Image: LIGO / Caltech / MIT / Sonoma State (Aurore Simonnet)

LIGO

An Introduction to Bayesian Parameter Estimation for Compact Binary Coalescences with bilby

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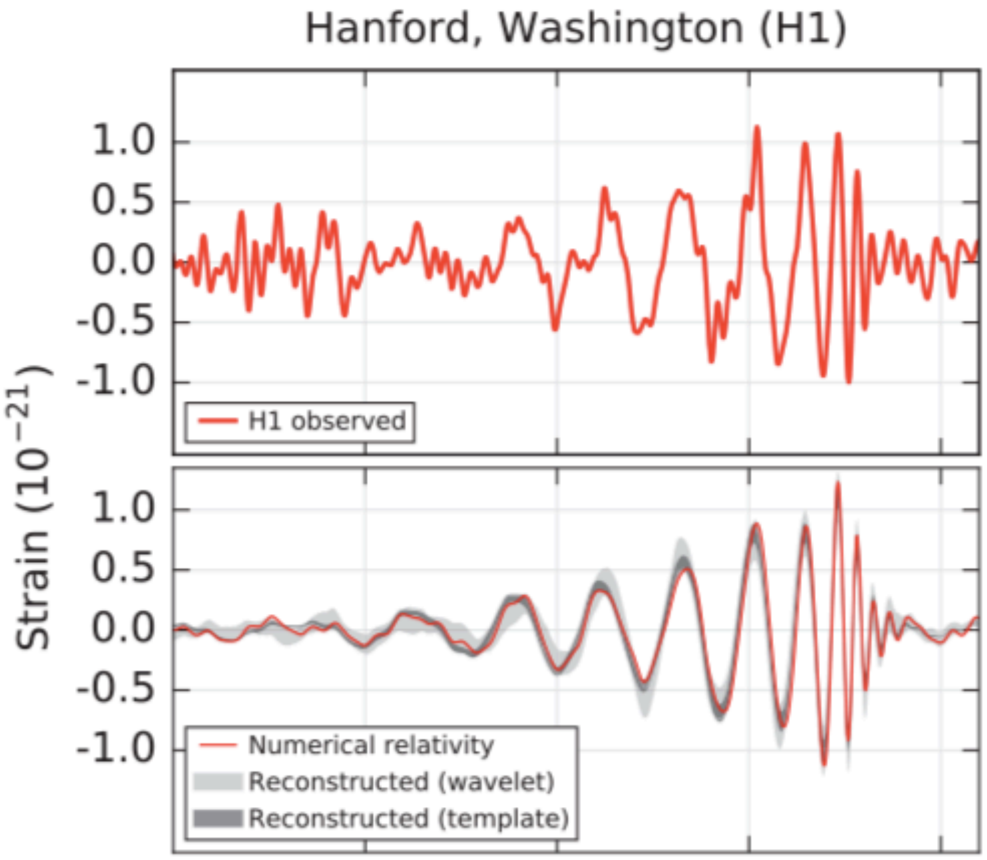
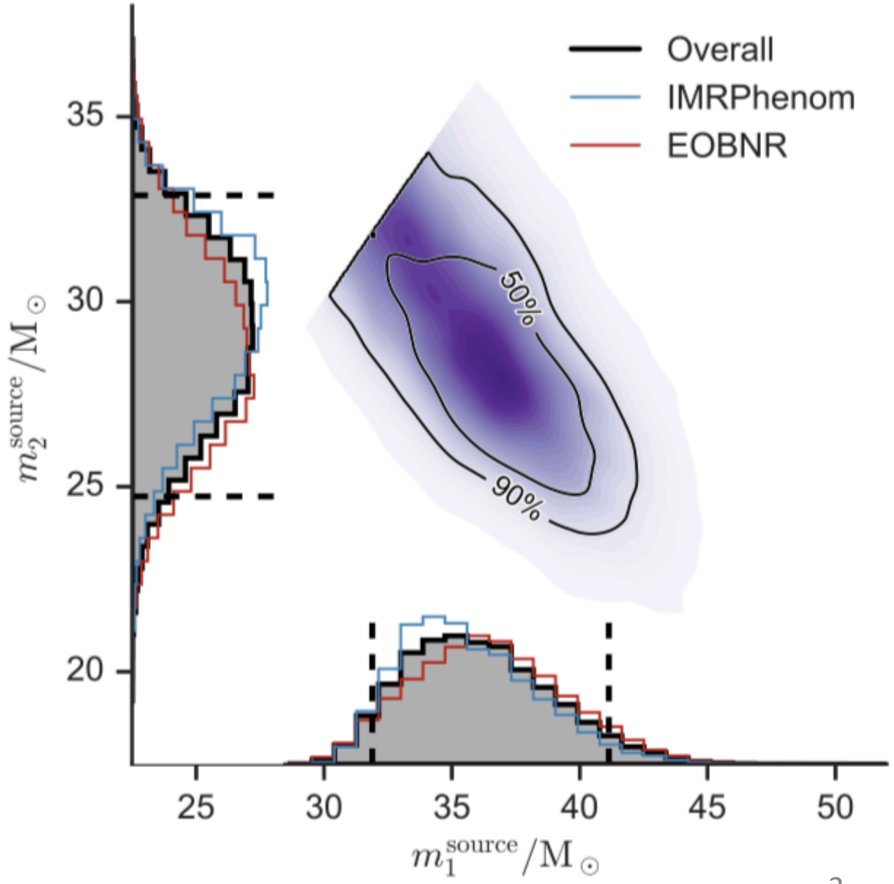
and Space Research

Motivation

- How do we go from this… to this?



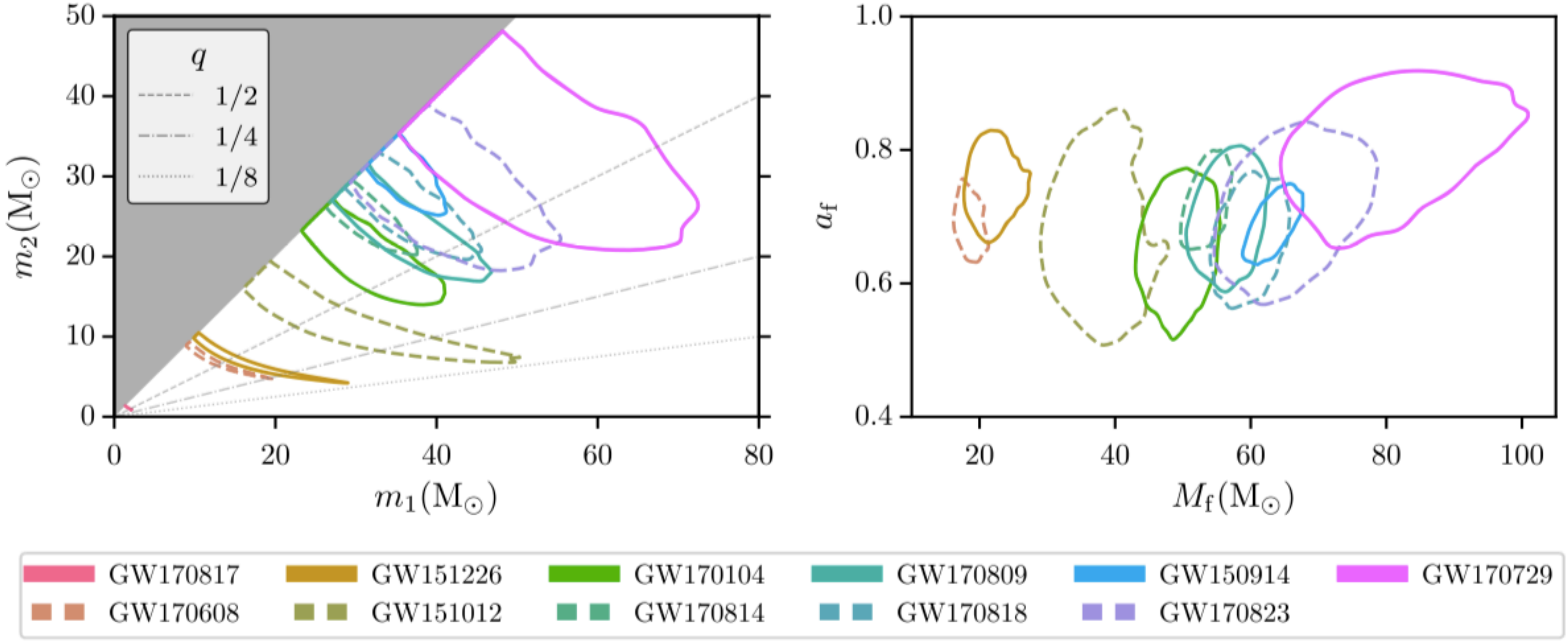
https://arxiv.org/abs/1602.03837



https://arxiv.org/pdf/1602.03840v2.pdf

Motivation

- Measure the properties of individual sources to learn about their evolutionary history



https://arxiv.org/pdf/1811.12907.pdf

Bayes' Theorem

$$\underbrace{p(\boldsymbol{\theta}|d, H)}_{\text{posterior}} = \frac{\overbrace{p(d|\boldsymbol{\theta}, H)}^{\text{likelihood}}\,\overbrace{p(\boldsymbol{\theta}|H)}^{\text{prior}}}{\underbrace{p(d|H)}_{\text{evidence}}}$$

posterior

parameters data model

likelihood prior

evidence

Bayes' Theorem Components

- Posterior – probability of the parameters θ given the data *d* and model *H*

$$p(\boldsymbol{\theta}|d, H)$$

- Likelihood – probability of the data *d* for parameters θ and model *H*

$$p(d|\boldsymbol{\theta}, H) \equiv \mathcal{L}(d|\boldsymbol{\theta}, H)$$

- Prior – initial probability of the parameters θ under model *H*

$$p(\boldsymbol{\theta}|H) \equiv \pi(\boldsymbol{\theta}|H)$$

Bayes' Theorem Components

- Evidence – normalization constant for the posterior, marginalized likelihood

$$p(d|H) \equiv \mathcal{Z}_H = \int \mathcal{L}(d|\boldsymbol{\theta}, H)\pi(\boldsymbol{\theta}|H)\ d\theta$$

Putting it all together:

$$p(\boldsymbol{\theta}|d, H) = \frac{\mathcal{L}(d|\boldsymbol{\theta}, H)\pi(\boldsymbol{\theta}|H)}{\mathcal{Z}_H}$$

The Gravitational-Wave Likelihood

- Under the assumption of Gaussian, stationary noise, the likelihood of observing data d given binary parameters θ is:

$$\mathcal{L}(d|\theta) \propto \exp\left(-\sum_k \frac{2|d_k - h_k(\theta)|^2}{T S_k}\right)$$

Segment duration

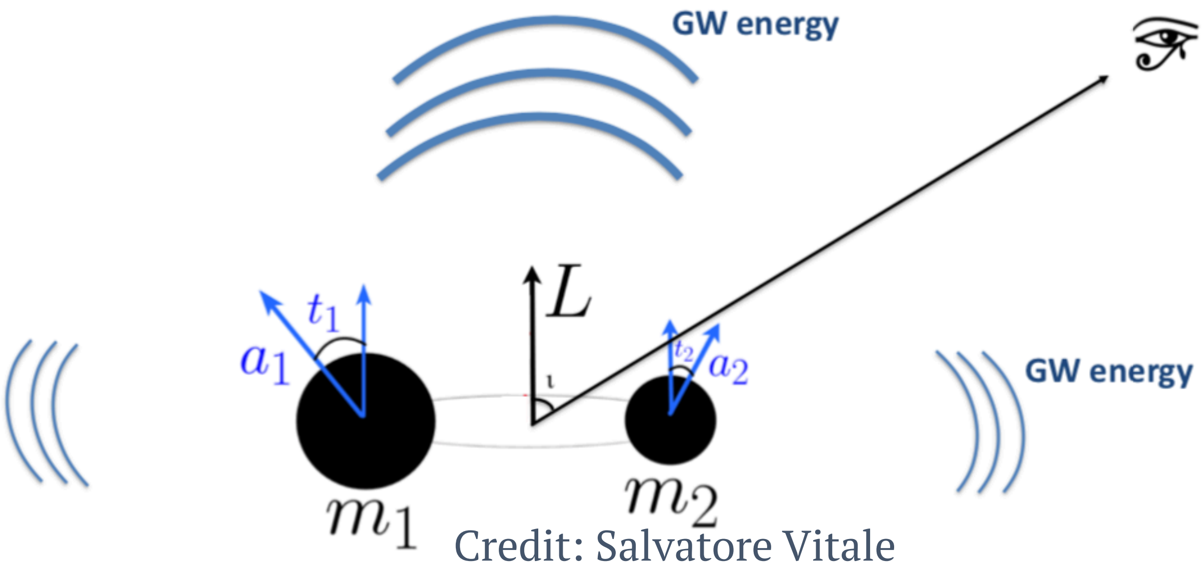
PSD

- $h(\theta)$ represents the gravitational waveform
- k subscript indicates frequency dependence
- Total likelihood is the product of the individual frequency bins

Binary parameters

- For compact binary coalescences, the astrophysical contribution is a waveform that depends on 17 parameters

Intrinsic:
Component masses
Component spins
(Tidal deformabilities)



Credit: Salvatore Vitale

Extrinsic:
Sky location
Distance
Inclination
Polarization
Reference phase
Time at coalescence

- Other models for other types of signals – sine gaussian wavelets, supernova waveforms, etc.

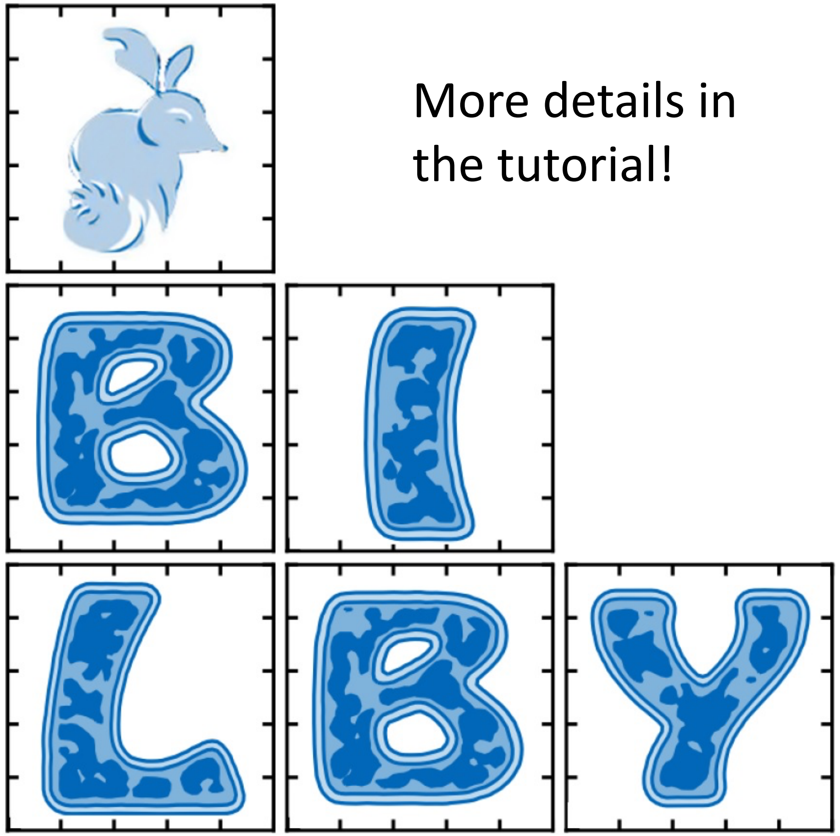
Priors

- Uniform in some parameterization of the mass
- Enforce $m_1 > m_2$
- Uniform in spin magnitudes
- Spin angles isotropic on the sphere
- Isotropic on the sky for right ascension and declination
- Uniform in luminosity volume ($\propto d_L^2$)

Sampling methods

- How do you actually obtain $p(\theta|d)$?
- Could evaluate the likelihood on a grid, but this isn't feasible with 17 parameters
- Instead use a stochastic sampler:
  - Markov Chain Monte Carlo (MCMC)
  - Nested sampling
- Obtain samples from the posterior probability distribution

Bilby



More details in the tutorial!

- The **B**ayesian **I**nference **Lib**rar**y** is a software package designed to enable parameter estimation for compact binary coalescences and more general problems
- Emphasis on modularity, transparency, and ease of use
- Wrapper for many different external samplers including dynesty, pymultinest, cpnest, emcee, ptemcee, and others
- Can analyze real data from LIGO and Virgo or simulated signals

Additional bilby resources

- Gitlab repo: https://git.ligo.org/lscsoft/bilby
- Documentation: https://lscsoft.docs.ligo.org/bilby/
- Slack workspace: bilby-code.slack.com
- GWTC-1 analysis: https://github.com/IsobelMarguarethe/Bilby-GWTC-1-Analysis-and-Verification/tree/v2.0
- Papers:
  - https://arxiv.org/abs/1811.02042
  - https://arxiv.org/abs/2006.00714

Backup

Measuring source properties from the waveform

$$\tilde{h}_+(f) = \frac{1}{2}\mathcal{A}_{\mathrm{GW}}(f)(1+\cos^2\iota)\cos\phi_{\mathrm{GW}}(f)$$

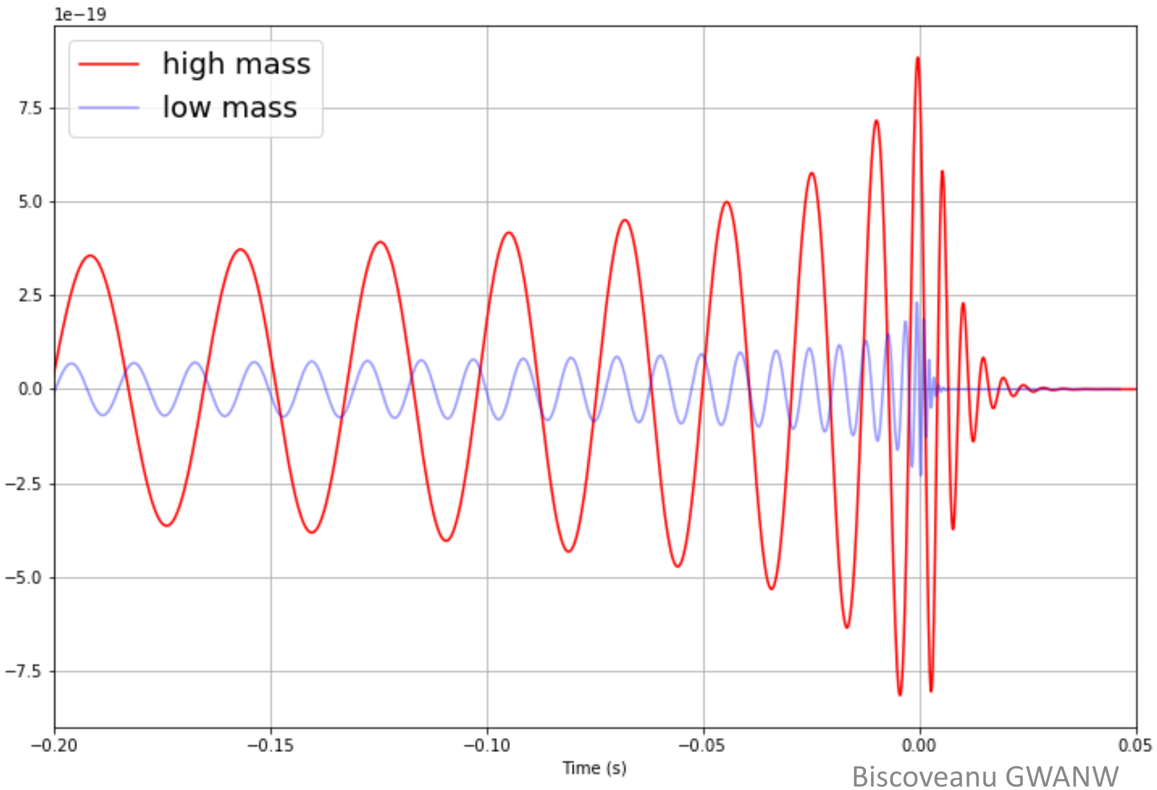
$$\tilde{h}_\times(f) = \mathcal{A}_{\mathrm{GW}}(f)\cos\iota\sin\phi_{\mathrm{GW}}(f)$$

$$\tilde{h}(f) = F_+\tilde{h}_+(f) + F_\times\tilde{h}_\times(f)$$

- Two polarizations
- Dependence on mass, spins, distance, etc. encoded in amplitude and phase
- Antenna patterns depend on the detector geometry and encode the effect of the extrinsic parameters

Effect of Mass

- Bigger mass → bigger amplitude
- Final mass measured from ringdown

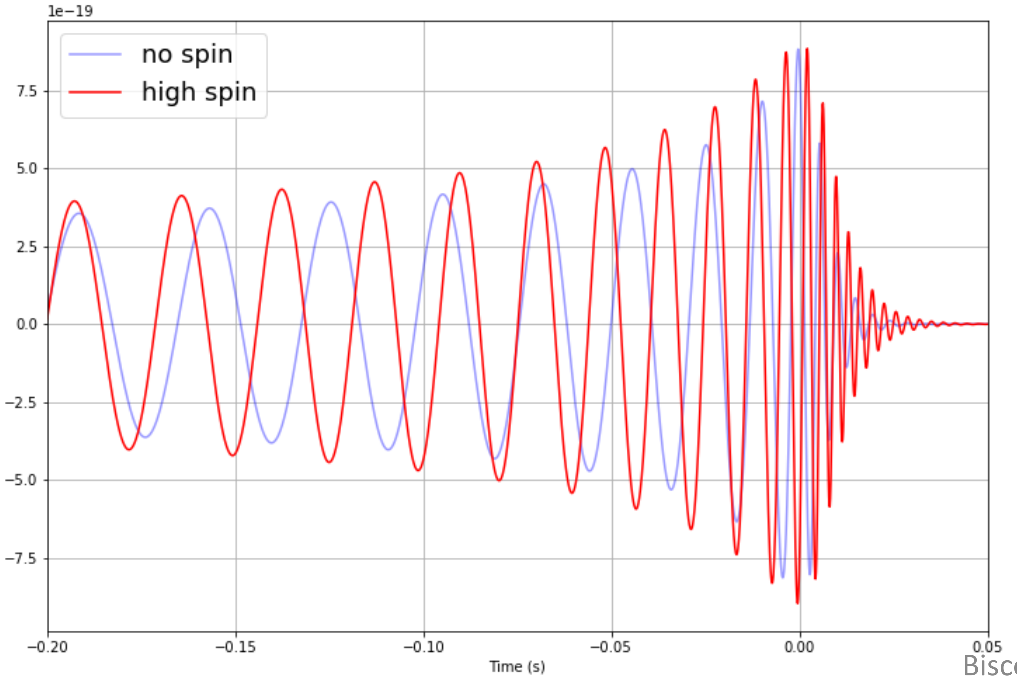


$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

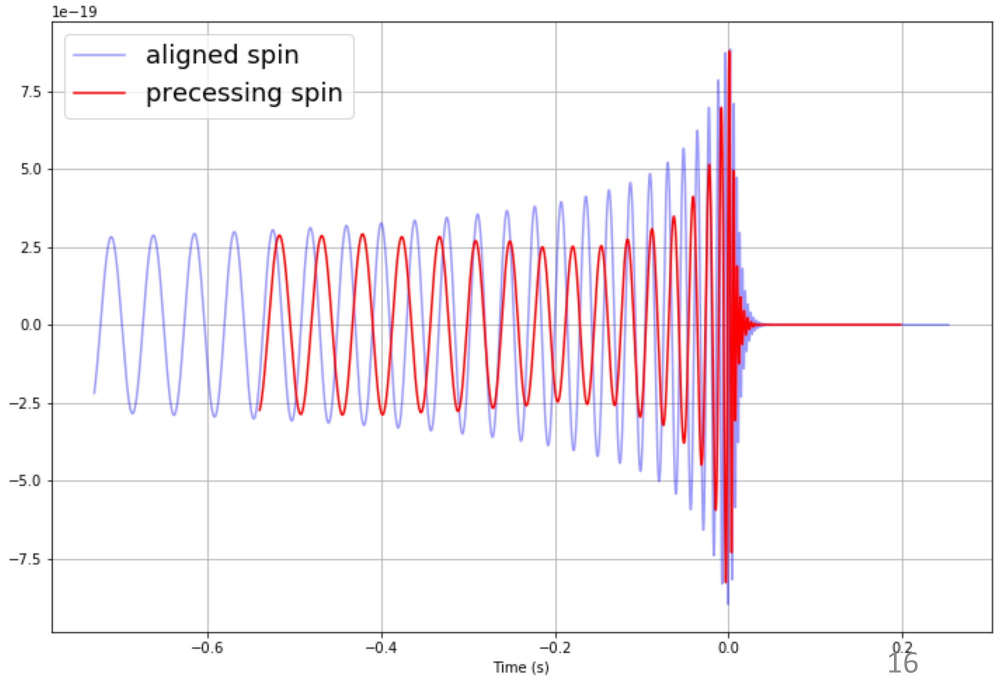
$$\mathcal{A}_{\mathrm{GW}} \propto \frac{\mathcal{M}^{5/6} f^{-7/6}}{d_L}$$

Effect of Spin

- More positive aligned spin → orbital hangup
- Takes longer for the system to merge

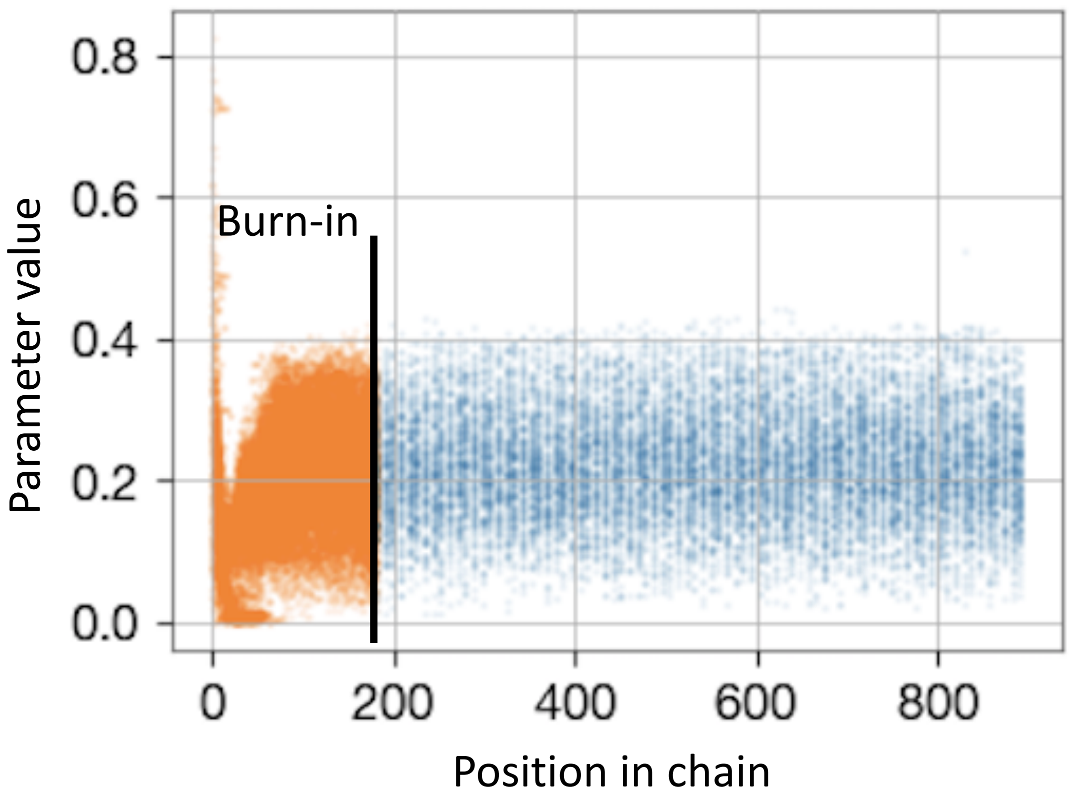


- Precessing spins → amplitude modulations
- Spins misaligned to orbital angular momentum

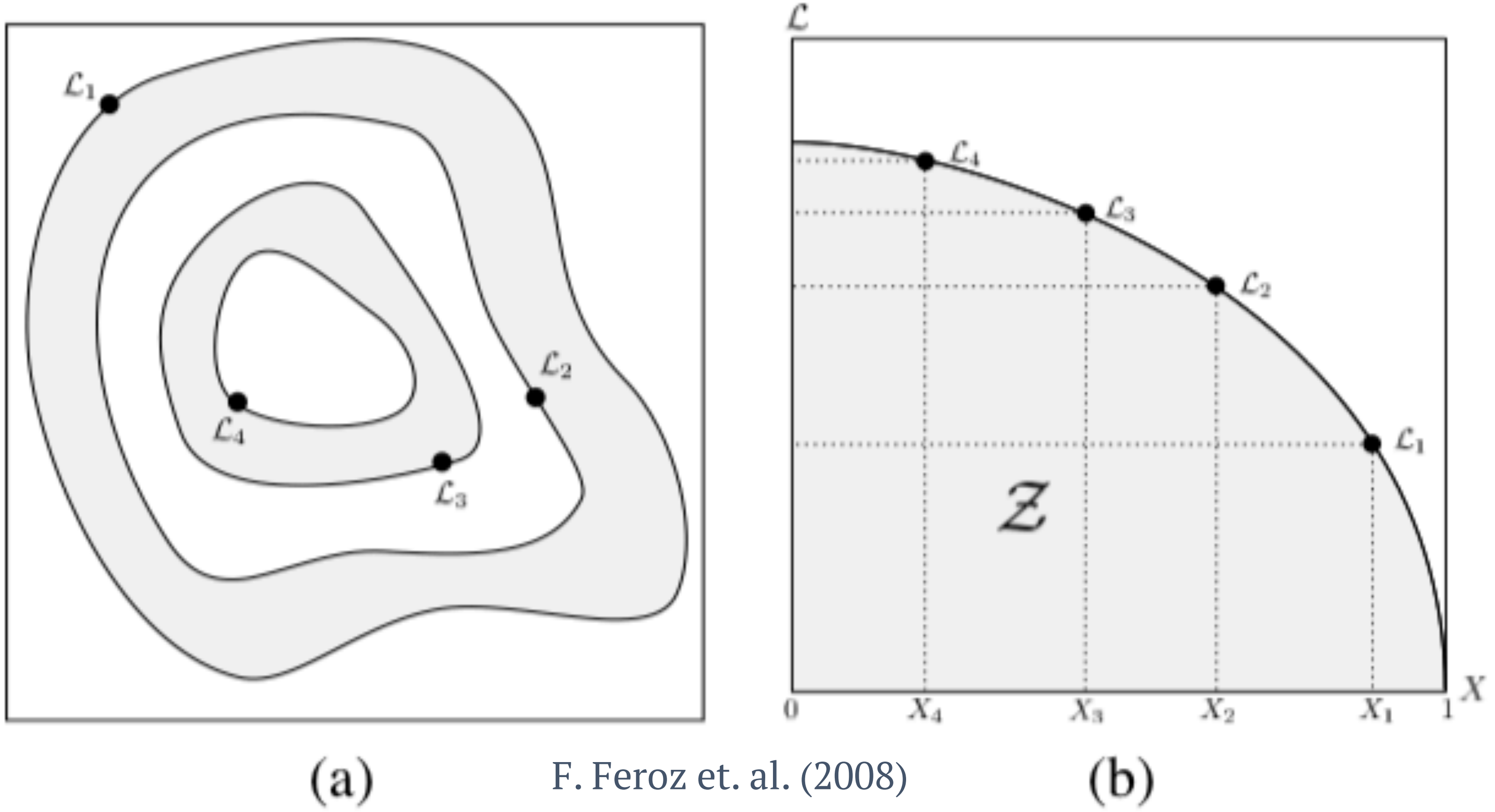


MCMC

- Particles undergo a random walk through the parameter space, where the probability of jumping to a new location is dictated by the proposal density function
- Determining a suitable proposal density function is the hard part of sampling – a simple example is a Gaussian centered on the current location
- Burn-in period before the walkers “forget” their starting positions
- Adjacent samples in a chain are correlated – chains need to be thinned by the integrated autocorrelation time



Nested Sampling



F. Feroz et. al. (2008)

- Sprinkle a set of live points over the prior space
- Replace the live point with the lowest likelihood with a point with a higher likelihood
- Evidence is the product of the likelihood at the discarded point and the difference in the prior volume between iterations
- Obtain samples from the prior in the process of calculating the evidence
- Proceed until a termination criterion is reached