NANOGrav tutorials

or

How to look for nanohertz gravitational waves

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Gravitational Wave
Astronomy Northwest
Student Workshop
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1. Pulsar Timing Array

A window to the nanohertz gravitational-wave sky

Gravitational wave landscape

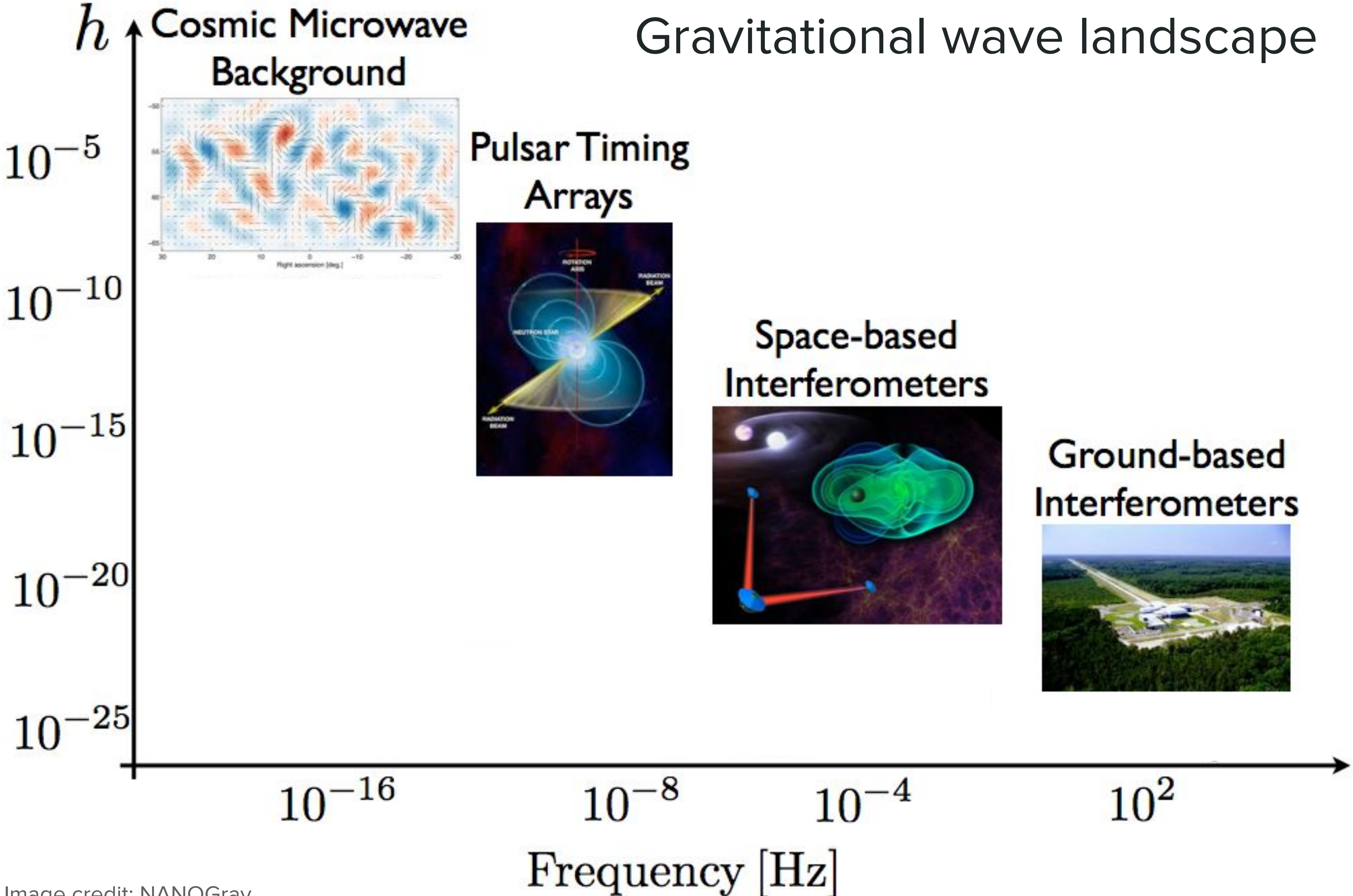
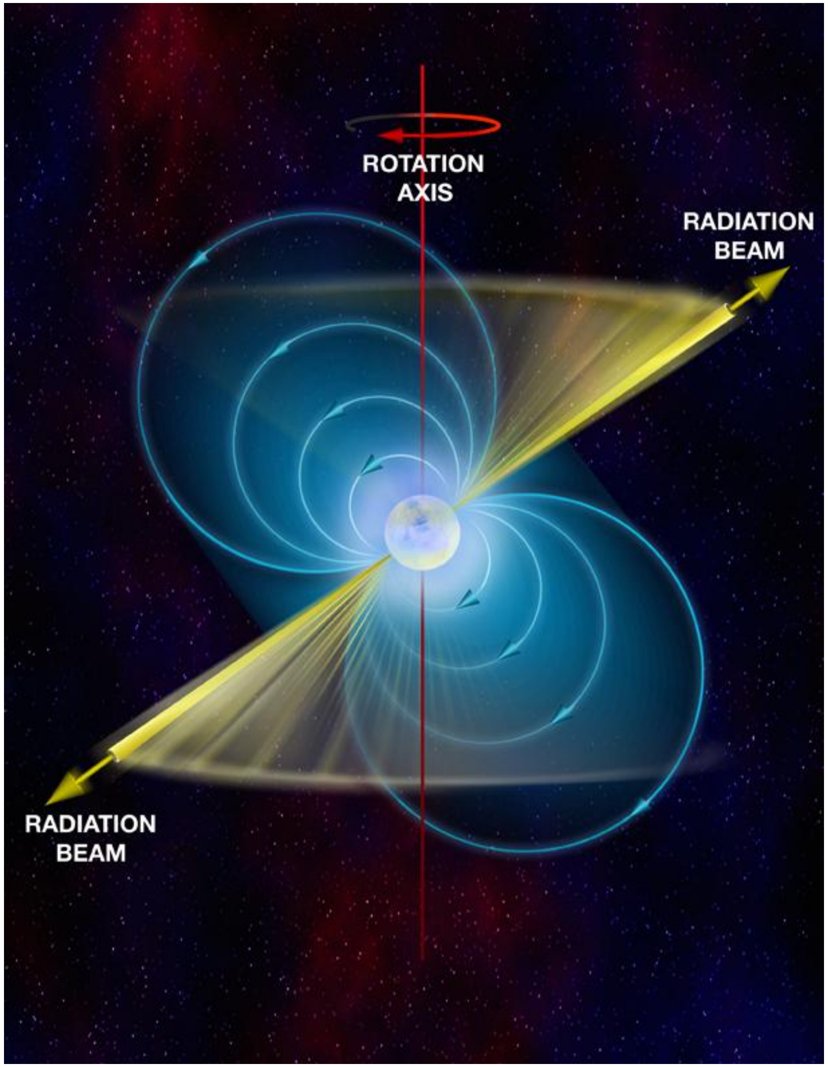
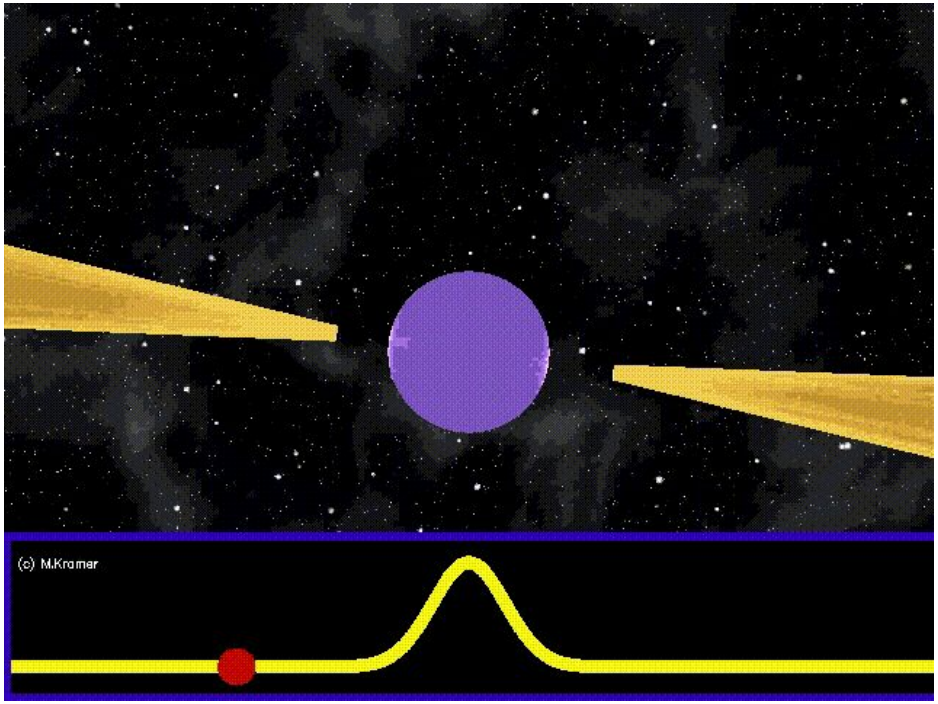


Image credit: NANOGrav

Pulsars





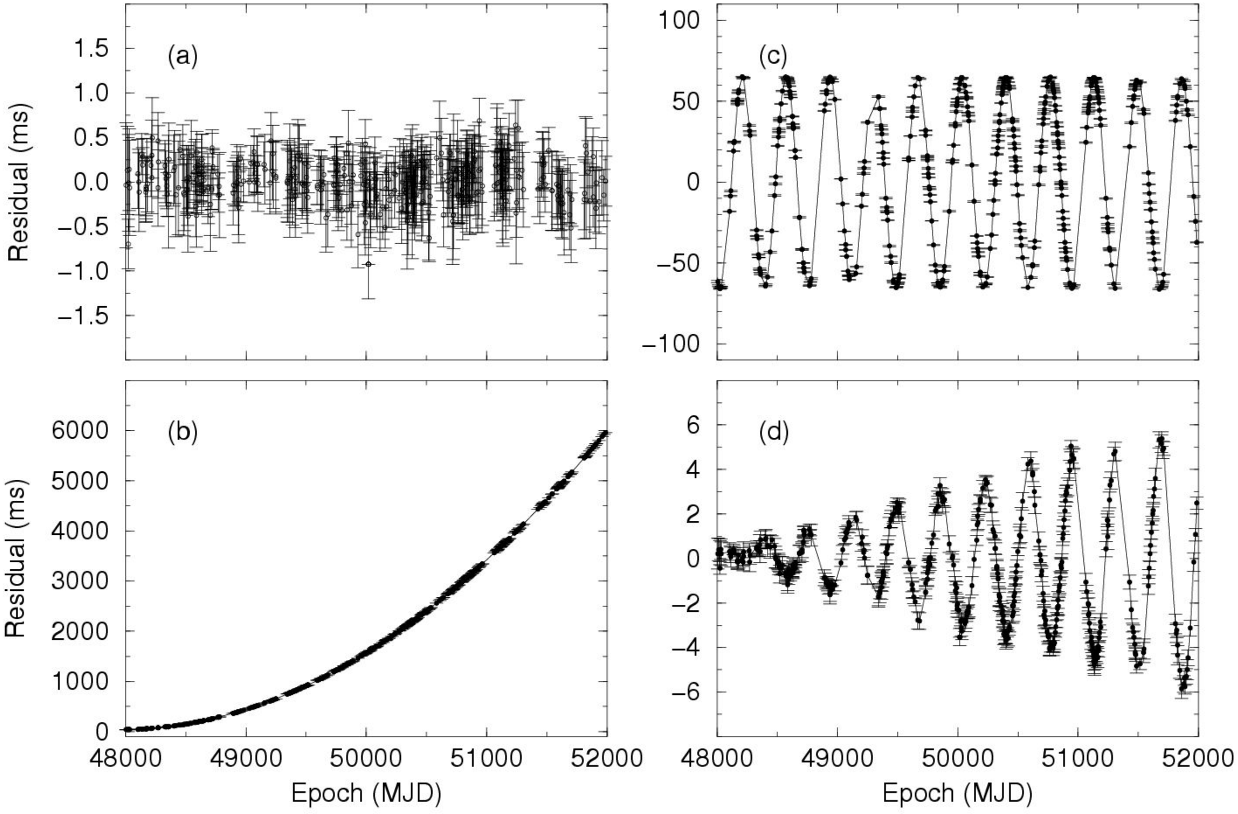
Extreme precision!

Spin period of PSR J0437-4715:

0.00575745193671259±0.000000000000000002s

Image credit: (left) Bill Saxton, NRAO/AUI/NSF; (right) M. Kramer

Pulsar Timing



Timing model

(a) good

(b) wrong frequency derivative

(c) wrong sky position

(d) wrong proper motion

<u>Other effects:</u>

Earth rotates and orbits, interstellar dispersion, NS system has a proper motion, pulsar spin-down, etc.

\+ Gravitational waves

Image credit: Handbook of Pulsar Astronomy, Lorimer and Kramer

Pulsar Timing Arrays

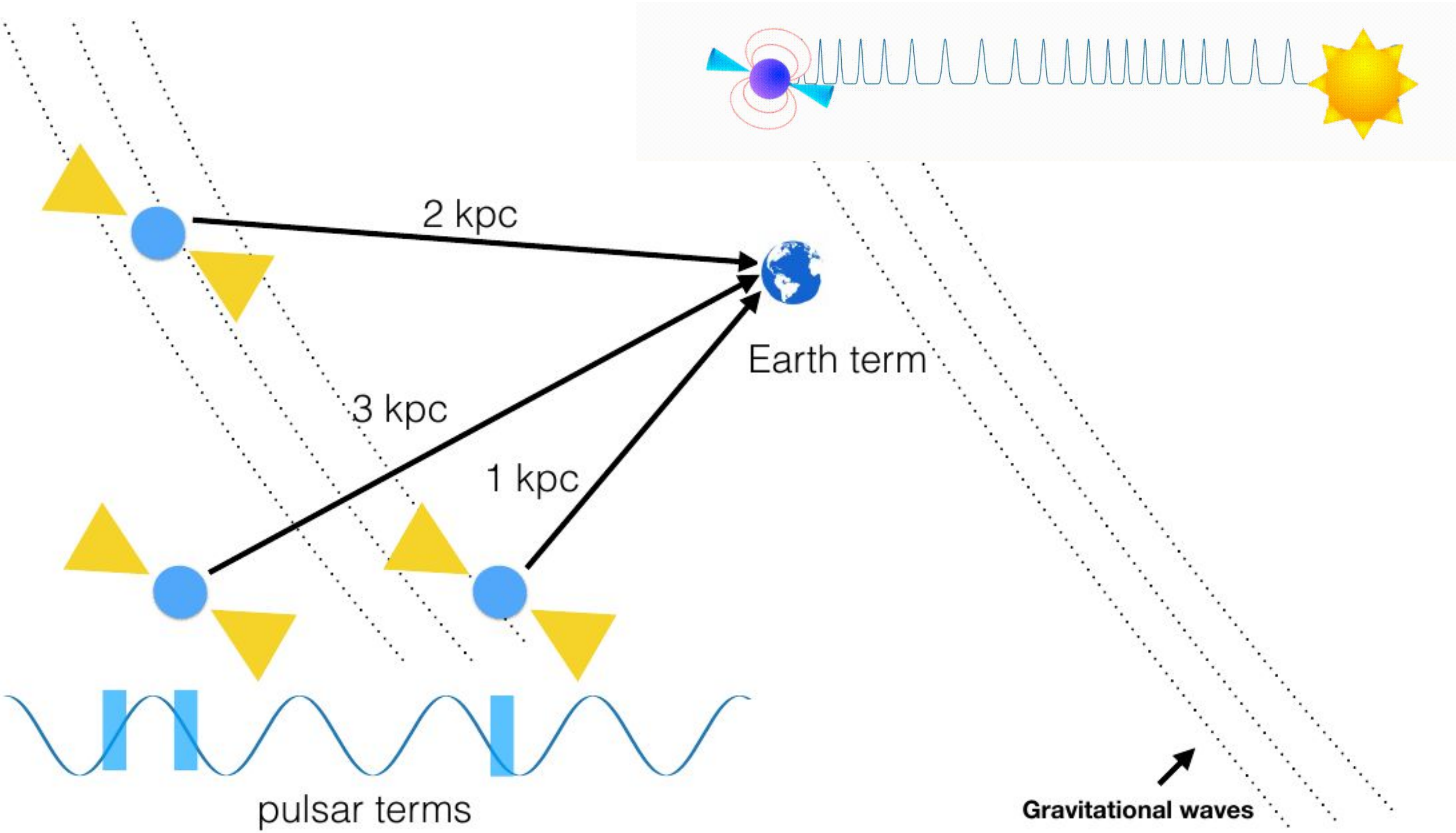


Image credit: Burke-Spolaor, Taylor et al. (2018) arXiv:1811.08826

NANOGrav

- North American Nanohertz Observatory for Gravitational Waves
- ~140 members (~65 faculty, ~30 postdocs, and ~45 graduate students)
- 55 institutions in the US and Canada
- + about 100 undergraduate students annually through the STARS (Student Teams of Astrophysics Researchers) program

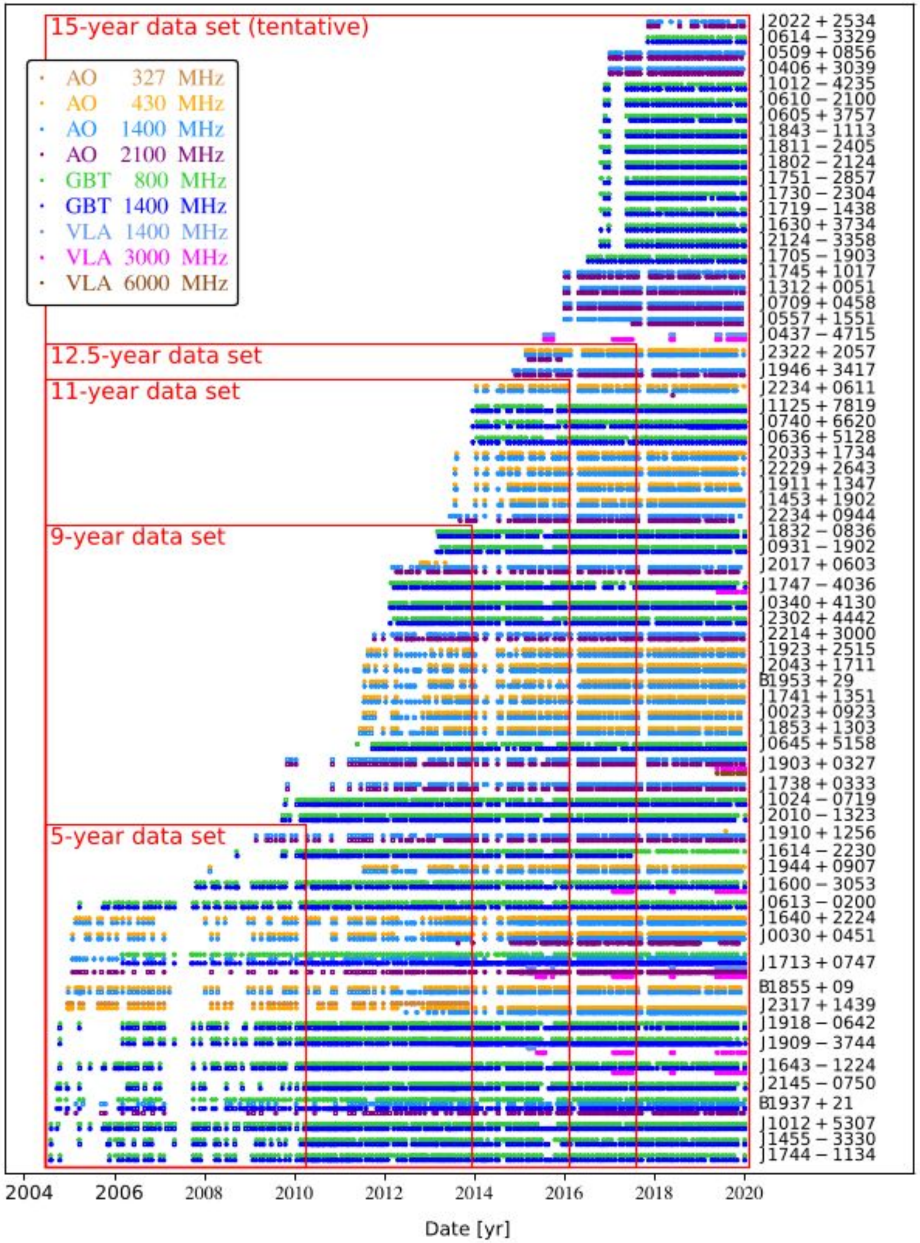


Image credit: (left) David Nice; (right) NANOGrav

Green Bank Telescope

Arecibo Observatory

Potential nHz GW signals

- Deterministic signals
  - Continuous GWs from individual supermassive black hole binaries (SMBHBs)
  - GW memory
  - Generic GW transients (aka bursts)

- **Stochastic GW backgrounds** ← subject of this tutorial
  - SMBHBs
  - Cosmic strings
  - Cosmological background

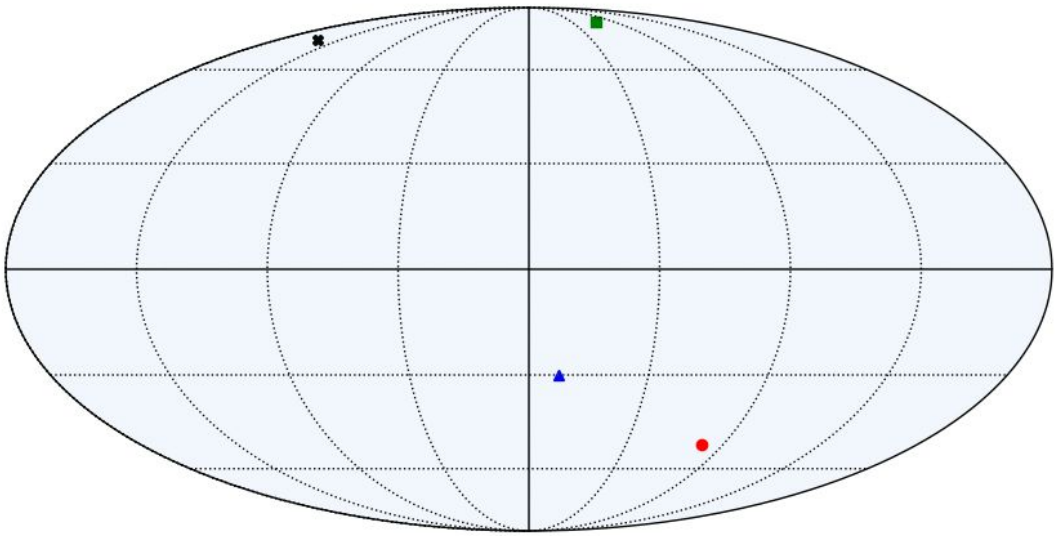
Isotropic stochastic GW background

Stochastic background = superposition of weak signals

Detection progression:

1) All our detectors (pulsars) show the same “noise”

2) Characteristic quadrupolar correlations between pulsars (Hellings & Downs)



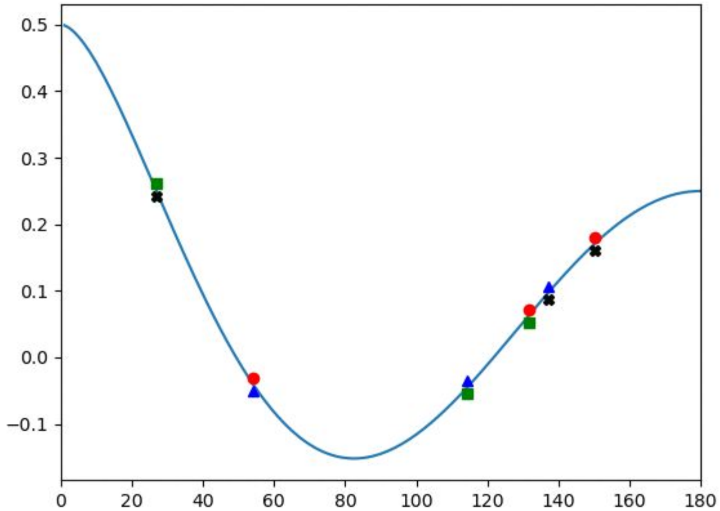


Image credit: Neil Cornish

3. Bayesian methods

Intro to Bayesian PTA model

Posterior **Likelihood** **Prior**

$$p(\theta \mid d) = \frac{p(d \mid \theta)p(\theta)}{p(d)}$$

Model Parameters **Data** **Evidence**

PTA model

$$\vec{t}_{\mathrm{TOA}} = \vec{t}_{\mathrm{det}} + \vec{t}_{\mathrm{stoch}}$$

$$\vec{\delta t} \equiv \vec{t}_{\mathrm{TOA}} - \vec{t}_{\mathrm{det}}\left(\vec{\beta}_0\right)$$

$$\delta\vec{t} = M\vec{\epsilon} + F\vec{a} + U\vec{j} + \vec{r}$$

- Timing Model ($M\vec{\epsilon}$)
- Red Noise ($F\vec{a}$)
- Correlated Noise ($U\vec{j}$)
- Gaussian White Noise ($\vec{r}$)

PTA model

$$\vec{r} = \vec{\delta t} - M\vec{\epsilon} - F\vec{a} - U\vec{j}$$

$$p(\vec{\delta t} \mid \vec{\epsilon}, \vec{a}, \vec{j}, \phi) = \frac{\exp\left(-\frac{1}{2}\vec{r}^T N^{-1} \vec{r}\right)}{\sqrt{\det(2\pi N)}}$$

Marginalization

Params

$$p(\{\vec{\delta t}\} \mid \vec{\eta}) = \frac{\exp\left(-\frac{1}{2}\vec{\delta t}^{\mathrm{T}} \boldsymbol{C}^{-1} \vec{\delta t}\right)}{\sqrt{\det(2\pi \boldsymbol{C})}}$$

Check out: “The Nanohertz Gravitational Wave Astronomer” by Steve Taylor (https://arxiv.org/abs/2105.13270)

Noise parameters

- White noise
  - EFAC (scale factor)
  - EQUAD (quadrature)
  - ECORR (correlated)
- Red noise (power law)
  - Amplitude
  - Spectral Index

$$\rho(f) = \frac{A^2}{12\pi^2}\frac{1}{T}\left(\frac{f}{1\mathrm{yr}^{-1}}\right)^{-\gamma}\mathrm{yr}^2$$

- Timing Model
  - Marginalized
- Power law total params:
  - 3 WN per backend
  - 2 IRN per pulsar
  - 1-2 CRN (fixed?)

PTA model (Enhanced Numerical Toolbox Enabling a Robust PulsaR Inference SuitE)

https://github.com/nanograv/enterprise

Pulsar #1
White Noise
Red Noise
Timing Model

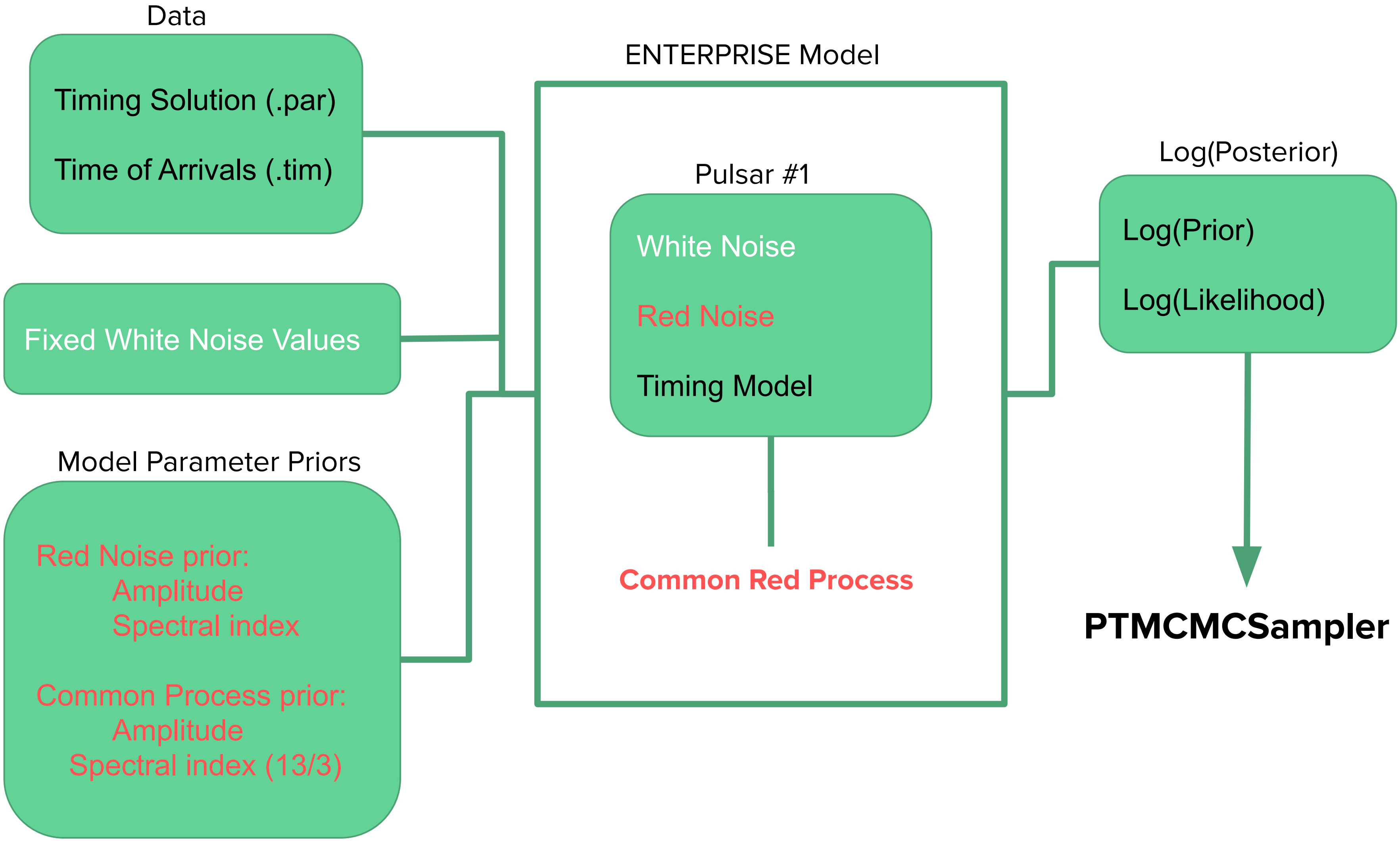
Pulsar #2
White Noise
Red Noise
Timing Model

Pulsar #3
White Noise
Red Noise
Timing Model

Common Red Noise

Even with white noise parameters **fixed**, PTAs can have over 100 parameters.

Single pulsar model



Introduction to data analysis

- Model for the signal
- Way to explore parameters of that model

➡ Markov Chain Monte Carlo (MCMC)

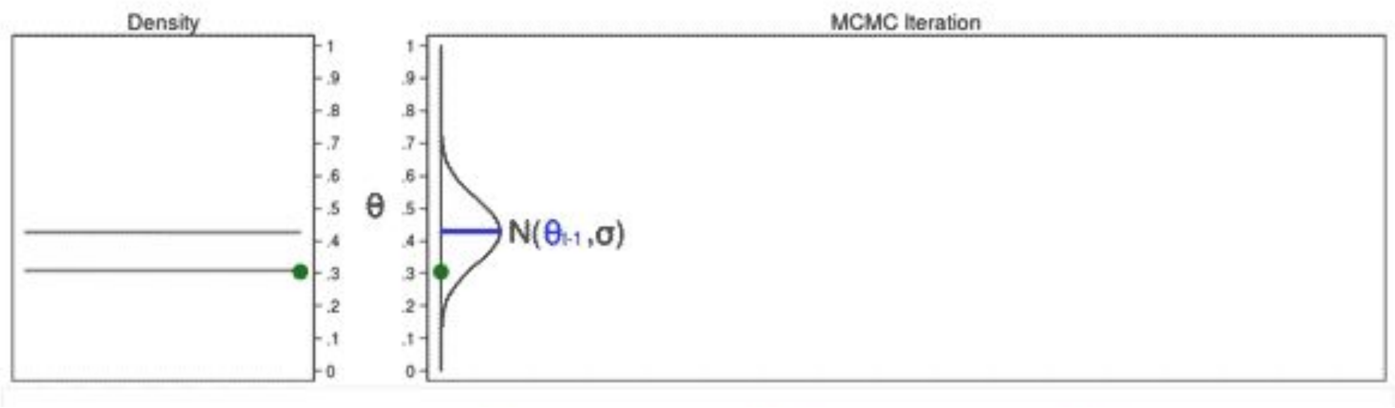


Image credit: Chuck Huber, blog.stata.com/author/chuber/

MCMC: Markov chain Monte Carlo

Goal: Sample from a complicated posterior!

Metropolis-Hastings algorithm

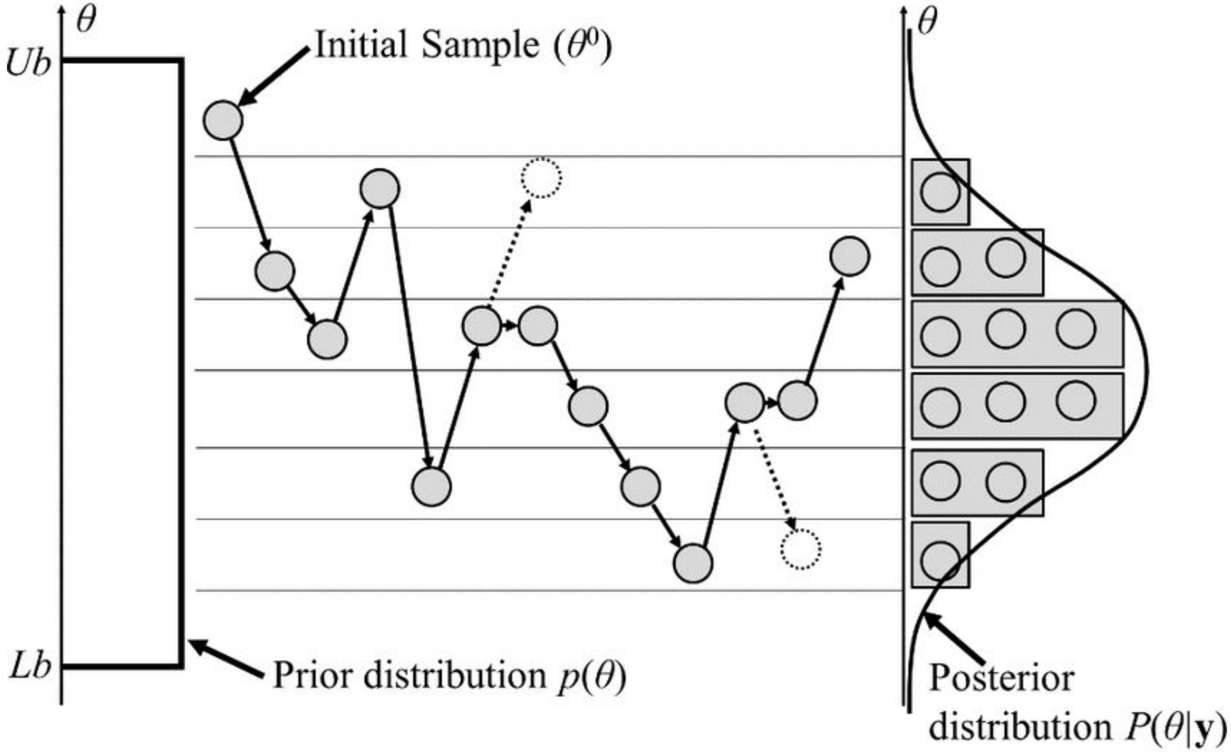
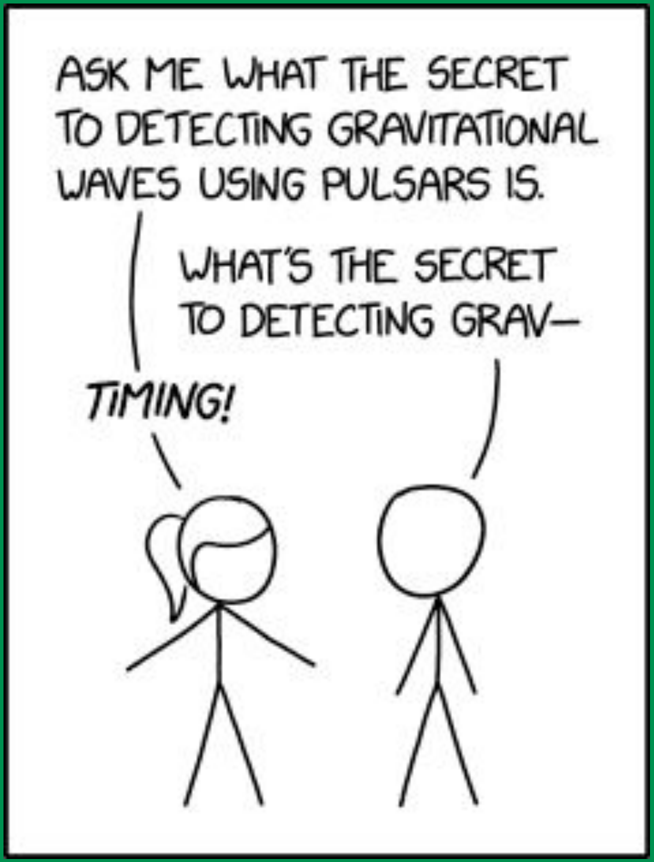


Image credit: Jaewook, Woosuk, Joo-Ho. (2015). Energies. 8. 5538-5554. 10.3390/en8065538.

Summary

- Using Bayesian model to search PTA data for SGWB
- Software to easily do this is called ENTERPRISE
- Feed in .par, .tim files + parameter priors
- Sample with PTMCMCSampler to get posteriors
- Let’s try it...

Questions?



xkcd.com/2358/

Colab notebooks:

Single-pulsar Bayesian analysis of J1909-3744

Creating and analyzing a single-pulsar simulated dataset