Parameter Estimation with

An introduction to Bayesian inference with CBCs

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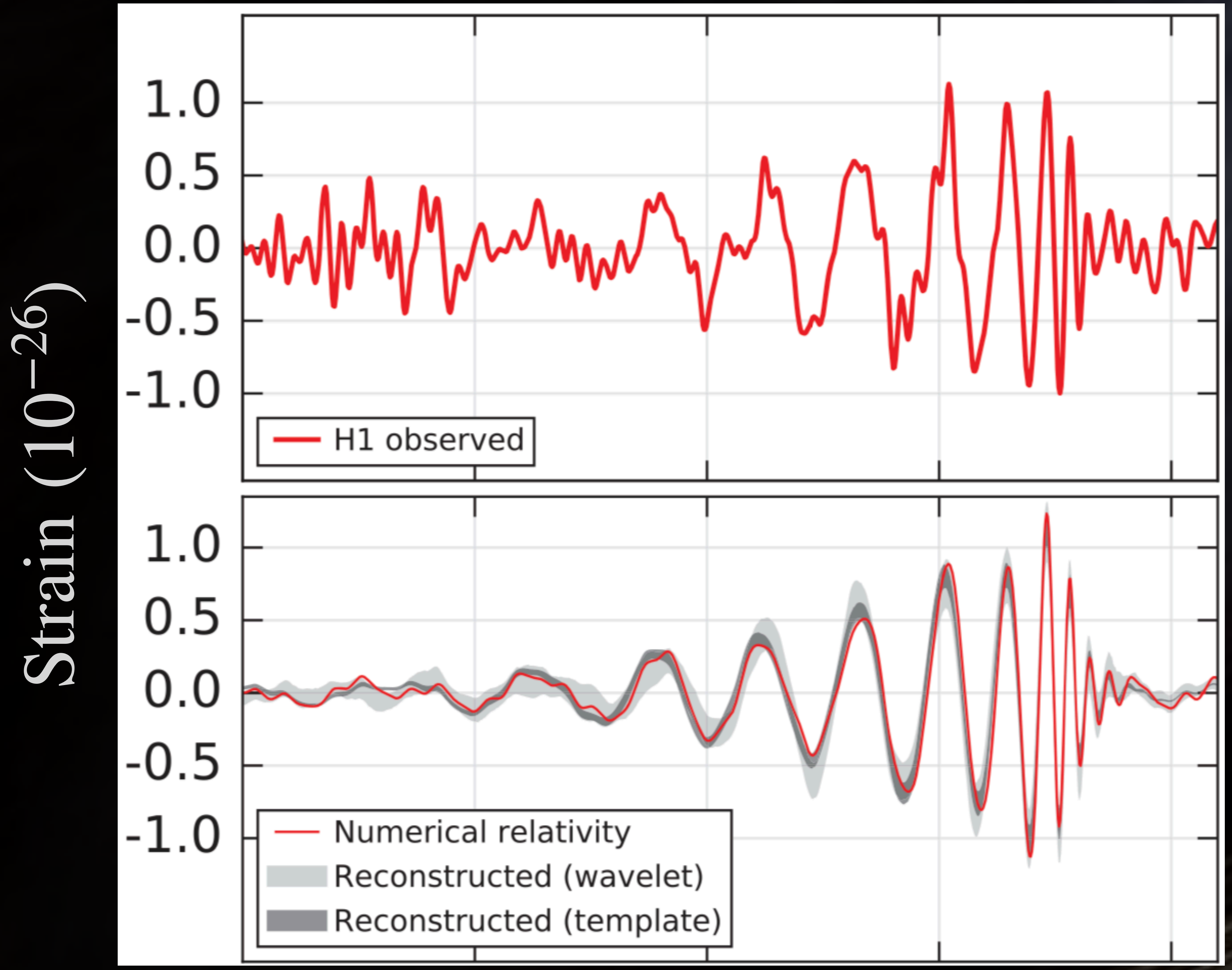
Open Data Workshop #4 - May 12, 2021

LIGO OzGrav MONASH University

Image credit: Mark Myers / OzGrav

From data to astrophysics

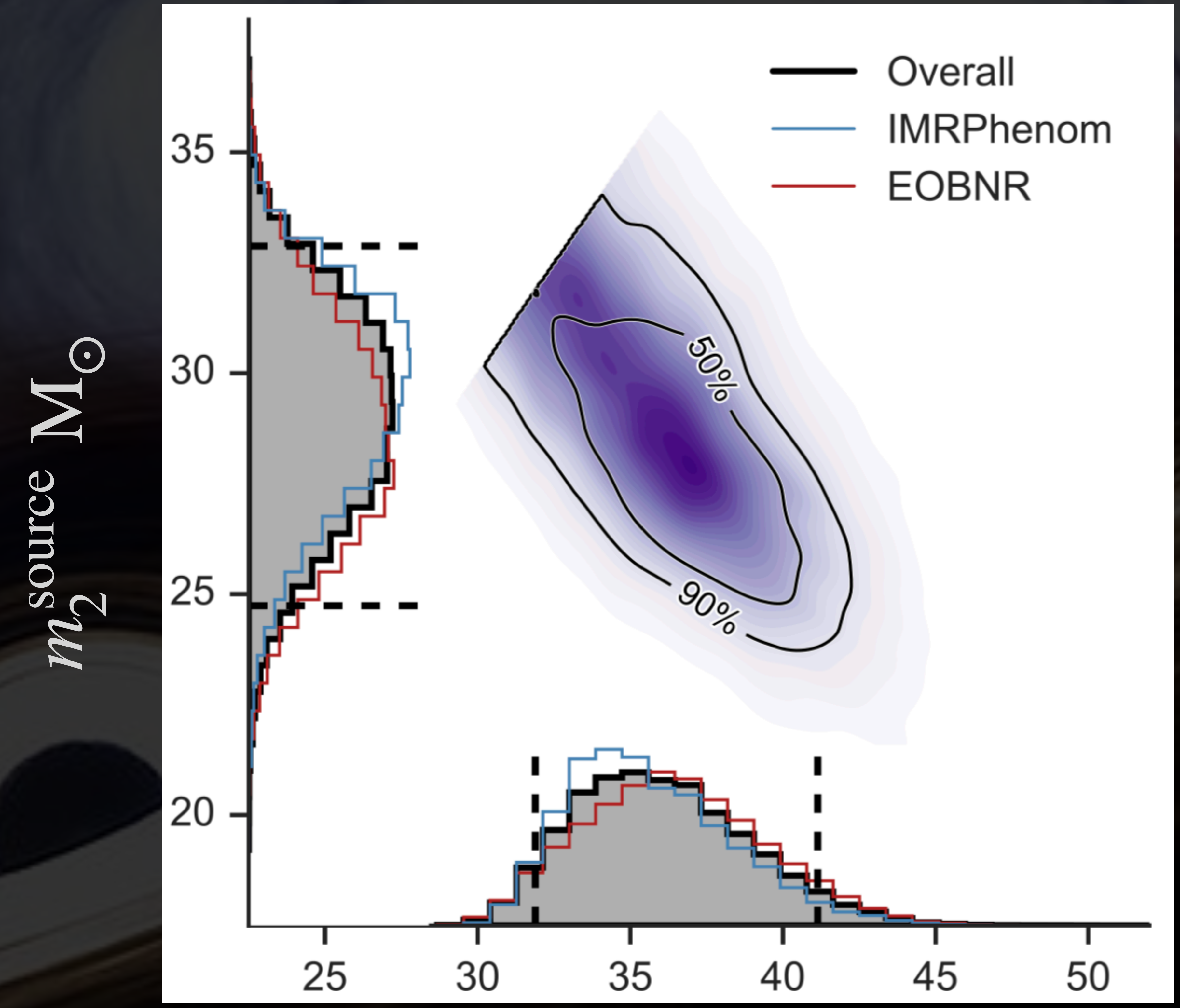
LVC (2016) arXiv:1602.03837



Hanford, Washington (H1)

?

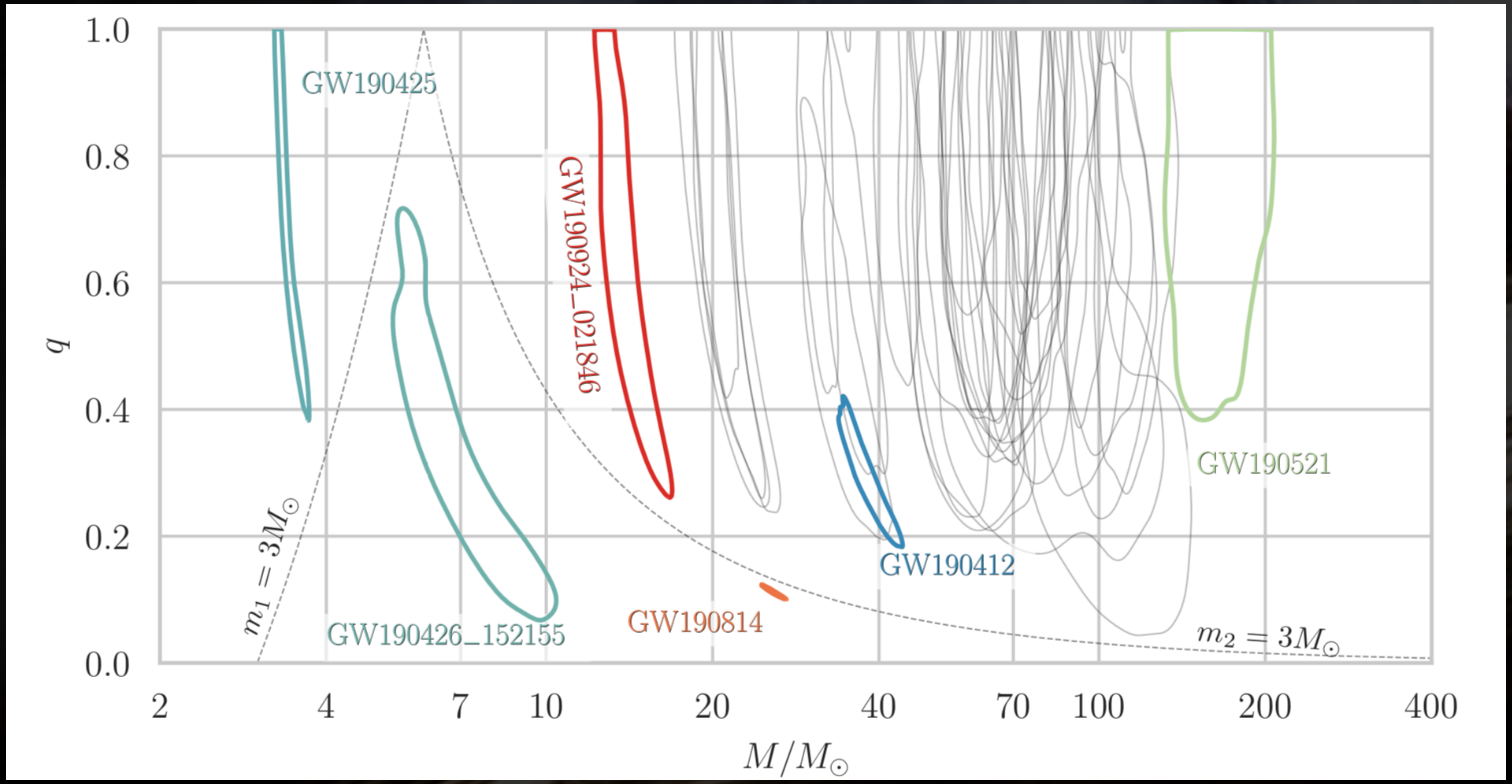
LVC (2016) arXiv:1602.03840



m_1^{source} $\mathrm{M}_\odot$

Parameter Estimation: Event properties

- Parameter estimation programs (e.g. BILBY), employing Bayesian inference to extract source properties (e.g. mass, spin) from gravitational-wave signals.

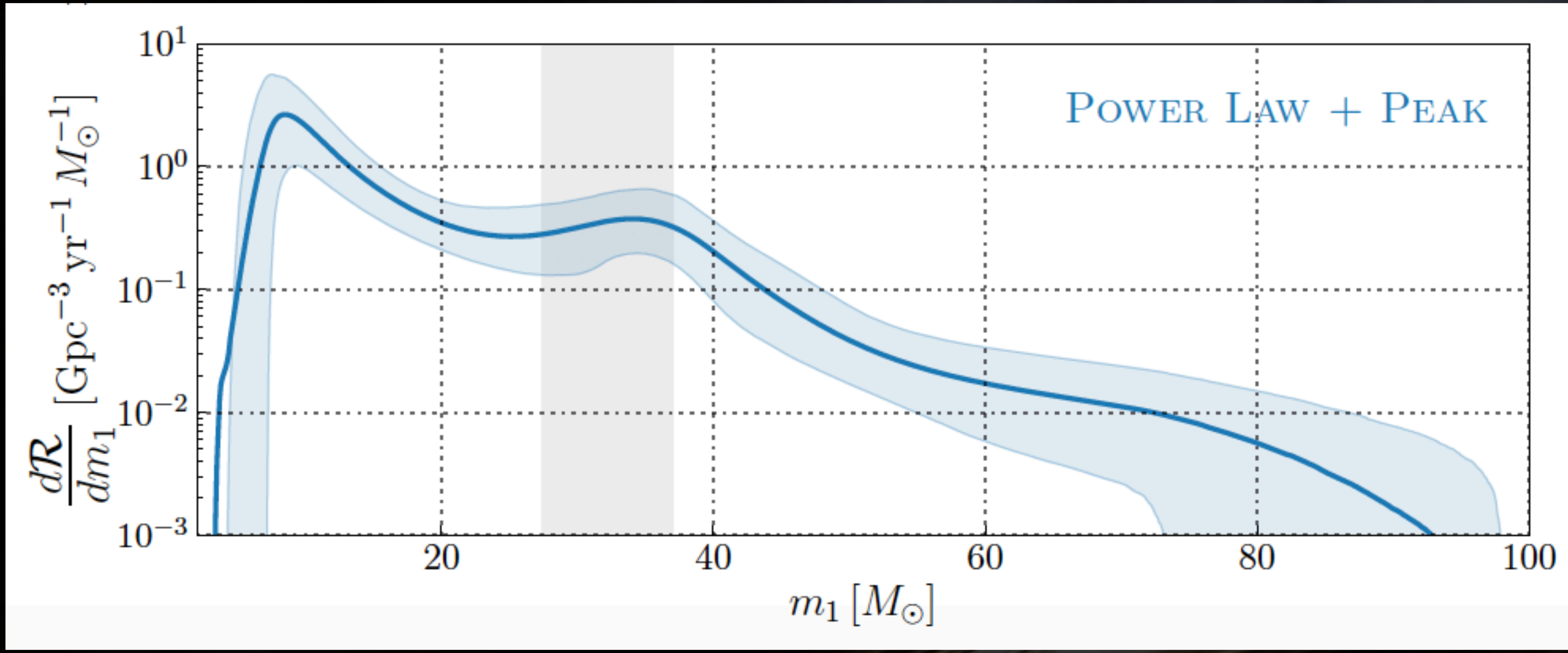


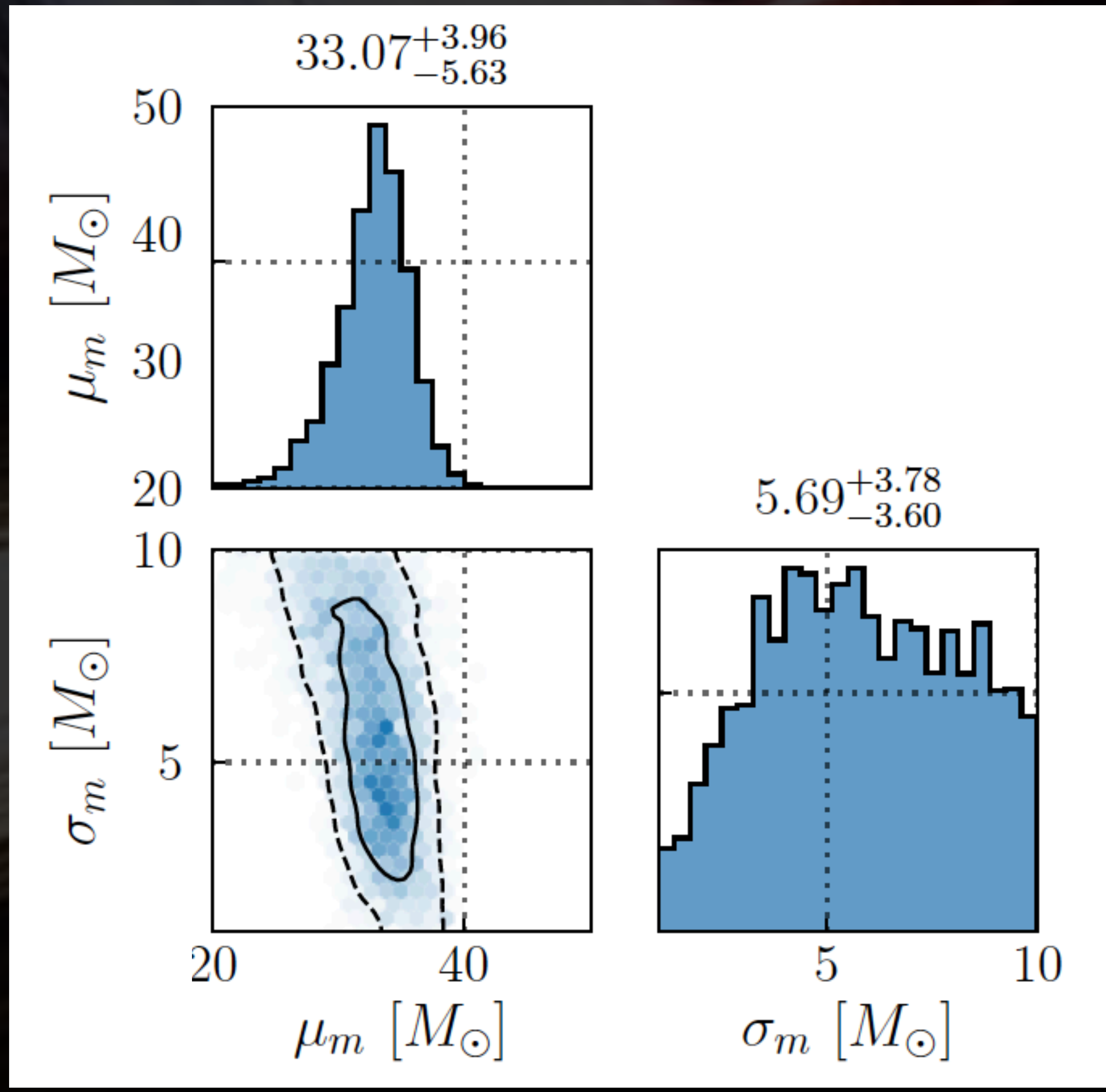
LVC (2021) arXiv:2010.14527

Parameter Estimation: Population properties

- Using population studies (employs hierarchical Bayesian inference), we can extract the ***shape*** of the distributions.

LVC (2021) arXiv:2010.14533





Bayesian Inference

- Bayesian inference is a method in which **Bayes' theorem** is used to determine the probability for a hypothesis that updates with information.
- Data d, parameters θ and model or signal hypothesis M

Likelihood

$$\underbrace{p(\theta|d,M)}_{\text{Posterior}} = \frac{p(d|\theta,M)\,p(\theta|M)}{p(d|M)}$$

Prior

Evidence

Bayesian Inference

- Bayesian inference is a method in which **Bayes' theorem** is used to determine the probability for a hypothesis that updates with information.
- Data d, parameters θ and model or signal hypothesis M

Likelihood

$$p(\theta|d, M) = \frac{\mathcal{L}(d|\theta, M)\, \pi(\theta|M)}{\mathcal{Z}(d|M)}$$

Posterior

Prior

Evidence

Using notation in Thrane and Talbot (2019)
arXiv:1809.02293

Bayesian Inference

Posterior: $p(\theta|d, M)$

Probability distribution source properties θ given the data d.

Likelihood: $\mathcal{L}(d|\theta, M)$

The probability of the detectors measuring data d, assuming a signal (i.e. model hypothesis M) with source properties θ.

Prior: $\pi(\theta|M)$

Incorporates any a *priori* knowledge about the parameters; range & shape

Evidence: $\mathcal{Z}(d|M) \equiv \int d\theta \mathcal{L}(d|\theta, M)\pi(\theta|M)$

A measure of how well the data is modelled by the hypothesis; acts as a normalisation constant; important in model selection; marginalised likelihood.

Introducing BILBY

- BILBY = **B**ayesian **I**nference **Lib**rary; a software package designed to enable parameter estimation.
- User-friendly, modular and adaptable!
- Analyse compact binary coalescences & more!

git.ligo.org/lscsoft/bilby

Gravitational-wave parameter estimation

$$\mathcal{L}(d|\theta, M)$$

Understanding contributions to the GW signal

$$\pi(\theta|M)$$

Having an initial belief on the distributions for GW parameters

$$\mathcal{Z}(d|M)$$

Doing model selection and calculating Bayes factors

Understanding signal and noise

$$d = n + h(\theta)$$

data

detector noise
noise contribution

waveform
astrophysical contribution

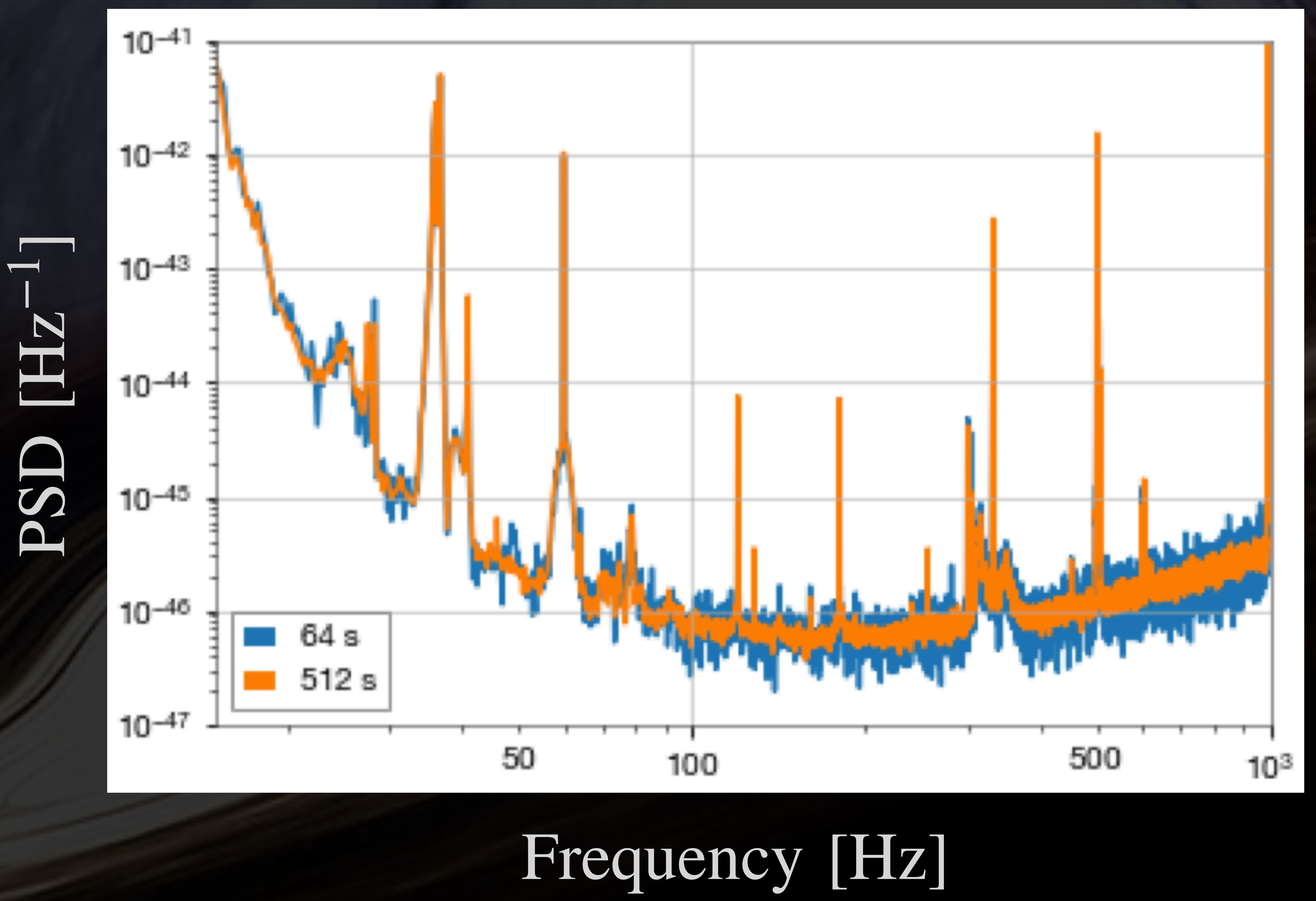
- Noise assumed to be stationary and Gaussian and is not known, but rather estimated by the power spectral density (PSD).
- Astrophysical contribution is dependant on the properties of the binary (indicated by dependance on θ

Off source PSD estimation

Involves averaging over segments

- This method is also called periodogram or Welch method
- Must exclude analysis segment
- Splits data into short segments; window and calculate $|d_i|^2$ for each segment.
- Periodograms are averaged together
- Not ideal for very long segments of data

Credit: S Biscoveanu

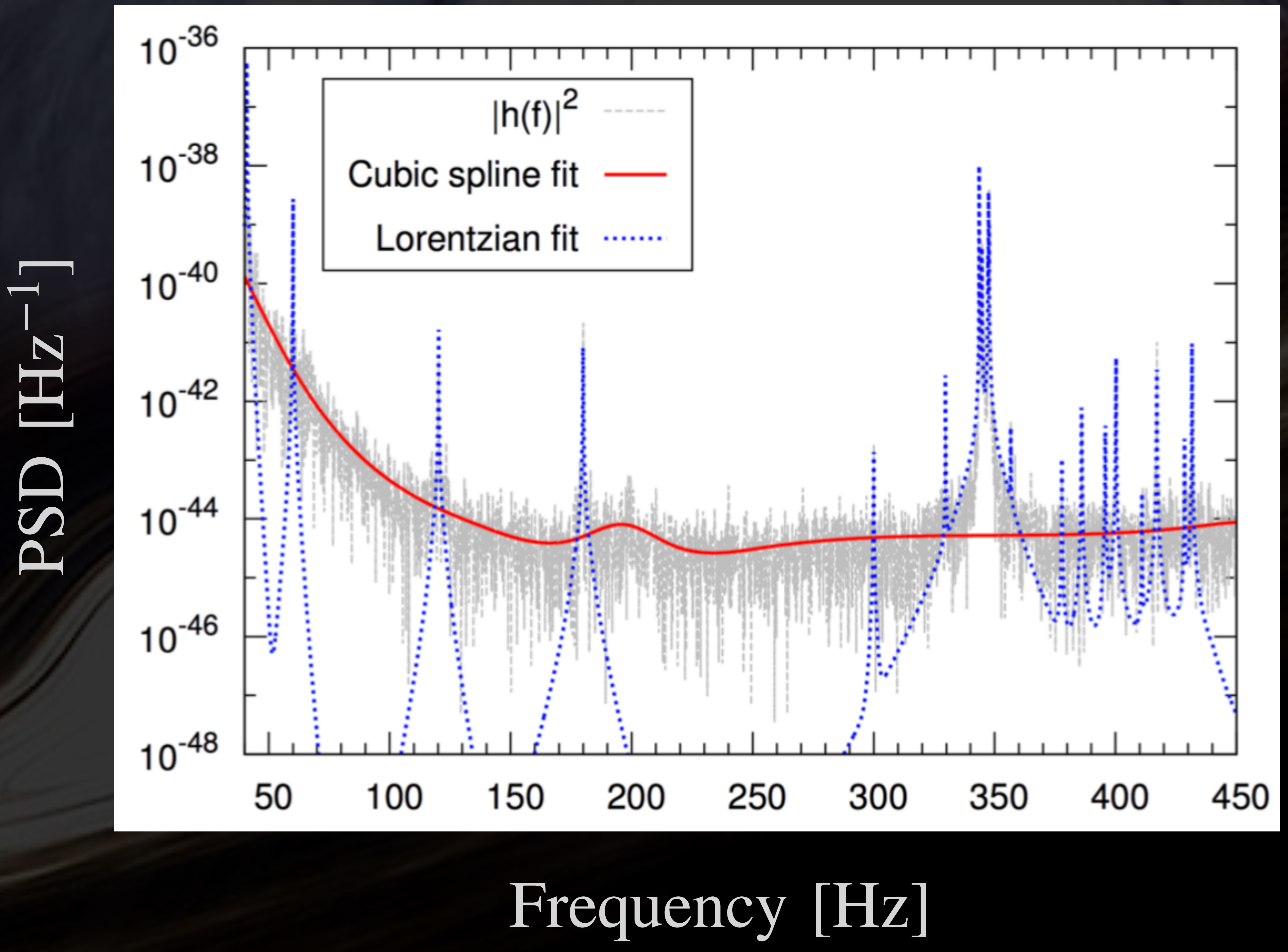


On source PSD estimation

Involves building a model for the PSD

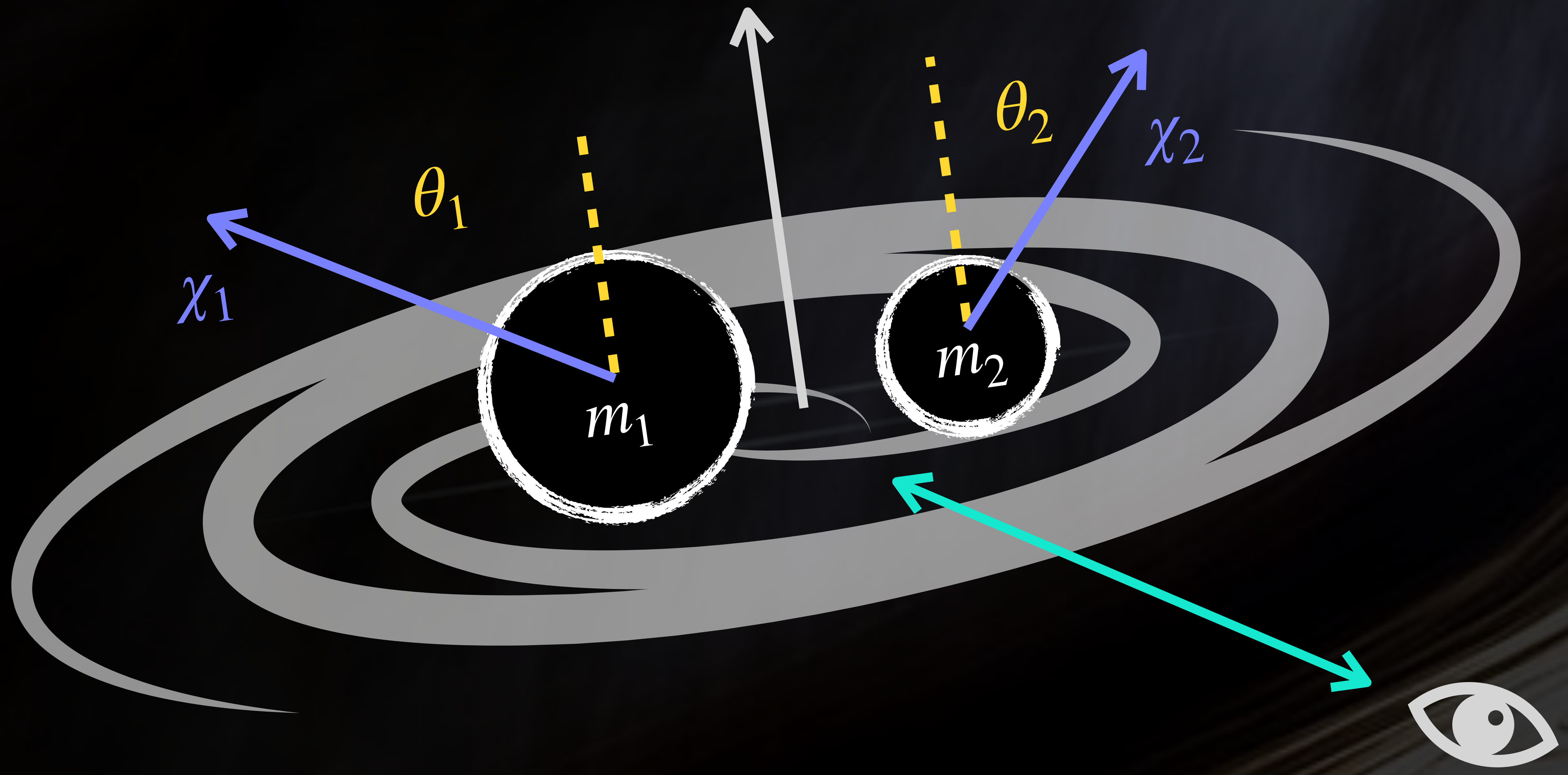
- Models PSD as a sum of a broadband spline and narrowband Lorentzians using the BayesLine algorithm
- Using data from the analysis segment, infers the spline and Lorentzian parameters that best describe the PSD
- More expensive, but needs less data

Littenberg & Cornish (2014) arXiv:1410.3852



Understanding the astrophysical properties

- 15 BBH parameters + 2 more parameters for BNS (tidal parameters)
- The GW signal has information on the intrinsic and extrinsic properties of the source.



Intrinsic	Extrinsic
• Masses	• Distance
• Spins	• Inclination
• Tidal deformations	• RA/Dec
	• Coalescence time
	etc.

Understanding the astrophysical properties

$$h(\theta) = F_{+}(\mathrm{RA}, \mathrm{DEC}, \psi) h_{+}(\theta) + F_{\times}(\mathrm{RA}, \mathrm{DEC}, \psi) h_{\times}(\theta)$$

plus polarisation

cross polarisation

antenna response functions

detector geometry

$$h_{+}(\theta) = \frac{1}{2} \mathcal{A}_{\mathrm{GW}}(f) \left(1 + \cos^2 \iota\right) \cos \phi_{\mathrm{GW}}(f)$$

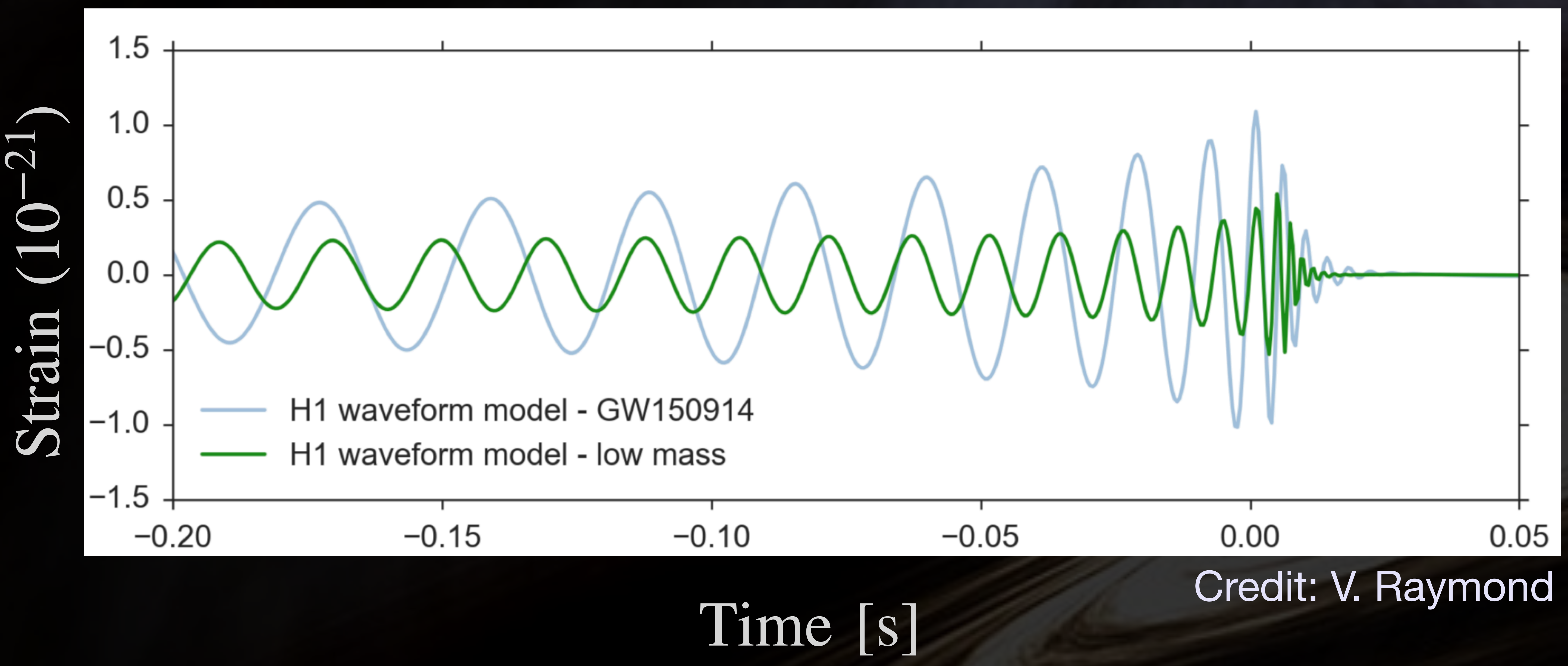
$$h_{\times}(\theta) = \mathcal{A}_{\mathrm{GW}}(f) \cos \iota \sin \phi_{\mathrm{GW}}(f)$$

Gravitational-wave signal: mass

- Amplitude of waveform is proportional to the chirp mass
- Increasing mass, increases amplitude of waveform

$$\mathcal{A}_{\mathrm{GW}} \propto \frac{\mathcal{M}^{5/6} f^{-7/6}}{d_L}$$

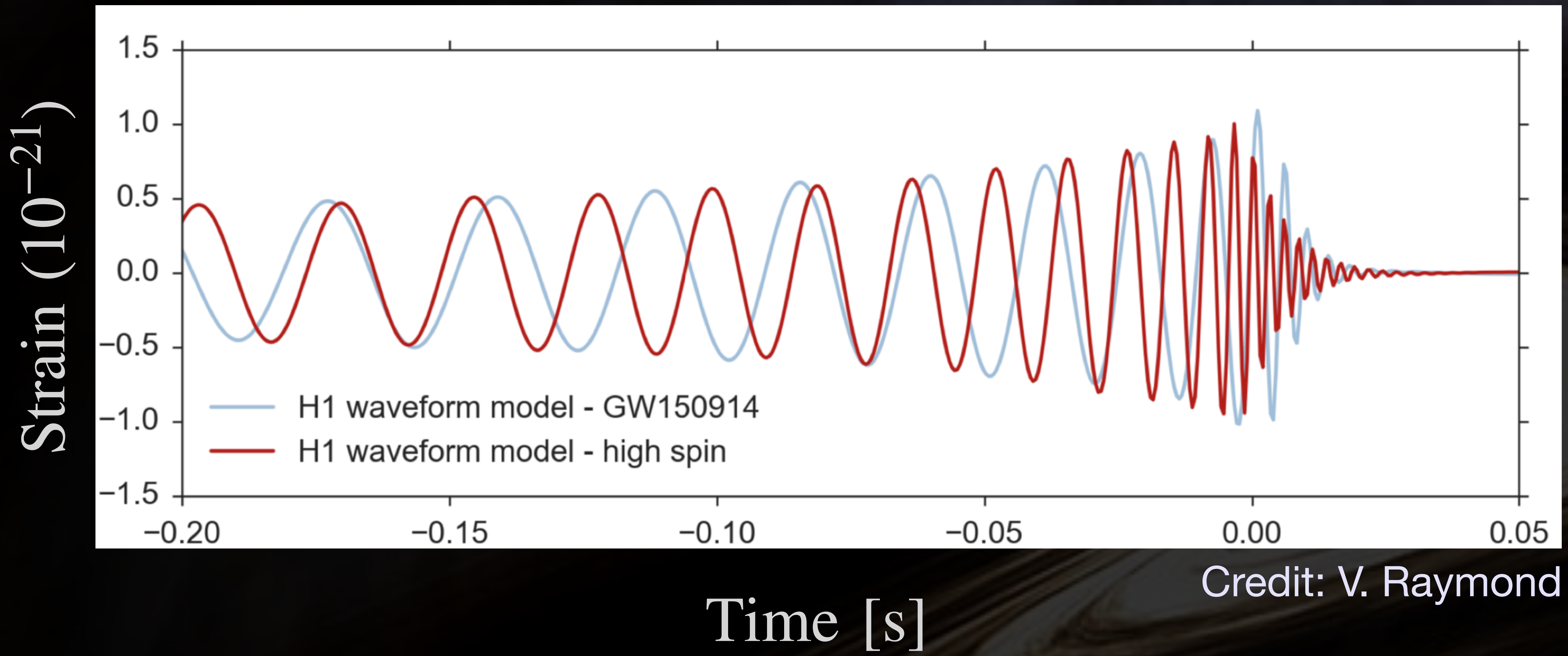
$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$



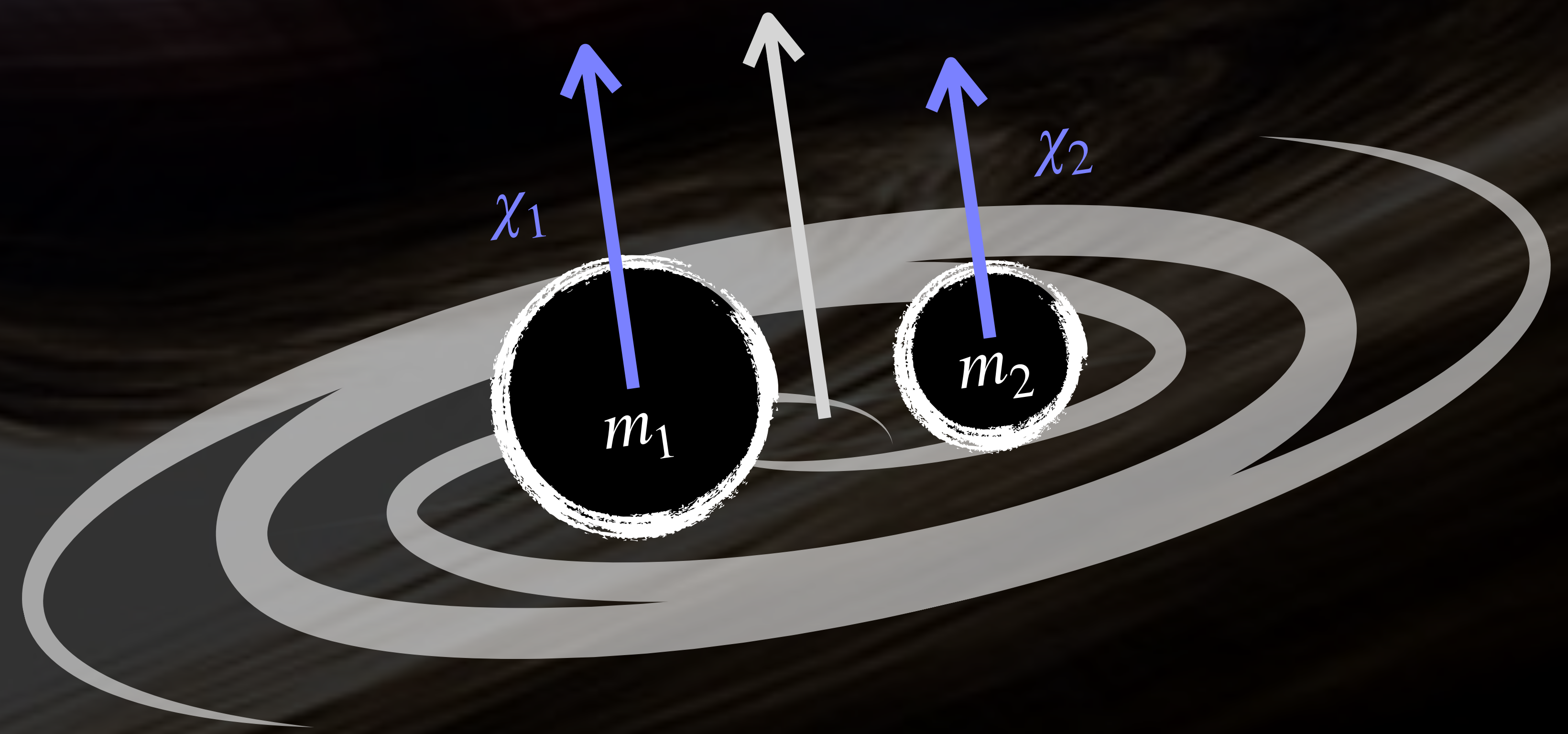
Credit: V. Raymond

Gravitational-wave signal: spin magnitudes

- Aligned spins, increasing magnitude results in orbital hangup.
- This means systems take longer to merge.

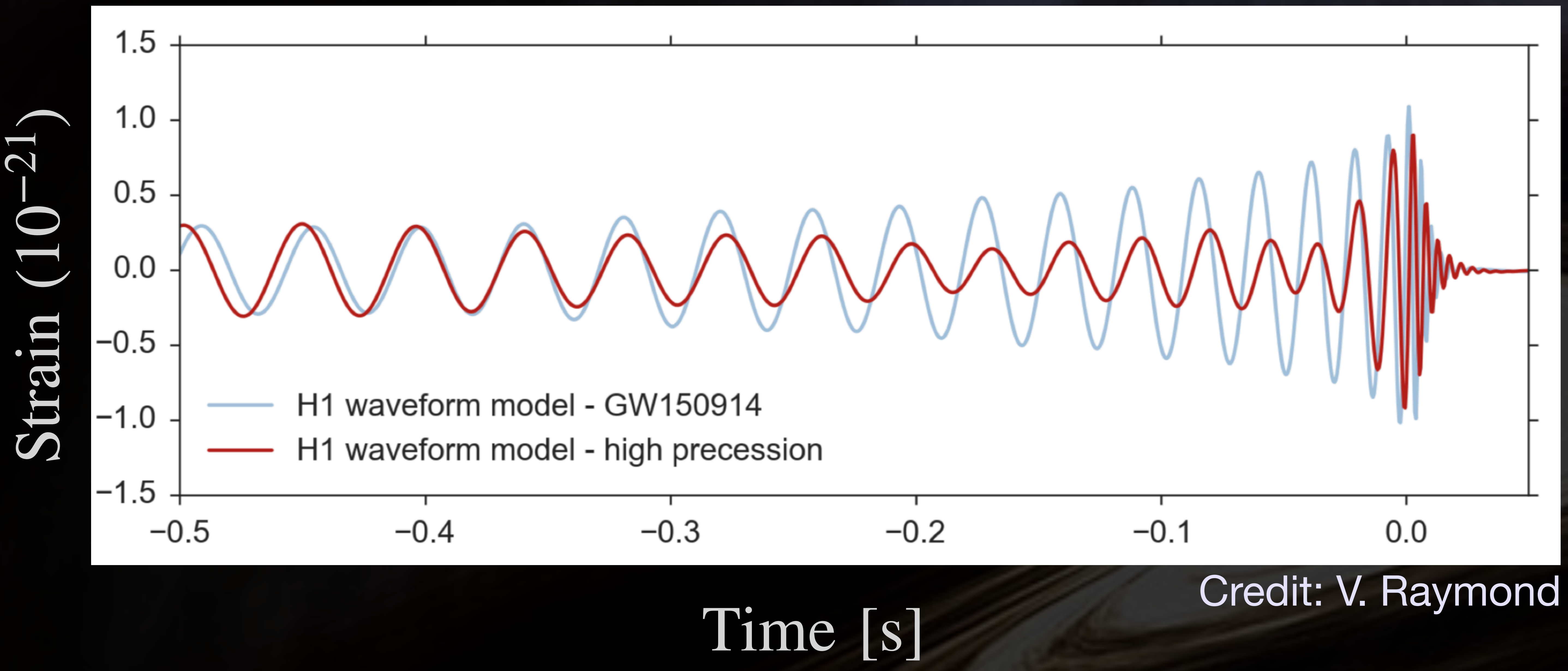


Credit: V. Raymond

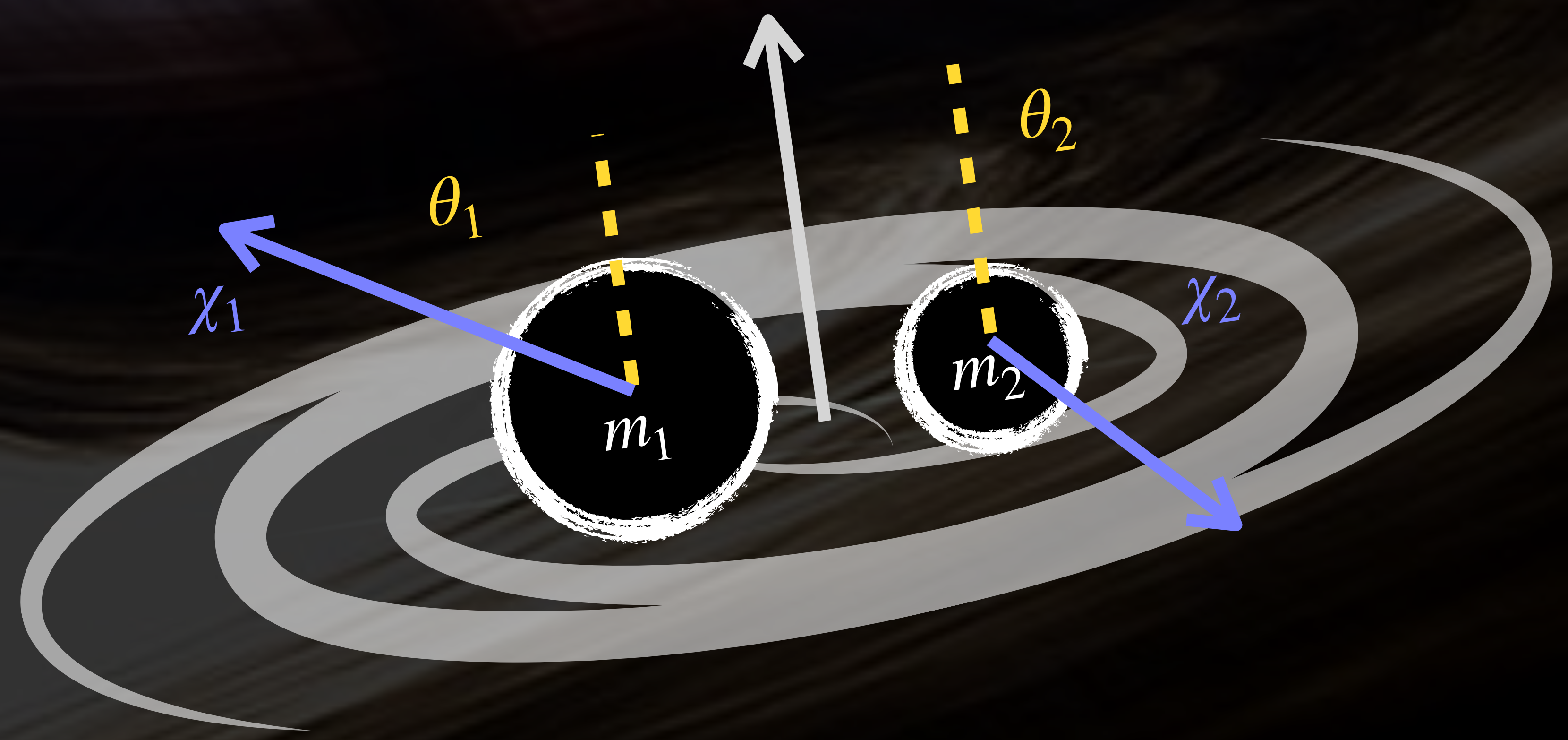


Gravitational-wave signal: spin orientations

- For mis-aligned spins, we have in-plane spin components resulting in precession
- We see a modulation in the amplitude.



Credit: V. Raymond



Gravitational-wave likelihood

The residual between the data and best-match waveform template should also follow a unit Gaussian about the square root of the PSD when there is a signal.

$$\mathcal{L}\left(d_i|\theta\right) = \frac{1}{2\pi P_i}\exp\left(-2\Delta f\frac{\left|d_i - h_i(\theta)\right|^2}{P_i}\right)$$

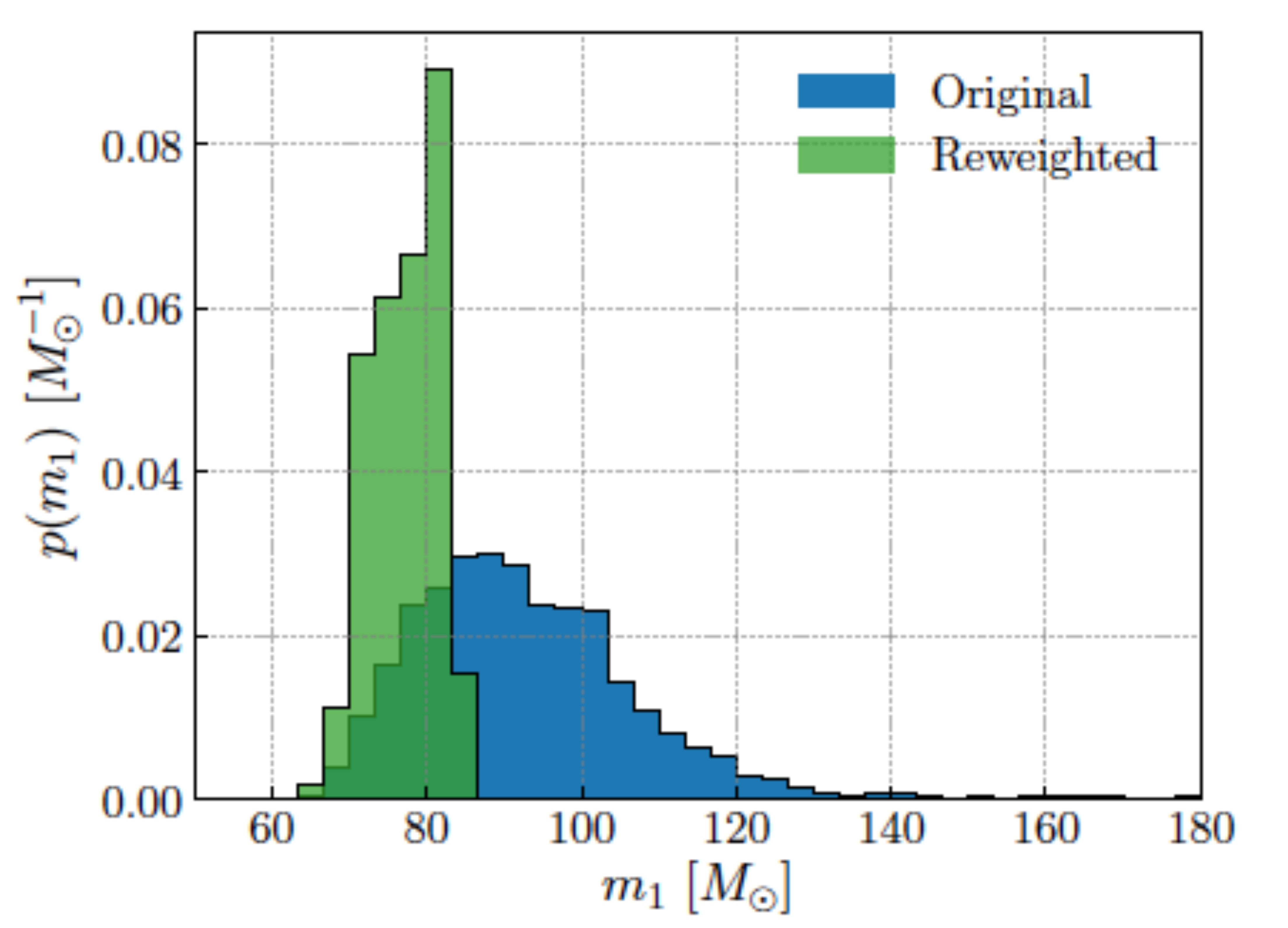
PSD

frequency resolution

$$\mathcal{L}\left(d|\theta\right) = \prod_i^N \mathcal{L}\left(d_i|\theta\right)$$

Prior distributions

- Describes the prior belief of the range and shape of the distribution.
- Some are 'obvious' choices (e.g. sky location isotropic), some are convenient (uniform in mass).
- Useful to have uniform distributions since they are 'uninformative' and are convenient for reweighing.



LVC (2021) arXiv:2010.14533

Model selection

- Calculating the evidence for the signal & noise allows you to calculate a Bayes factor.

$$\mathrm{BF}^S_N = \frac{\mathcal{Z}_S}{\mathcal{Z}_N}$$

- You can do the same thing on a population level with different models. Having a $BF \gtrsim 3000$ or $\ln BF \gtrsim 8$ is considered significant.
- Allows you to do model selection and determine which models best fit your data.

Sampling methods

We have the likelihood, prior and evidence. How do we calculate the posterior?

We use stochastic samplers to obtain samples for the posterior distribution

- Markov Chain Monte Carlo (MCMC)
- Nested sampling

Why? Because evaluating likelihood on a grid is not feasible for 15-17 dimension space!

Use BILBY!

- Analyse your favourite event from open data!
- Uses external samplers including dynesty, pymultinest, cpnest, emcee, ptemcee, and others.
- Can use this package to analyse real and simulated GW data.

Hands on session: Tutorials 3.1 and 3.2 using Bilby to do Parameter Estimation

git.ligo.org/lscsoft/bilby

Population inference

The new likelihood is the original likelihood marginalised over the original parameters:

$$\mathcal{L}(d \mid \Lambda) = \int d\theta \mathcal{L}(d \mid \theta)\pi(\theta \mid \Lambda)$$

hyper-parameter

hyper-prior

$$\mathcal{L}_{\text{tot}}(\vec{d} \mid \Lambda) = \prod_i^N \int d\theta_i \mathcal{L}\left(d_i \mid \theta_i\right) \pi\left(\theta_i \mid \Lambda\right)$$

$$\mathcal{L}_{\text{tot}}(\vec{d} \mid \Lambda) = \prod_i^N \int d\theta_i p\left(\theta_i \mid d_i, \varnothing\right) \mathcal{Z}_{\varnothing}\left(d_i\right) \frac{\pi\left(\theta_i \mid \Lambda\right)}{\pi\left(\theta_i \mid \varnothing\right)}$$

See Thrane and Talbot (2019) arXiv:1809.02293

Resources

https://lscsoft.docs.ligo.org/bilby/ - Bilby documentation

More on Bayesian Inference

Veitch et. al. (2015) arXiv:1409.7215

Thrane and Talbot (2019) arXiv:1809.02293

More on Bilby

Ashton et. al. (2018) arXiv:1811.02042

Romero-Shaw et. al. (2020) arXiv:2006.00714