two black holes of 62* $M_\odot$ *and 65* $M_\odot$ *at 3663Mpc. The form displays an overlayed modification of amplitude due to the BH spins. The top graph shows the strain of the GW plus polarization. The bottom graph displays the strain each detector of the gravitational wave detector network would detect due to sky position.*

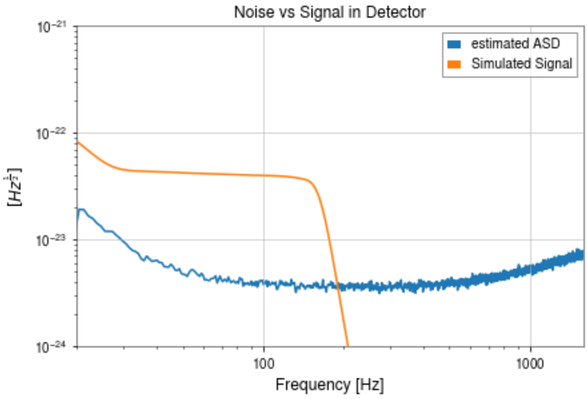

Figure 4: *The strain in the frequency domain and the estimated amplitude spectral density of the noise in the detector corresponding to the merger event modeled in figure 2.*

2.2. Detector Frame

Though the intrinsic parameters of a GW waveform can be included in the waveform model, the detector antenna response will vary. In the bottom graph of figure 2 the right ascension, declination, and inclination, etc., of the event elicits differing response in the three detectors. This particular source would be most favorably oriented with respect to the Livingston detector, producing the strain with the largest amplitude.

In addition to detector sensitivity at any given sky location, the waveforms must be red shifted. This affects the amplitude of the strain detected, the frequency the GWs are observed at, and by extension the red shifting of the masses. The LIGO detector can record frequencies from 10 Hz up to 10 kHz [8], at a SNR of about 12 [7]. A merger far enough away emitting waves can have them shifted out of, or into, the LIGO band depending on the initial source frequency. Therefore the study will have to take into account the initial frequency produced by the merger, determine if this is visible to the detectors, and then determine if any of the following conditions affect its detectability: The red-shift on signals can reduce the frequency of the mergers to the region below the LIGO band, in which case they will no longer be visible. Additionally the amplitude of the signal is inversely proportional to the radial distance to Earth, so some signals may be diminished below the necessary SNR, eliminating them from possible discovery. Along with the signal reduction due to detector antenna response to non-optimally oriented events (optimal events are directly over a detector and face-on) [19], as mentioned above. Using these conditions to correct the waveforms, a final sensitivity distance can be calculated.

2.3. Next Steps

Merging binaries behave like a standard candle, with knowledge of a few intrinsic variables, like mass, spin, etc., we can determine their initial amplitude using general relativity. Similarly with knowledge of radial distance, r_{max}, of the merger it is possible to predict the amplitude the detectors here on Earth will receive. Therefore the waveforms will be generated using waveform models at a distance of $\frac{1}{r_{max}}$ for varying binary masses to maximize the signal. The next steps here will be to be sample waveforms on a large scale, using Monte Carlo sampling, and categorize the efficiencies with which they can be detected based on select variables. Then using Bayesian statistical methods the maximized waveforms will be subtracted from data to construct Gaussian noise likelihoods corresponding to the parameters. From there the SNR can be constructed for the simulations, thereby determining a horizon distance estimate for varying stellar masses in the LIGO detector network.

3. References

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