System Identification of LIGO Suspensions and Interferometer

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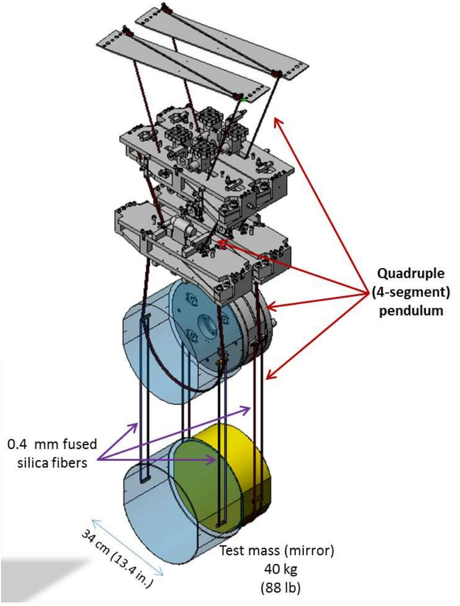
Flow of the Presentation

- Objective
- In the context of LIGO
- System Identification
- Measurement
- Single Loop Suspension
- Simple Harmonic Oscillator
- Acknowledgements & References

Objective

Knowing the transfer function with high accuracy and precision



System: LIGO Quadruple Mirror Suspensions
Actual System: Simple Harmonic Oscillator & LIGO Single Loop Suspension

System Identification and Optimal Control

Step 1

Maximise Fisher Information Matrix of the system and obtain parameter estimates

(⇒ minimise covariance)

Step 2

Obtain the optimal transfer function of the system

(based on parameter estimates)

Fisher Matrix: Inverse of the covariance matrix (minimise uncertainty)

Image Credits: ligo.caltech.edu/page/about-aligo

Identification in LIGO

SysID in LIGO

- Interferometer Calibration
- Quadruple Suspension
- Seismic Isolation

- What are the parameter values?
- Is the estimation good?
- How good is good enough?
- What are the uncertainties?

Interferometer Calibration

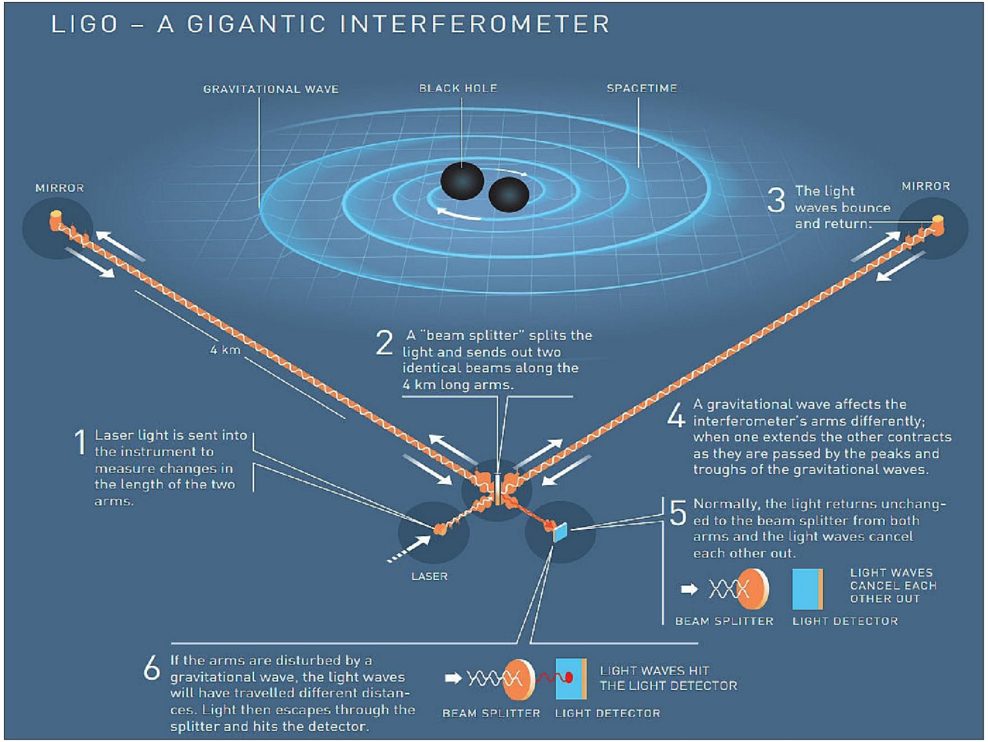


Image Credits: https://osr.org/blog/kids/observing-gravitational-waves-at-ligo/

- Parameters: poles and zeros
- Need to know the strain channel calibration up to **0.1% accuracy**
- Calibrate to identify BH signals and mergers

$$T(s) = a_m \frac{(s-Z_1)\cdot(s-Z_2)\cdot\ldots\cdot(s-Z_m)}{(s-P_1)\cdot(s-P_2)\cdot\ldots\cdot(s-P_n)}$$

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Quadruple Suspensions

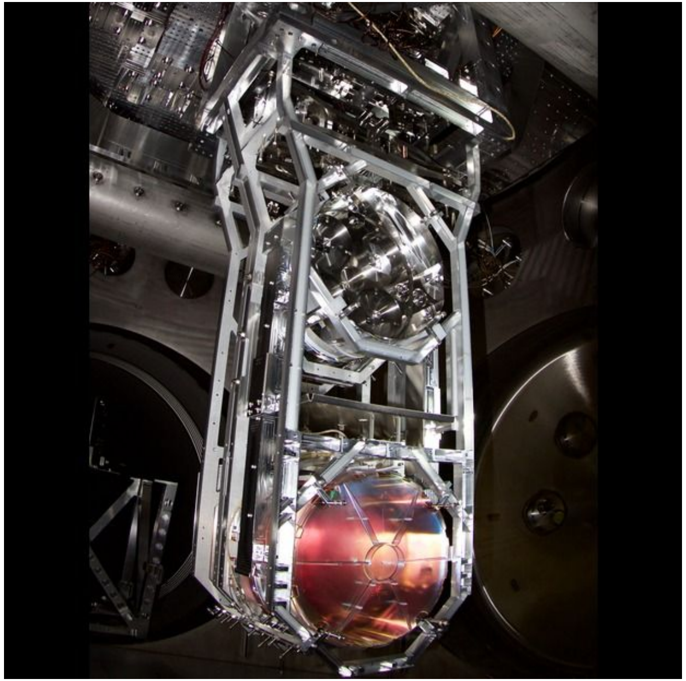
- Is the pendulum working correctly?
- Are expected parameters = obtained parameters?
- Is the cross coupling as expected?

Optimal SysID based transfer function for fault diagnosis!

Example of a MIMO system:

$$G(s) = \begin{bmatrix} \frac{s-1}{(s+2)(s+3)} & \frac{4}{s+3} \\ \frac{s}{(s+2)(s+3)} & \frac{2}{s+2} \end{bmatrix}$$

Off diagonal terms of a tf matrix



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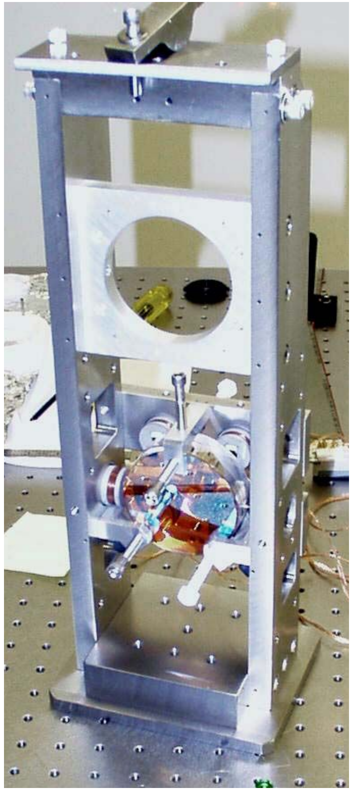
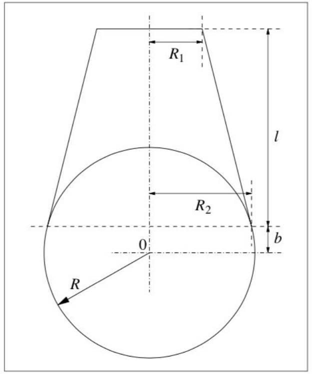
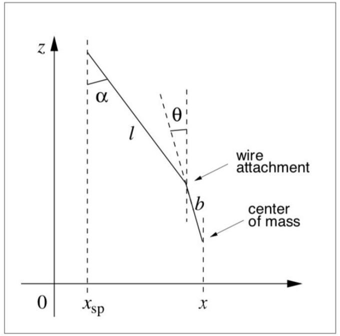
LIGO Single Loop Suspension



Dynamics and Frequency response





$$\ddot{x} + \gamma_x \dot{x} + \omega_x^2 x = \omega_x^2 (x_{sp} + b\theta)$$

$$\ddot{\theta} + \gamma_\theta \dot{\theta} + \omega_\theta^2 \theta = \frac{\omega_\theta^2}{l+b}(x - x_{sp})$$

$$\ddot{\phi} + \gamma_\phi \dot{\phi} + \omega_\phi^2 \phi = \omega_\phi^2 \phi_{sp}$$

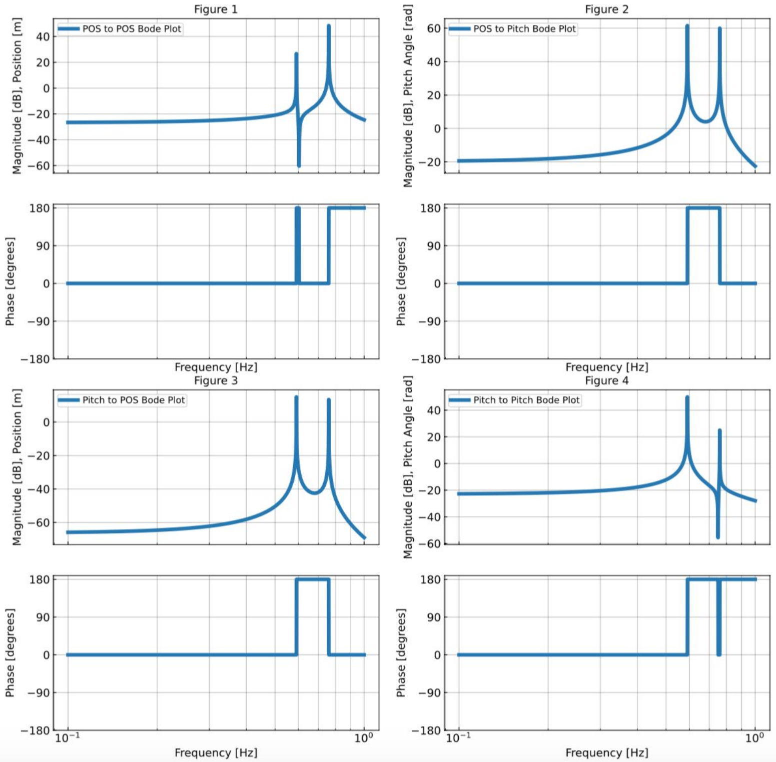
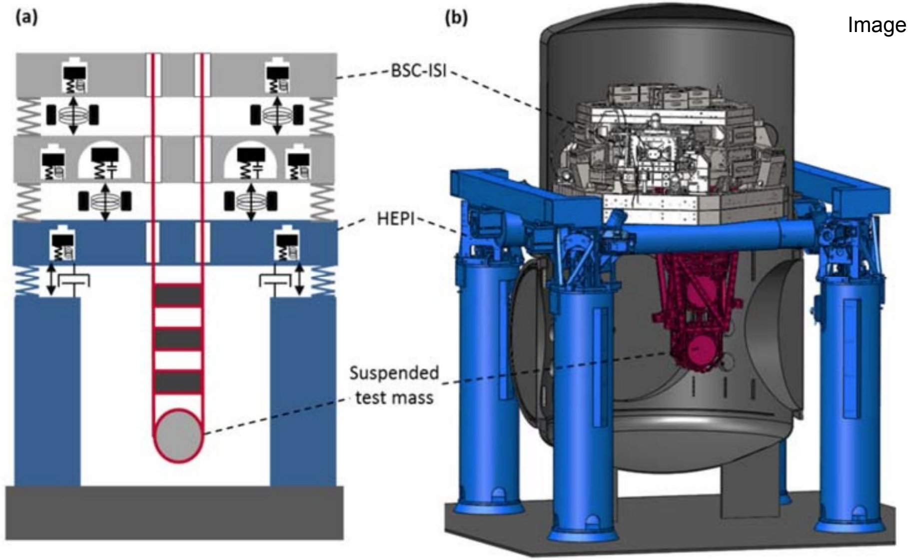


Image Credits: https://www.ligo.org/multimedia/gallery/opt-images/sos.jpg

Seismic Isolation

- Active vibration isolation system above quadruple pendulum
- Operation: Low frequency regime



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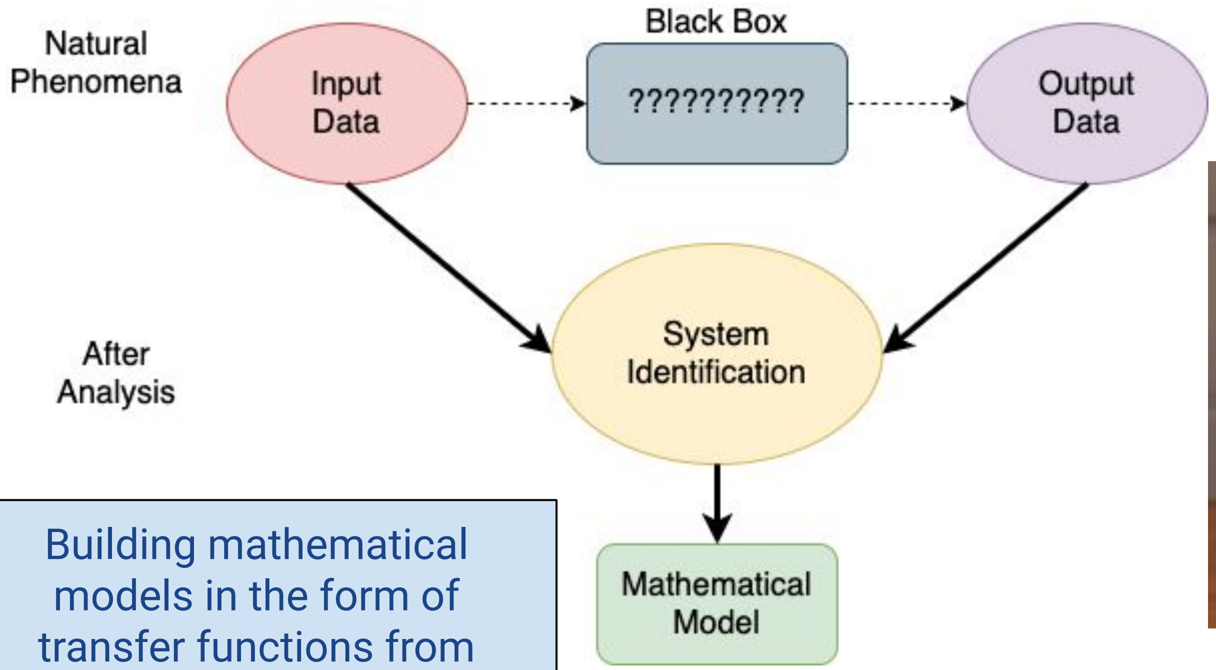
(a) schematic and (b) CAD model of the isolation systems supporting the core optics in the BSC chambers.

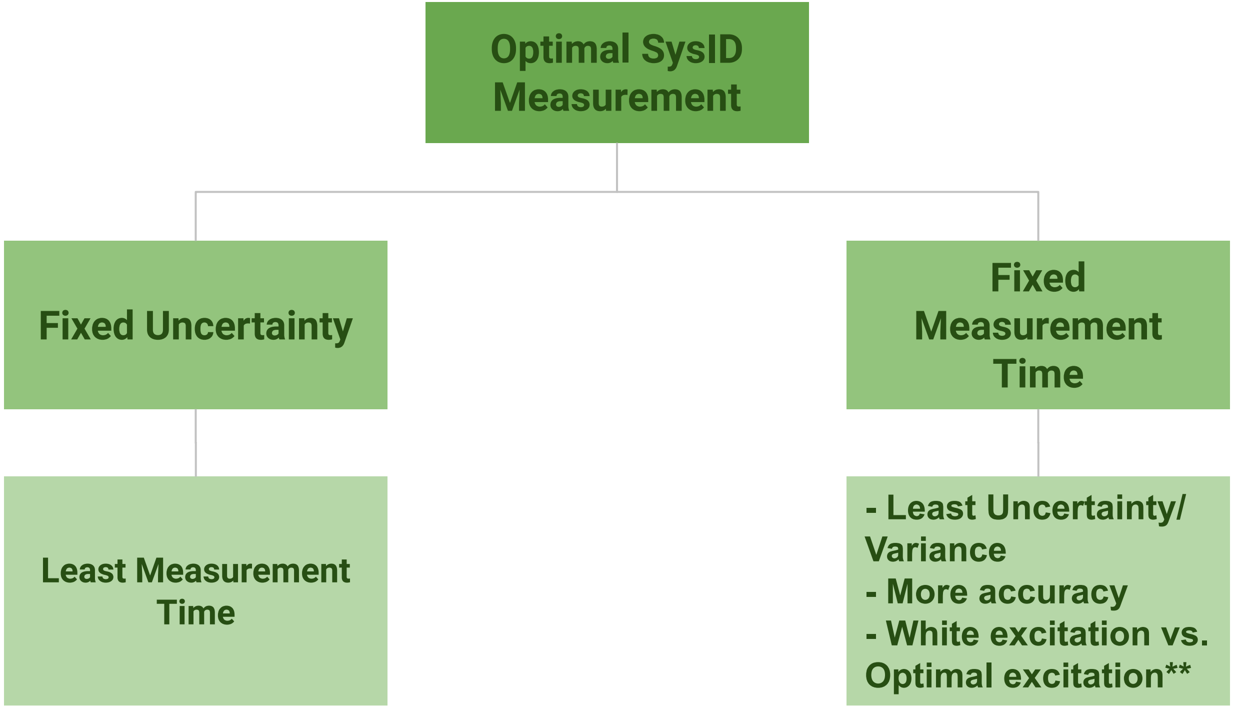
System Identification (SysID)



Building mathematical models in the form of transfer functions from observed input-output data

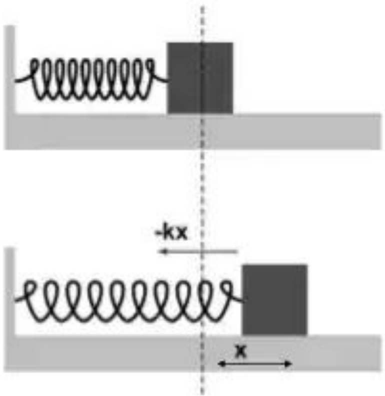
Image Credits: Google Images

Measurement

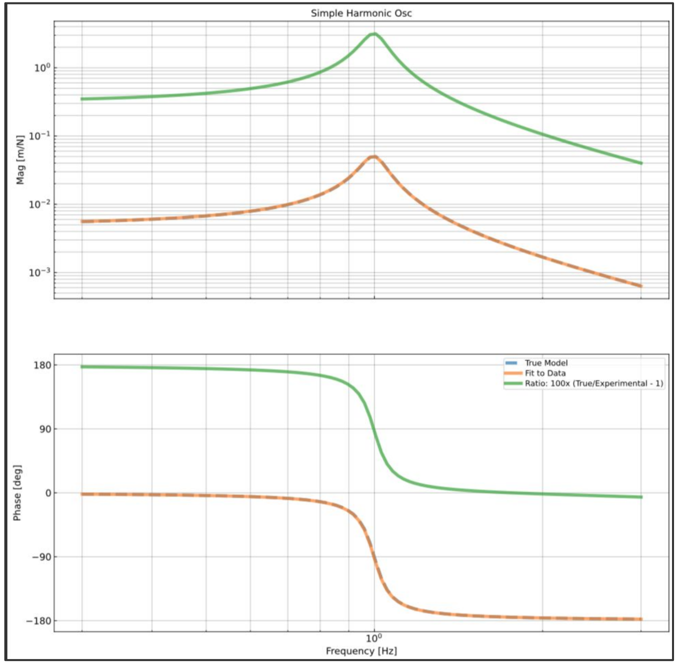


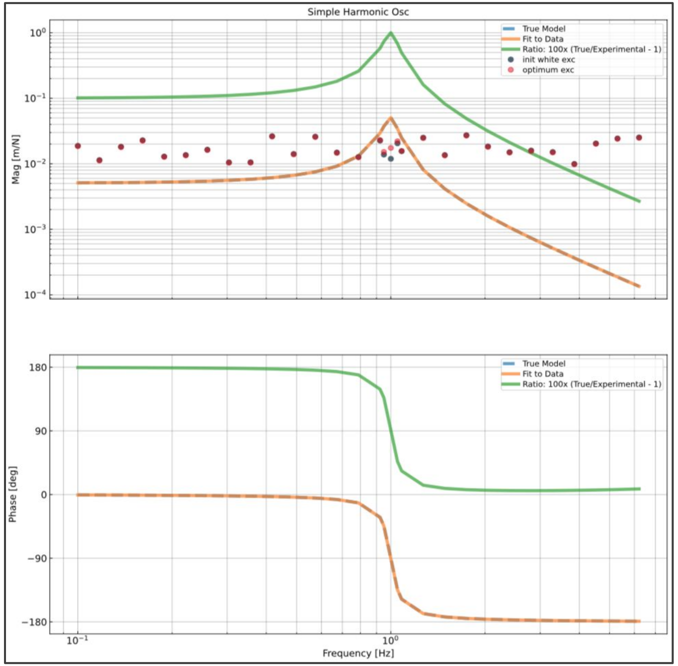
Simple Harmonic Oscillator

$$m\ddot{x} + c\dot{x} + kx = F(t)$$



$$G(s) = \frac{X(s)}{F(s)} = \frac{\frac{1}{m}}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$





Results
Optimal > White
(Excitation)

Parameter	True Value	White Exc	Uncertainty (White)	Optimal Exc	Uncertainty (Optimal)
Q	10	9.97345	0.0185351	9.997	0.00690703
Res. Freq. [Hz]	0.628319	6.27966	0.00056758	6.28006	0.000215723

Acknowledgments & References

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