LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
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Back-action evasion for PT-symmetric interferometer

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1 Introduction/Background

Future gravitational and astrophysical research calls for the broadband and high-frequency sensitivity of gravitational wave detectors. Conventional resonant detectors are subject to bandwidth-peak sensitivity trade-off. The idea to circumvent this limitation, i.e. to improve the bandwidth without sacrificing the peak sensitivity, is called White Light Cavity (WLC). The PT-symmetric interferometer with coherent quantum feedback [1] is a stable realization of WLC (which is called sWLC for short), compared with the direct attachment of a filter cavity with anomalous dispersion [2] (uWLC for short). However, the original proposal [1] hasn't considered the back-action noise caused by the radiation pressure on the test mass.

In this project, we work with the PT symmetric interferometer. We aim to explore a more complete PT-symmetric structure to improve the low-frequency noise spectrum by back-action evasion with an effective negative mass. The effective negative mass will be possibly achieved using parametric amplification and optical damping formed by multiple additional pumpings. and hence, have a larger bandwidth with sacrificing less of the sensitivity than we would for a conventional trade-off between the bandwidth and sensitivity.

1.1 Bandwidth Sensitivity Trade-off

In a setup with an arm cavity and a strain signal that comes to detector, we see a trade-off between the bandwidth and the peak sensitivity of the strain noise at the detector as a result of Energetic Quantum Limit (EQL) which is represented by the inequality [1]

$$\int_0^{+\infty} d\Omega/(2\pi) S_h^{-1}(\Omega) \leq \Delta\mathcal{E}^2/(4\hbar)^2 \tag{1}$$

If we want a larger bandwidth, we would have to have a flatter peak sensitivity and for a sharper peak sensitivity, we would have a smaller bandwidth for the strain noise being detected, hence the trade-off.

1.2 Anomalous Dispersion

To deal with the restraints of the EQL, we need to create anomalous dispersion [2]. The arm cavity from the initial setup has phase delay. We attach a filter cavity, to the arm cavity, with phase advance to compensate for the phase delay. From the phase point of view, the two cavities can be thought as the time reversal of each other. As a result, we have a zero and constant phase for much larger bandwidth for light circulating inside the two cavities. This can be achieved by anomalous dispersion, which is a parametric interaction and helps us realize the white light cavity. Assuming that the arm cavity with the phase delay has resonant frequency ω_0, we pump the filter cavity with the detuned light at the frequency $\omega_0 + \omega_m$ [2]. As a result of this parametric interaction, we get to have a larger bandwidth, at the same level of sensitivity as before. However, this parametric interaction is unstable.

1.3 PT-symmetric Interferometer

Even though the unstable White Light Cavity can overcome the bandwidth sensitivity trade-off, we seek to stabilize the system, because unstability brings technical complications with the feedback control of the system. To stabilize the system we will use a coherent quantum feedback with PT-symmetry where we place the detector at the end of the filter cavity to get the readout. It can be seen from FIG. 1 that in the unstable case (uWLC) modes $\hat{b}$ and $\hat{c}$ are at the filter cavity and attached to the arm cavity (mode $\hat{a}$) from which we get the readout. On the other hand, for sWLC, we get the readout from the filter cavity (mode $\hat{b}$) which also satisfies the PT-symmetry.

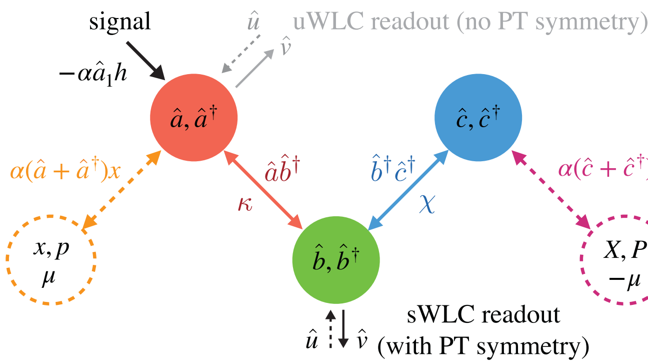

Figure 1: The stable White Light Cavity (sWLC) scheme [1]. The arm cavity has mode $\hat{a}$, the filter cavity has mode $\hat{c}$ and the detector has mode $\hat{b}$. Mode $\hat{a}$ couples to mode $\hat{b}$ with a coupling rate κ and mode $\hat{c}$ couples to mode $\hat{b}$ with coupling rate χ. Mode $\hat{b}$ is coupled to stable WLC (sWLC) readout. This system is PT symmetric when $\chi{=}\kappa$. $\hat{u}$ is the input vacuum noise and $\hat{v}$ is the output field.

Both for the uWLC and sWLC, the interaction Hamiltonian is expressed as [1]

$$\hat{V}_{\text{int}} = i\hbar\kappa(\hat{a}\hat{b}^\dagger - \hat{a}^\dagger\hat{b}) + i\hbar\chi(\hat{b}^\dagger\hat{c}^\dagger - \hat{b}\hat{c}). \tag{2}$$

1.4 Coherent Quantum Feedback

Looking at the equations of motion for modes, $\hat{a}$, $\hat{b}$, and $\hat{c}$ [1]

$$\dot{\hat{a}} = -\kappa\hat{b} + i\alpha h \tag{3}$$

$$\dot{\hat{c}}^\dagger = \chi\hat{b} \tag{4}$$

$$\dot{\hat{b}} = -\gamma_R\hat{b} + \kappa\hat{a} + \chi\hat{c}^\dagger + \sqrt{2\gamma_R}\hat{u} \tag{5}$$

and ignoring the signal part of mode $\hat{a}$ (iαh), we see that in the PT symmetric case, the equations of motion for mode $\hat{a}$ and $\hat{c}^\dagger$ are equal and opposite. Therefore, the following expression

$$\frac{d}{dt}(\chi\hat{a} + \kappa\hat{c}^\dagger) = i\chi\alpha h \tag{6}$$

is exclusively dependent on the signal [1]. In this equation, we obtain an internal mode (the expression inside the time derivative) which responds to the signal only, providing a coherent quantum feedback inside the system (between the modes $\hat{a}$ and $\hat{c}$). The internal mode is also a part of the readout mode ($\hat{b}$), but it is not influenced by the noise from the readout.

In this project we aim to use the coherent quantum feedback to stabilize our system.

1.5 Backaction

Backaction is due to the photons imparting momentum to the mirrors in the system. We can also call this radiation pressure. Moreover, the inserted momentum will influence the position measurement at later times. The backaction noise will increase when we increase the pumping power of the laser. In our project, we observe backaction when the strong field interacts with the test mass μ with mode (x,p) attached to mode $\hat{a}$. Due to backaction, both the conventional and any WLC noise spectra have a tail at low frequencies [1] where radiation pressure noise is effective, which can be seen in FIG. 2.

2 Objectives

2.1 Backaction Evasion

In this project, our objective is to add back-action evasion module to realize the full PT-symmetric structure of FIG. 1 which has not been described in prior work [1]. To preserve the PT-symmetry structure of the system, (since $\hat{a}$ and $\hat{c}$ are PT-symmetric) we aim to attach another mode (X,P) with negative mass $-\mu$ to mode $\hat{c}$. The P-symmetry is done by attaching (X,P) to $\hat{c}$ and T-symmetry is preserved by the negative mass since $\dot{X} = \frac{P}{-\mu}$. Conserving the PT-symmetry of the system causes backaction evasion, which flattens the tails of uWLC and sWLC noise spectrum curves at low frequencies.

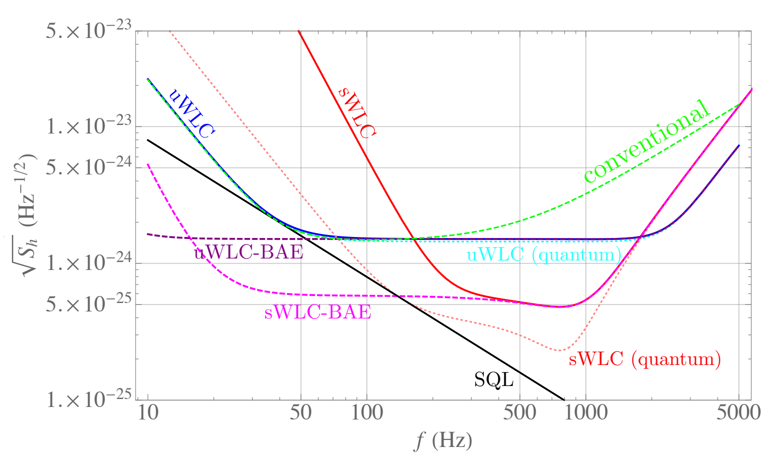

Figure 2: GW noise spectra for sWLC and uWLC both with and without backaction evasion (BAE) [1]. We can see that the tails of uWLC and sWLC at low frequencies have been flattened more due to backaction evasion.

3 Approach

3.1 Negative Mass

The complete description of the system includes the optomechanical interaction between test mass (x, p) and the cavity mode $\hat{a}$. The total Hamiltonian is given by

$$\hat{H} = \alpha(\hat{a} + \hat{a}^\dagger)x + \frac{\hat{p}^2}{2\mu} + \hat{V}_{\text{int}}, \tag{7}$$

where μ is the reduced mass of the cavity mirrors, and $\hat{V}_{\text{int}}$ is given in Eq. (2) where the PT-symmetry was initially constructed by adding a filter cavity to the arm cavity. Considering the interaction between mode $\hat{a}$ and the test mass (x, p), the PT-symmetry is no longer reserved. To restore the PT-symmetry of the system, we need to add a negative mass counterpart and attach it to mode $\hat{c}$:

$$\hat{V}_{\text{aux}} = \alpha(\hat{c} + \hat{c}^\dagger)X + \frac{\hat{P}^2}{-2\mu}. \tag{8}$$

Attaching (X, P) to mode $\hat{c}$will conserve the P-symmetry, while the negative mass will conserve the time reversal symmetry.

To implement this negative mass in our system, we propose to add an optomechanical auxiliary mode ($\hat{d}$, $\hat{d}^\dagger$), which is expressed by the Hamiltonian

$$\hat{V}_{\text{aux}} = \alpha X_{\text{ZPF}}(\hat{c}\hat{d} + \hat{c}\hat{d}^\dagger + \hat{c}^\dagger\hat{d} + \hat{c}^\dagger\hat{d}^\dagger) + \frac{1}{-2\mu}\left(\frac{\hbar}{2iX_{\text{ZPF}}}\right)^2(\hat{d}^2 - \hat{d}\hat{d}^\dagger - \hat{d}^\dagger\hat{d} + \hat{d}^{\dagger^2}) \tag{9}$$

where $\hat{c}^\dagger\hat{d}^\dagger$ is the non-degenerate squeezing generator which can be generated by blue tuned pumping, $\hat{c}\hat{d}^\dagger$ is the beam splitter generated by red detuned pumping, and $\hat{d}^2 + \hat{d}^{\dagger^2}$ are the squeezing terms.

Besides this optomechanical system, other realizations such as a nonlinear crystal realization can potentially achieve similar results.

4 Project Schedule

In this project, we aim to simulate the results for this Hamiltonian and its effects on the GW noise spectra for sWLC (just like in FIG. 2) in Mathematica and Finesse. We will calculate the reasonable values for the parameters such as the coupling rates κ and χ, and the readout rate γ_R using analytical solutions and Mathematica simulations.

Week#	Task
1-2	Hamiltonian equations under single-mode and resolved-sideband approximations.
3-4	Achieve the negative mass with optomechanical modes and pumping.
4-5	Simulate the system beyond the approximations using Mathematica.
6-7	Noise budgeting with reasonable physical parameters.
8-9	Add PT-symmetry and negative mass module into Finesse.
10	Report the results.

References

[1] Li, Xiang, et al, *Broadband sensitivity improvement via coherent quantum feedback with PT symmetry*. arXiv preprint arXiv:2012.00836 (2020).

[2] Miao, Haixing, et al, *Enhancing the Bandwidth of Gravitational-Wave Detectors with Unstable Optomechanical Filters*. Phys. Rev. Lett. 115, 211104

[3] Chen, Yanbei, *Macroscopic quantum mechanics: theory and experimental concepts of optomechanics*. Journal of Physics B: Atomic, Molecular and Optical Physics 46.10 (2013): 104001.