NCal Torque to Force Calculations

M.P.Ross, J. Kissel, L. Datrier, T. Mistry

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1 Introduction

This document describes the effect of torques on the Newtonian Calibrator injections during the third observing run [1]. In general gravitational calibrators cause torques as well as forces on the test mass. These torques can couple into the strain read out and must be taken into accounted for.

These calculations are only valid for the LIGO NCal during O3 [1]. Both the NCal geometry and the beam offset may be different for future observing runs.

2 Geometry

The NCal was installed on the BSC pier closest to the ETMX test mass, as shown in Figure 1. This ensured maximal SNR for the injections and was the only pre-existing structure that was rigid enough to attach the rotor. One downside to this location is that it induces relatively large torques on the test mass.

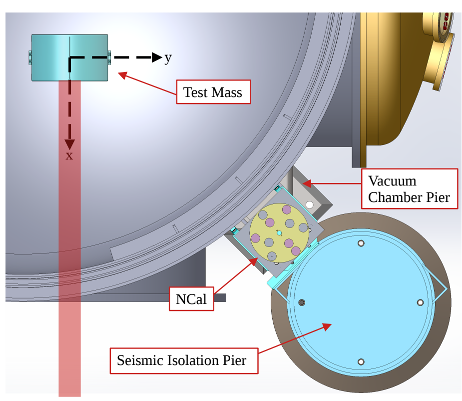

Figure 1: CAD rendering of the NCal rotor and test mass. [1]

The primary way the torques couple into the injections is due to the main interferometer beam spot being off center. Due to this off center beam spot, angle changes about the y- and z-axis induce an arm length change in the interferometer. This is then interpreted as force in the NCal analysis and skews the results if not accounted for.

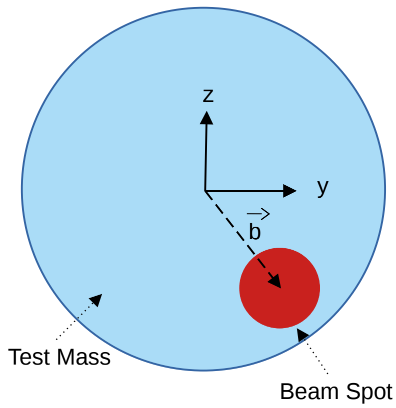

Figure 2: Geometry of the main interferometer beam spot on the test mass.

3 FEA Model

We use the finite element model developed for the NCal force predictions [1] to calculate the torques caused by the NCal about all three directions. We do this calculation using the mean values of parameter in Table 1 of [1]. We do not assess the uncertainties on the torques but it is expected be on the order of the uncertainty of the force, $\sim 0.7 - 0.9\%$, and thus much smaller than the uncertainty from the beam spot position.

Below are the amplitudes of the torques about the x, y, and z axes at $2f$ and $3f$ frequencies along with their phases relative to the force at that frequency.

2f

$\tau_x = 0.0138$ fNm at $76.3°$
$\tau_y = 4.2203$ fNm at $11.8°$
$\tau_z = 270.3682$ fNm at $26.6°$

3f

$\tau_x = 0.0108$ fNm at $-30.4°$
$\tau_y = 2.5403$ fNm at $-51.4°$
$\tau_z = 158.4345$ fNm at $-32.1°$

4 Apparent Force

Since there is no immediate way that torque about the x-axis (roll) can couple to strain readout we ignore it and focus on the torque about the y-axis (pitch) and z-axis (yaw).

Assuming that the test mass is a free mass:

$$\theta_i(\omega) = -\frac{\tau_i(\omega)}{I_i \omega^2} \tag{1}$$

where θ_i is the angle about the i-axis, τ_i is the torque about the i-axis, I_i is the moment of inertia of the test mass, and ω is the angular frequency of motion. The arm length change due to this shift in angle follows:

$$x(\omega) = b_j \tan(\theta_i(\omega)) \approx b_j \ \theta_i(\omega) \tag{2}$$

where b_j is the relevant beam offset. This is then interpreted as a force via:

$$\tilde{F}_x(\omega) = -M\omega^2 x(\omega) \tag{3}$$

where M is the mass of the test mass. Plugging equation 1 and 2 in gives:

$$\tilde{\tilde{F}}_x(\omega) = \frac{M b_j}{I_i} \tau_i(\omega) \tag{4}$$

Since we have two directions that the test mass can torque this is actually two separate equations:

$$\tilde{F}_x^z(\omega) = \frac{M b_y}{I_z} \tau_z(\omega) \tag{5}$$

$$\tilde{F}_x^y(\omega) = \frac{M b_z}{I_y} \tau_y(\omega) \tag{6}$$

where b_y and b_z are the beam offset in the y- and z-directions, τ_y and τ_z are the torque about the y- and z-axis (pitch and yaw), and the superscripts on the force denote which torque is causing the apparent x-direction force.

The mass of the test mass is $M = 39.66$ kg [1] and its moments about the y and z axes are $I_y = 0.419$ kg m^2 and $I_z = 0.410$ kg m^2. The beam offsets are estimated to be $b_y = 13.2$ mm and $b_z = -15.7$ mm with uncertainty of $\sigma_b = 1$ mm. Plugging these in gives:

$$\frac{M b_z}{I_y} = -1.486 \pm 0.095 \text{ m}^{-1} \tag{7}$$

$$\frac{M b_y}{I_z} = 1.277 \pm 0.097 \text{ m}^{-1} \tag{8}$$

Using this value we get the following apparent forces:

2f

$$\tilde{F}_x^y = -0.0063 \pm 0.0004 \text{ pN at } 11.8°$$
$$\tilde{F}_x^z = 0.3454 \pm 0.0262 \text{ pN at } 26.6°$$

3f

$$\tilde{F}_x^y = -0.0038 \pm 0.0002 \text{ pN at } -51.4°$$
$$\tilde{F}_x^z = 0.2024 \pm 0.0153 \text{ pN at } -32.1°$$

Adding the contributions from the torque about y and z together as complex numbers, we get:

$$\tilde{F}_x^{2f} = 0.3393 \pm 0.0262 \text{ pN at } 26.9° \tag{9}$$

$$\tilde{F}_x^{3f} = 0.1988 \pm 0.0153 \text{ pN at } -31.7° \tag{10}$$

5 Correcting the Observed Force

The proper way to account for the apparent force caused by the torques is to correct each measured force by the values calculated in Equations 9 and 10. However, the relative phase between the force and the torque must now be taken into account.

$$F_x^{\text{corr}} = \sqrt{(\tilde{F}_x)^2 + (F_x)^2 - 2\tilde{F}_x F_x \cos(\alpha)} \tag{11}$$

where $\tilde{F}_x$ is the apparent force caused by the torque, F_x is the measured force, and α is the relative phase between the two.

The uncertainty can be calculated via:

$$\sigma^{\text{corr}} = \frac{1}{F_x^{\text{corr}}} \sqrt{(\tilde{F}_x - F_x \cos(\alpha))^2 \ \tilde{\sigma}^2 + (F_x - \tilde{F}_x \cos(\alpha))^2 \ \sigma^2} \tag{12}$$

where σ and $\tilde{\sigma}$ are the uncertainties for F_x and $\tilde{F}_x$, respectively.

The following table shows the uncorrected and corrected force amplitudes and uncertainties for each injected frequency:

$2f$ (Hz)	F_x (pN)	F_x^{corr} (pN)	$3f$ (Hz)	F_x (pN)	F_x^{corr} (pN)
8.32	–		12.45	$8.96^{\pm 0.31}_{\pm 3.46\%}$	$8.79^{\pm 0.32}_{\pm 3.61\%}$
10.34	$16.90^{\pm 2.33}_{\pm 13.79\%}$	$16.59^{\pm 2.33}_{\pm 14.05\%}$	15.51	$9.16^{\pm 0.45}_{\pm 4.91\%}$	$9.00^{\pm 0.46}_{\pm 5.07\%}$
15.60	$19.59^{\pm 0.30}_{\pm 1.48\%}$	$19.29^{\pm 0.30}_{\pm 1.57\%}$	23.40	$9.30^{\pm 0.37}_{\pm 3.98\%}$	$9.13^{\pm 0.37}_{\pm 4.10\%}$
17.11	$19.41^{\pm 0.46}_{\pm 2.37\%}$	$19.10^{\pm 0.47}_{\pm 2.44\%}$	25.66	$9.19^{\pm 0.57}_{\pm 6.20\%}$	$9.02^{\pm 0.57}_{\pm 6.36\%}$
19.11	$19.60^{\pm 0.21}_{\pm 1.07\%}$	$19.29^{\pm 0.22}_{\pm 1.12\%}$	28.67	$9.33^{\pm 0.24}_{\pm 2.57\%}$	$9.16^{\pm 0.24}_{\pm 2.66\%}$

References

[1] P1900244.