

LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY  
- LIGO -  
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Technical Note	LIGO-T2100239-	2021/08/31
<b>Low-noise Nonlinear Cavity for Cryogenic Interferometers</b>		
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## 1 Experimental Set up

The end goal of this project is measuring and eliminating frequency noise of the Degenerate Optical Parametric Oscillator (DOPO). The role of DOPO is converting the frequency of LIGO's laser frequency from 1064 nm to 2128 nm so they new frequency can be used in cryogenic LIGO [1]. Measuring the frequency noise of DOPO can be done through homodyne or heterodyne scheme as shown in figures 1 and 2. Though the two schemes are different, there are noise sources that are common to both of them such as the thermal noise of mirrors 1 and 2, the free-running laser noise, seismic noise, PD noise (dark noise and shot noise), PLL noise, and finally the spectrum analyser noise.

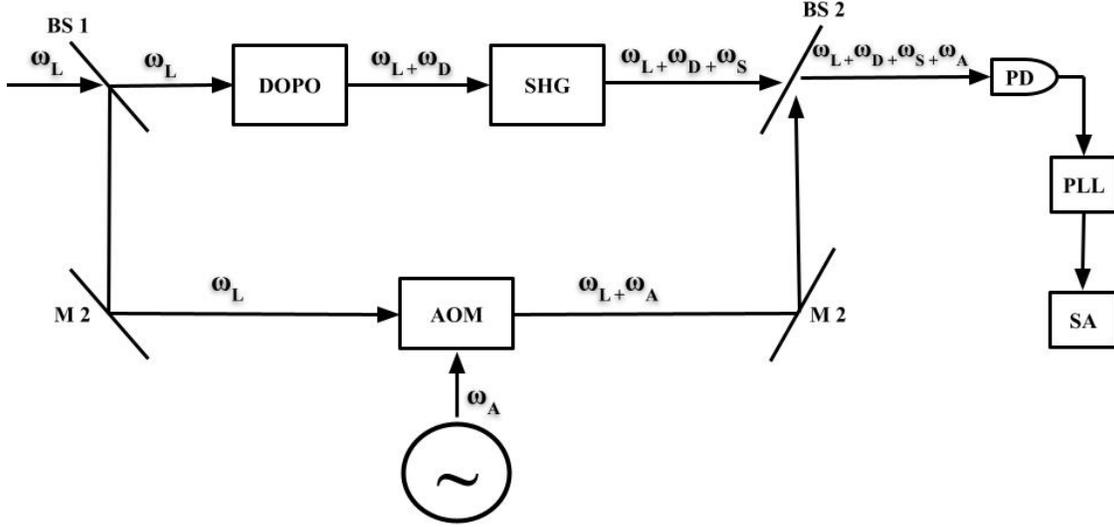


Figure 1: Heterodyne detection scheme to measure the frequency noise of DOPO. The first arm is the one including DOPO and SHG, whereas the second arm is the one including AOM. The mirrors, M1 and M2, are highly reflective. The angular frequency  $\omega_L$ ,  $\omega_D$ ,  $\omega_S$  and  $\omega_A$  represent laser frequency, DOPO's frequency, SHG's frequency, and AOM's frequency, respectively.

## 2 Noise Sources

In order to decide which scheme works best for measuring the frequency noise of DOPO, all the noise sources need to be considered. After that, the scheme with the higher signal-

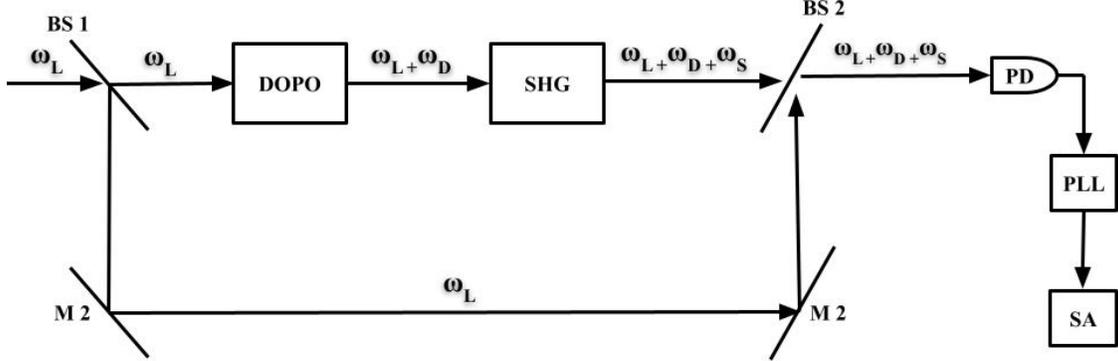


Figure 2: Homodyne detection scheme to measure the frequency noise of DOPO. The first arm is the one including DOPO and SHG, whereas the second arm is the one including AOM. The mirrors, M1 and M2, are highly reflective.

to-noise ratio will be considered more efficient for our purposes. There are many ways to classify noise sources in each setup. Here, we classified noise by electronic components or optical setup that produce them. In our analysis, we need to make sure that the power spectral density functions are valid in the 5 Hz - 5 KHz range because this is the range in which LIGO operates. Some of the noise sources that we plotted so far are shown below

## 2.1 Mirrors 1 and 2

The substrate of these mirrors is fused silica and the coating is made of fused silica ( $SiO_2$ ) and tantalum ( $Ta_2O_5$ ). The mirrors are round with a diameter of 1 inch, so the area is  $\pi r^2 = \pi(0.0127)^2$ . The thickness of the substrate is 0.005 m and the coating is  $4.72\mu m = 4.72 \times 10^{-6}$  m (2.75 $\mu$  m of silica + 1.97 $\mu$  m of tantalum). There are many forms of thermal noise in these mirrors. So far, we considered some types of thermal noise in the substrate of the mirrors. Specifically, we considered the thermo-refractive and thermo-optic noise.

- **Thermo-refractive Noise:** describes how temperature fluctuations induced by the laser change the refractive index of the mirrors' coating, which in turn changes the

resonant frequency of the mirrors.

$$S(f) = \frac{4a\beta^2}{\pi^3 w^4 f^2} \frac{\kappa k_B T^2}{\rho^2 C^2} \quad (1)$$

where  $w$  is the radius of the Gaussian beam,  $a$  is the thickness of the plate,  $\rho$  is the density of the substrate,  $C$  is the specific heat capacity, and  $\kappa$  is the thermal conductivity. [1].

- **Thermo-optic noise** represents the coherent sum of thermo-elastic and thermo-refractive noise. The coherent sum of the power spectrum of both sources of noise is given by

$$S_{TO}^{\Delta z} \simeq S_{TO}^{\Delta T} \left( \bar{\alpha}_c d - \bar{\beta} \lambda - \bar{\alpha}_s d \frac{C_c}{C_s} \right)^2 \quad (2)$$

where

$$S_{TO}^{\Delta T} = \frac{2\sqrt{2}}{\pi} \frac{k_B T^2}{r_G^2 \sqrt{\kappa C \omega}}$$

where  $\kappa$  is the thermal conductivity of substrate,  $r_G^2$  is the beam radius, and  $C$  is the heat capacity per volume. Additionally,  $\bar{\alpha}_c$  is the effective coefficient of thermal expansion of the coating,  $\bar{\beta}$  is the the effective thermo-refractive coefficient,  $d$  is the coating thickness, and  $C_s$  and  $C_c$  represent the heat capacity per volume of the substrate and coating respectively [2].

As shown in Figure 3, the thermo-refractive and thermo-optic noise introduced by the substrate of M1 and M2 are extremely small.

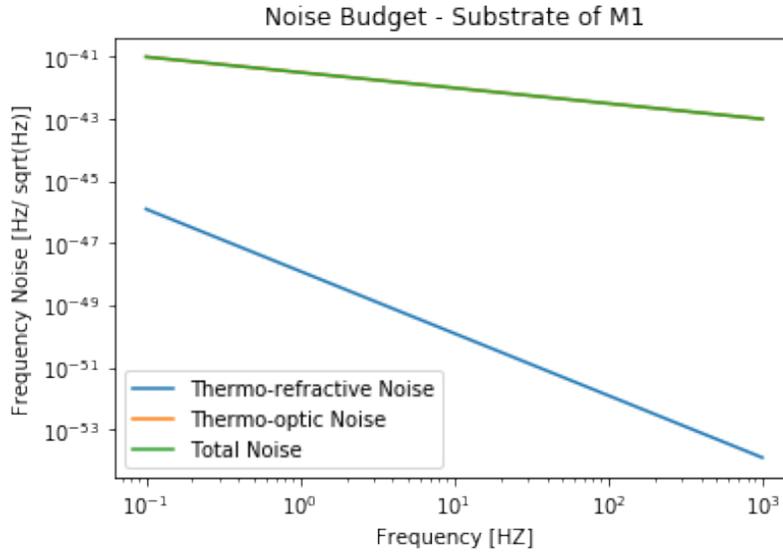


Figure 3: Noise Budget for the substrate of mirror 1. The noise introduced by mirror 2 will be identical to mirror 1.

## 2.2 Seismic Noise

The optical table’s transfer function suppresses seismic noise in the 1 - 100 Hz band. Its transfer function is proportional to

$$S(f) = \frac{1}{f^2} \quad (3)$$

## 2.3 Free-running frequency noise

The laser used to pump DOPO utilizes a non-planar ring oscillator (NPRO). The frequency noise associated with this laser is given by

$$S(f) = \frac{10^4}{f} \quad (4)$$

## 2.4 Total Noise

When these noise contributions (subsection 2.1 – 2.4) are compared, it is evident that the noise introduced by the substrate is several orders of magnitude smaller than other sources (laser and seismic noise). Furthermore, the laser beam bounces once off of mirrors 1 and 2, meaning that the losses caused by these mirrors are not significant. For these two reasons, the frequency noise introduced by these mirrors can be neglected in our analysis.

# 3 Next Steps

For the next steps, we need to

- Finish the noise budget of the substrate of mirrors 1 and 2 by incorporating Brownian and thermo-elastic noise. Additionally, thermal noise sources (Brownian, thermo-elastic, thermo-refractive, and thermo-optic noise) need to be calculated for the coating of the mirrors. If the total noise introduced by both mirrors is still 30 orders of magnitude less than other noise sources, we can omit them from our analysis.
- Determine the optimal temperature that allows for DOPO to operate as efficiently as possible.
- Consider thermal noise sources in the PPKTP crystal and DOPO cavity to decide how much frequency noise they introduce.

## 4 References

### References

- [1] R X Adhikari et al 2020 *Class. Quantum Grav.* 37 165003
- [2] Evans, M., Ballmer, S., Fejer, M., Fritschel, P., Harry, G., amp; Ogin, G. (2008). Thermo-optic noise in coated mirrors for high-precision optical measurements. *Physical Review D*, 78 (10). <https://doi.org/10.1103/physrevd.78.102003>