Cryogenic High-Q Mechanical Resonator

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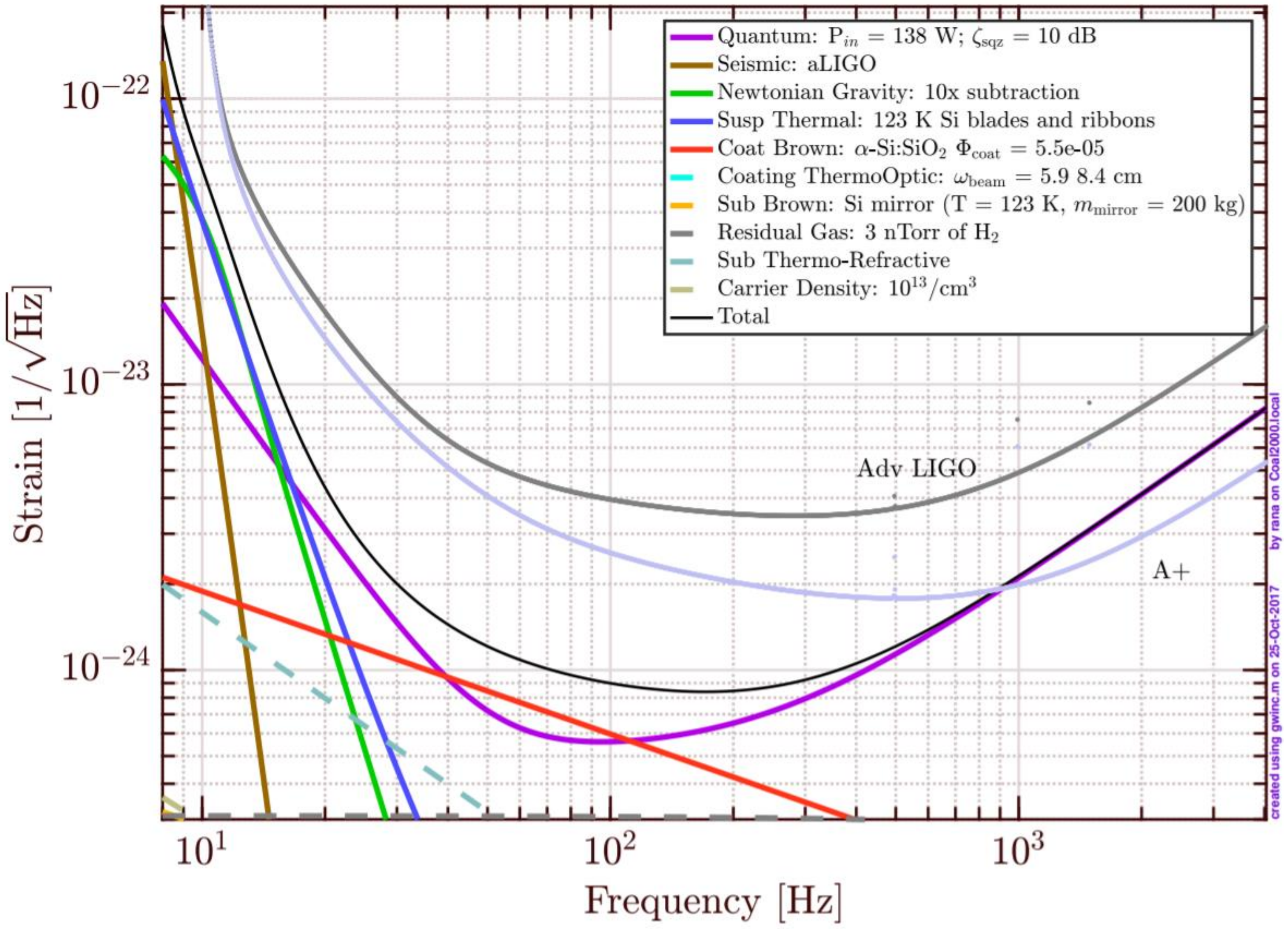
Outline

- Noise budget of LIGO Voyager
- Cryo-Q experiment
- Need for a metamaterial as a clamp
- N-coupled Spring-Mass System Analog
- Electrical Analog
- Finite Element Analysis
- Future work
- Acknowledgements

Noise Budget of LIGO Voyager



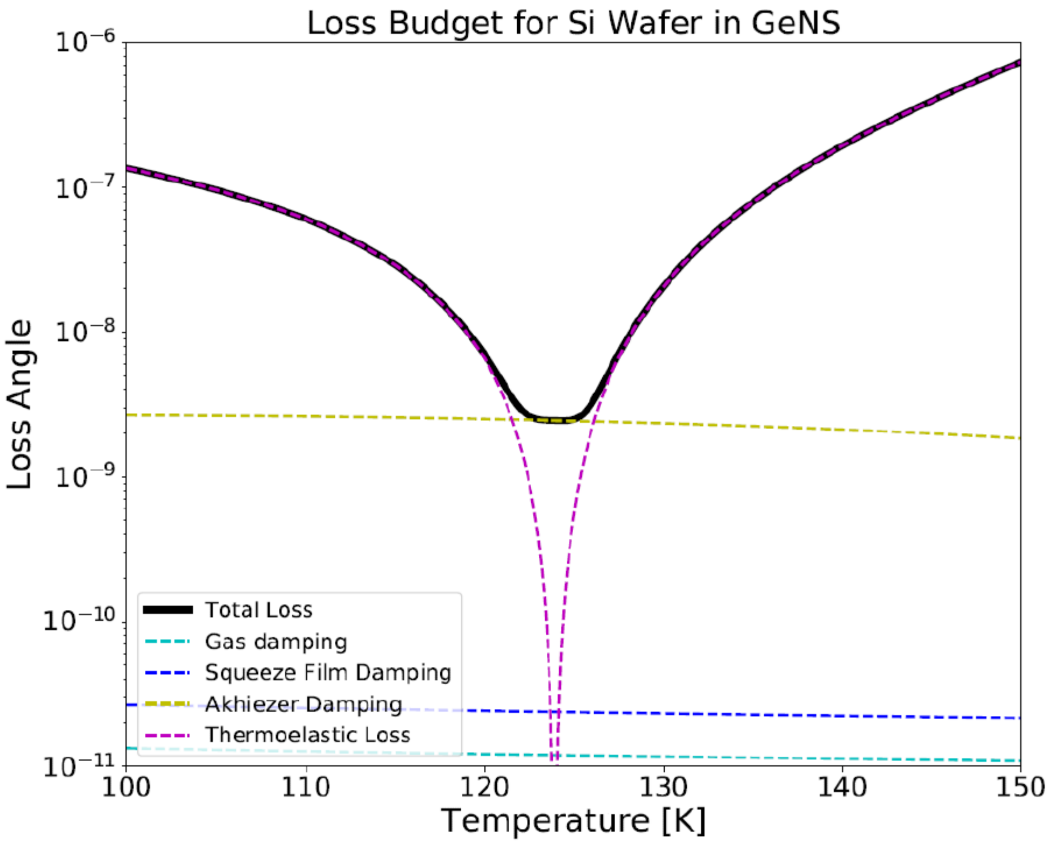
R. Adhikari et al., LIGO Voyager Upgrade: Design Concept, LIGO-T1400226-v9, LIGO Scientific collaboration (2018)

Cryo-Q Experiment

- Goal: To measure the quality factor of thin-film coating materials
- How: A Silicon disk, coated with the material, is oscillated and the quality factor is determined from the driving amplitude required to maintain the oscillations
- The experiment is conducted at ~123 K since the coefficient of thermal expansion is ~0, removing sources of thermo-elastic noise

Losses in Si Disk

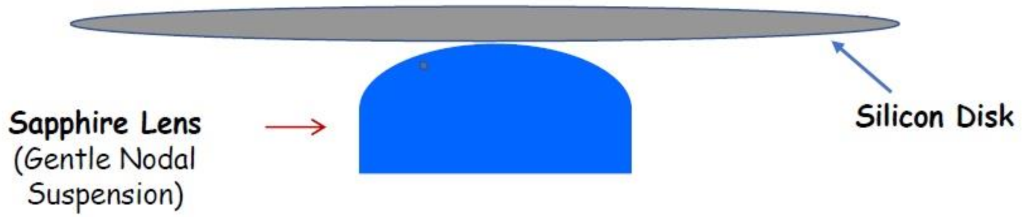
- Thermoelastic effect (α = 0 @ 123K)
- Phonon-phonon interaction
- Intrinsic losses
- Clamping losses
- Gas Damping



SURF 2019 presentation by Shubhabroto Mukherjee (DCC T1900384-v1)

Gentle Nodal Suspension

- The Gentle Nodal Suspension (GeNS) has a single point of contact with the disk which provides maximal mechanical isolation
- GeNS is unable to regulate temperature well due to the minimal contact area and pressure



SURF 2019 presentation by Shubhabroto Mukherjee (DCC T1900384-v1)

Need for a metamaterial as a clamp

- A metamaterial is a specially designed material which has properties that normal materials do not
- We want to use a metamaterial that does not couple the disk with the environment at the frequencies at which it vibrates, while effectively providing temperature control
- We want it to act as a bandstop filter, with the stopband frequency of 1 - 10 kHz, while higher frequencies correspond to heat transfer
- The speed of sound in Si is ∼5000 m/s, therefore the wavelength is ∼ 50 - 500 cm for frequencies of interest, but a practical length for the material is only few cm's, and is, hence, incomparable to the wavelength
- Due to the large wavelength, we cannot use periodic structures; therefore, we use locally resonant structures

Transfer Functions

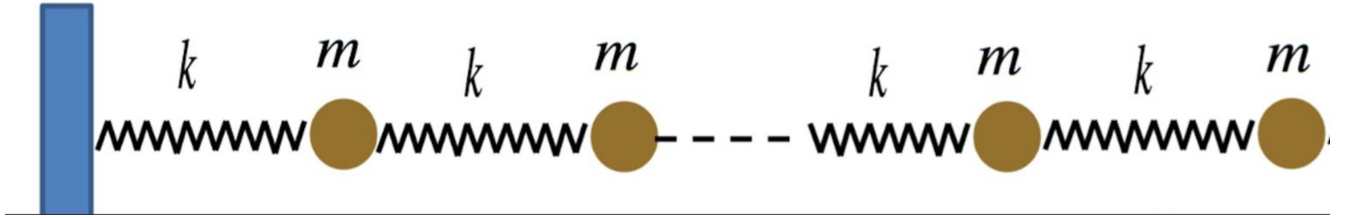
- A Transfer Function is a model that predicts the behavior of a system relative to its input

- Our goal for this project is to create a metamaterial with a coating that has the transfer function corresponding to the bandstop filter with the stop-band at 1-10 KHz

- Our transfer function is represented by a Bode Plot that shows the magnitude $\frac{a_N}{a_1}$ and phase of the Nth oscillator (normalized from the magnitude and phase of the first oscillator) for different input frequencies. It is effectively a frequency response model.

N-coupled Spring-Mass System Analog

- To gain an intuition for creating transfer functions of locally resonant structures, we first create a program to determine the transfer function of a N-coupled Spring-Mass system
- The common approach for finding the transfer function is to create a system of equations in the form of a matrix and invert it using numpy
- Due to the intractability of this method for large N, we used a less conventional method and defined recursive equations for the amplitude and created a program to calculate A_N for various ω.



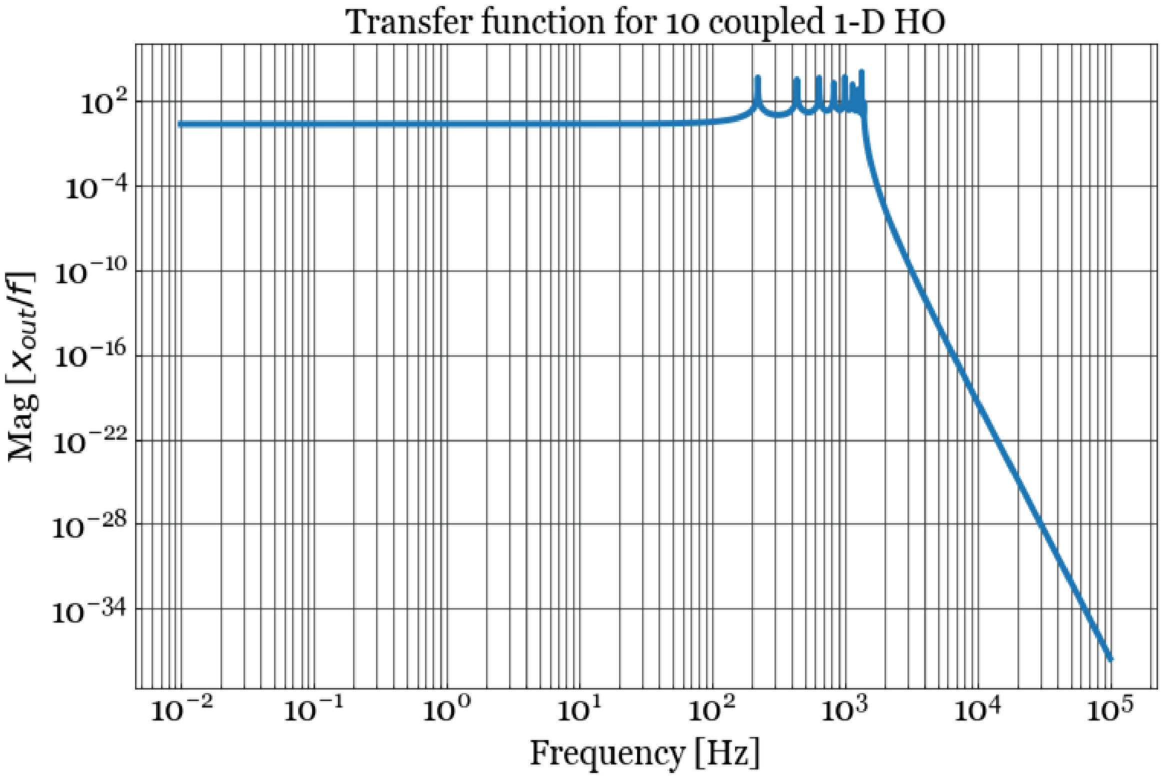
$$A_2 = A_1 \frac{2\omega_1^2 - \omega^2}{\omega_1^2}$$

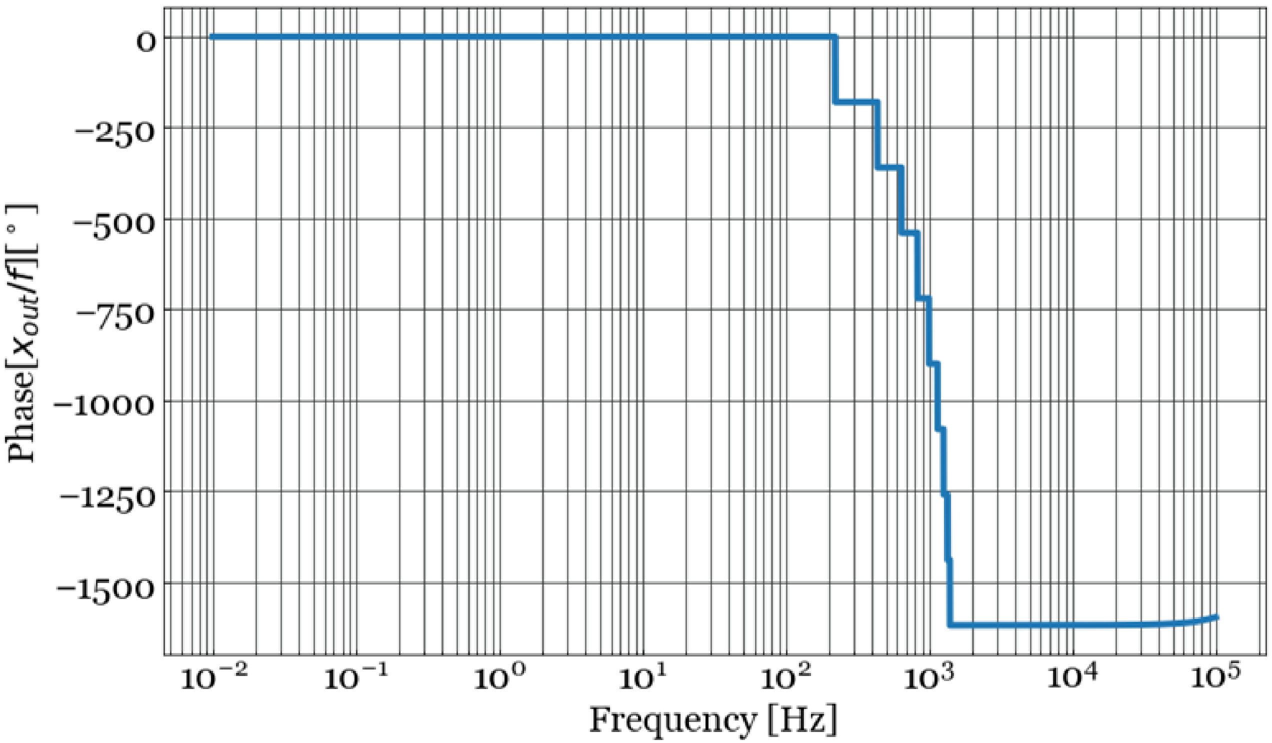
$$A_{j+1} = A_j \frac{2\omega_j^2 - \omega^2}{\omega_j^2} - A_{j-1}$$

$$A_N = A_{N-1} \frac{\omega_N^2}{2\omega_N^2 - \omega^2},$$

where $\omega_N = \sqrt{\frac{k_N}{m_N}}$

Transfer Function from the N-coupled Spring-Mass System



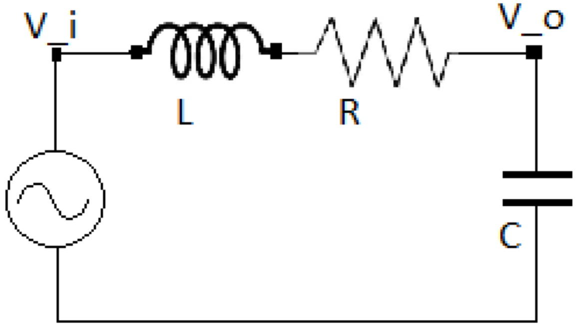


Transfer Function for an N=10 Spring-Mass System presented in the form of a bode plot

Electrical Analog

A driven damped oscillator is described by

$$\ddot{x}(t) + 2\zeta\omega_0\dot{x}(t) + \omega_0^2 x(t) = F(t)$$



And for the circuit shown,

$$(-\omega^2 LC + \imath\omega CR + 1)\tilde{V}_o(\omega) = \tilde{V}_i(\omega)$$

Both have the exact same structure, thus there is a direct analogy between mechanical systems and electrical networks.

Transmission Line

The voltage as a function of position and time is,

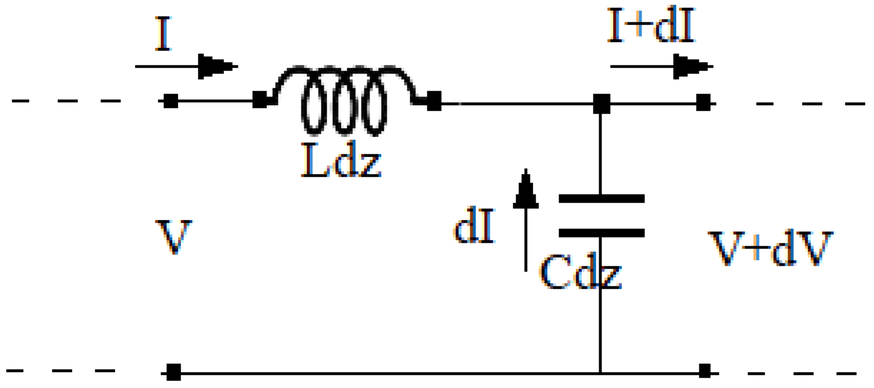
$$V = V_0 Re\left\{\exp(\imath(\omega t - \beta z))\right\}$$

propagation factor,

$$\beta = \omega\sqrt{LC}$$

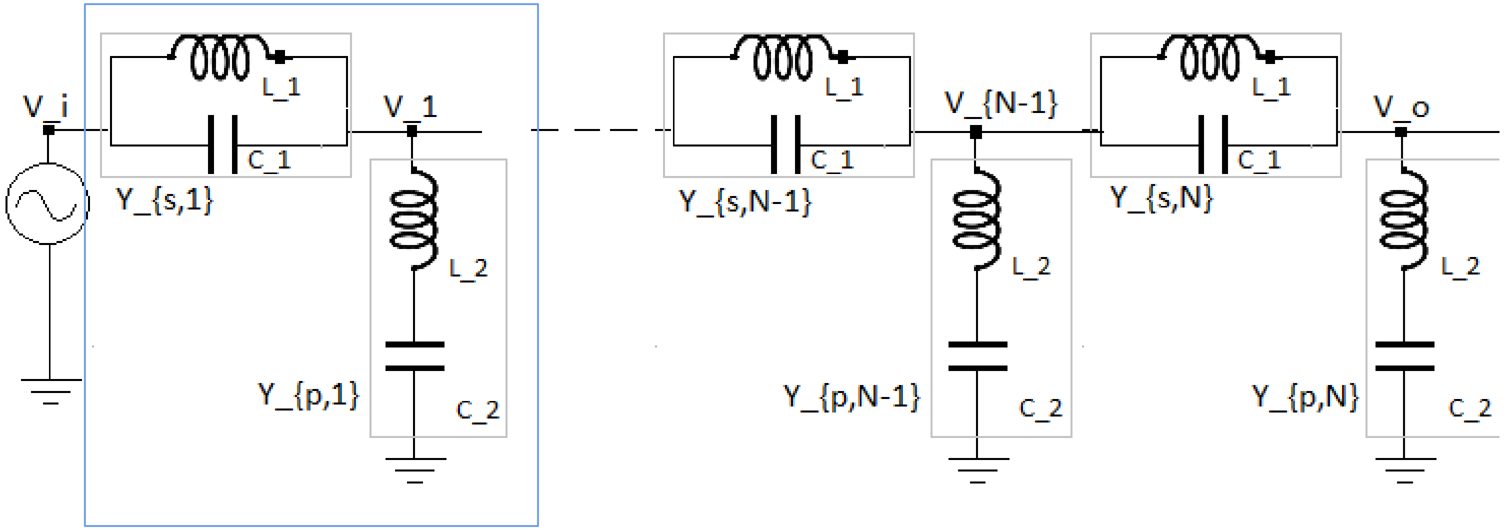
Characteristic impedance,

$$Z_0 = \sqrt{L/C}$$



Appropriate Circuit for our Requirement

Can use lumped element model since length is much less than the wavelength. For one unit:



$$Z_s = \frac{L_1/C_1}{sL_1 + 1/sC_1} = \frac{sL_1}{s^2 L_1 C_1 + 1}$$

$$Y_p = \frac{sC_2}{s^2 L_2 C_2 + 1}$$

$$H(s) = \frac{1/Y_p}{Z_s + 1/Y_p} = \frac{1}{Z_s Y_p + 1} = \frac{\left(s^2 L_1 C_1 + 1\right)\left(s^2 L_2 C_2 + 1\right)}{\left(s^2 L_1 C_1 + 1\right)\left(s^2 L_2 C_2 + 1\right) + s^2 L_1 C_2}$$

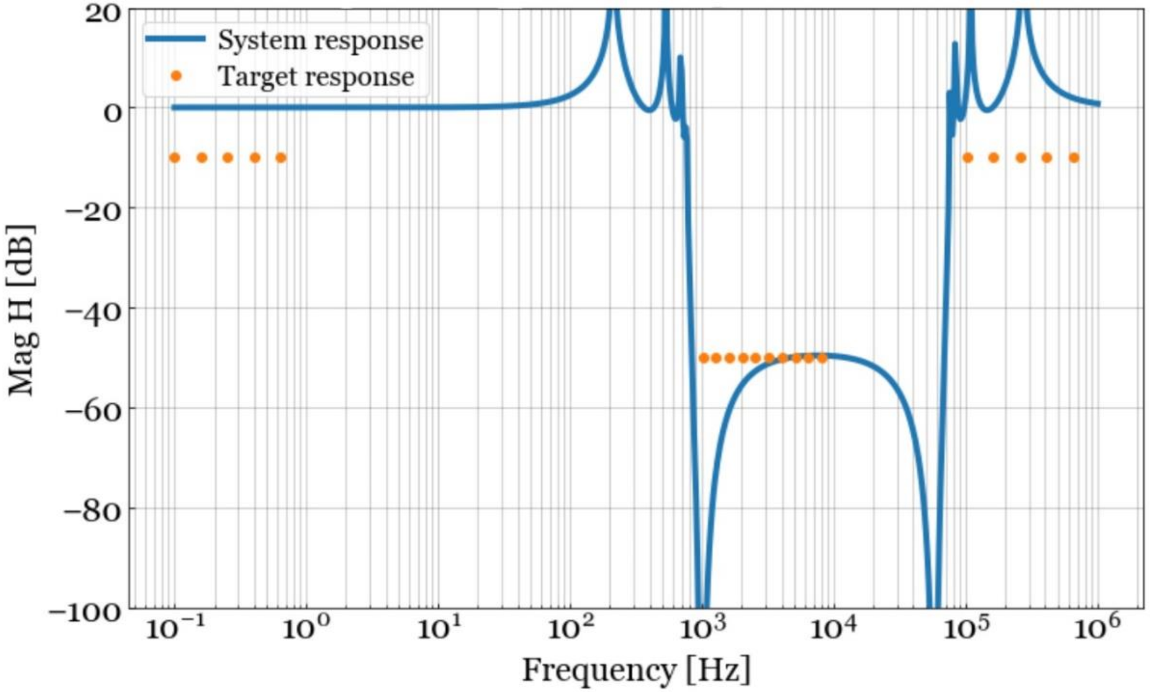
Nodal Analysis

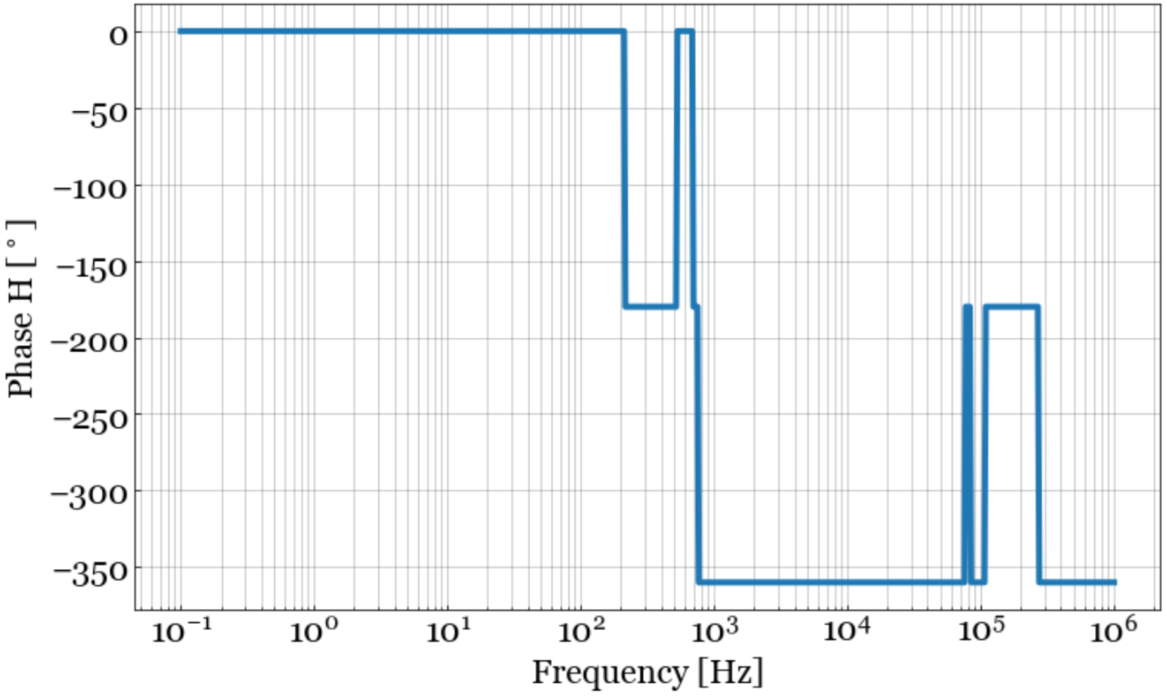
$$Y_{mat}\begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_{N-1} \\ V_o \end{pmatrix} = \begin{pmatrix} Y_{s,1}V_i \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}$$

$$Y_{mat} = \begin{pmatrix} Y_{s,1}+Y_{p,1}+Y_{s,2} & -Y_{s,2} & 0 & \cdots & 0 \\ -Y_{s,2} & Y_{s,2}+Y_{p,2}+Y_{s,3} & -Y_{s,3} & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ & & & & 0 \\ \vdots & & \ddots & \ddots & -Y_{s,N} \\ 0 & \cdots & 0 & -Y_{s,N} & Y_{s,N}+Y_{p,N} \end{pmatrix}$$

$$H = \frac{V_o}{V_i} = \{Y_{mat}^{-1}\}_{N,1} Y_{s,1}$$

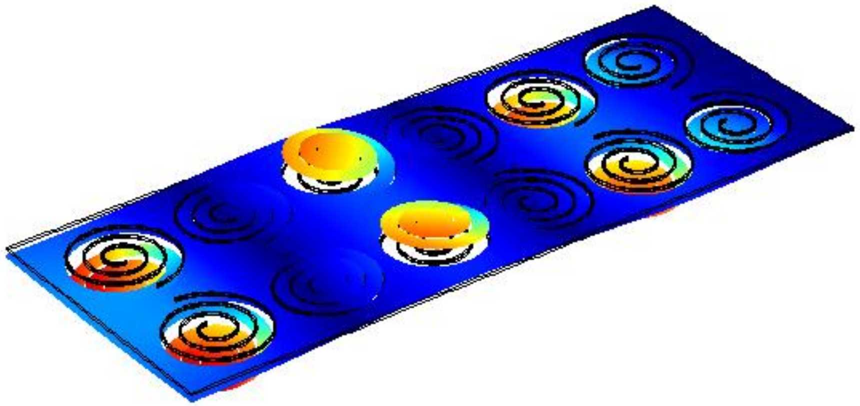
Response

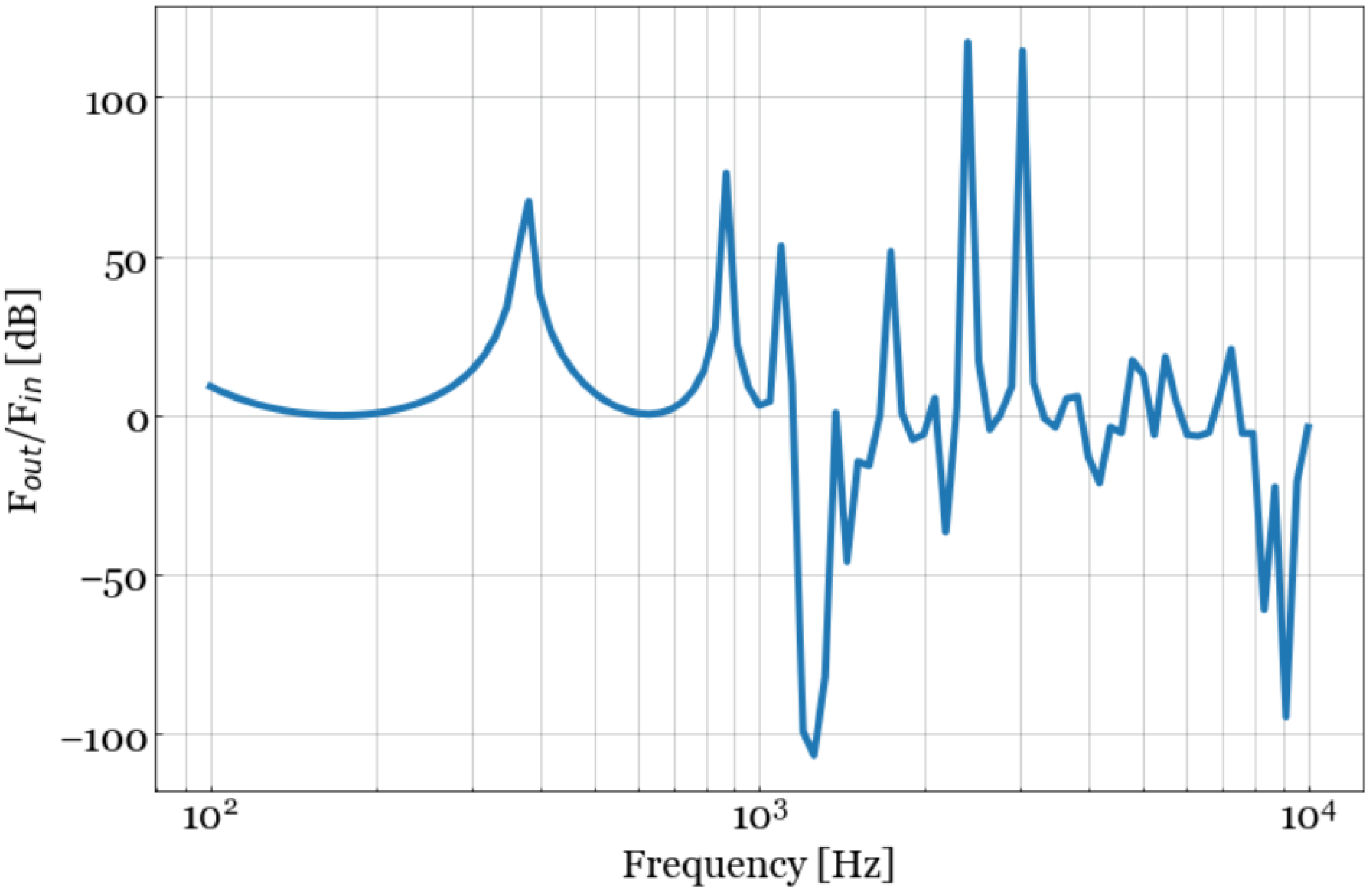




$$\text{Number of units} = 4$$
$$1/\sqrt{L_1 C_1} = 1.00 \text{ kHz}$$
$$1/\sqrt{L_2 C_2} = 10.01 \text{ kHz}$$
$$L_1 = 4.05 \text{ mH}$$
$$C_2 = 2.75\ \mu\text{F}$$

Finite Element Analysis





Beam dimensions:
Length = 5cm
Width = 2cm
Thickness = 0.2mm

Spiral:
Radius = 4mm
Turns = 3

Summary

- Suitable electrical circuit with ‘transmission line like’ topology found
- Idea for a local resonator based metamaterial proposed

Future Work

- Mechanically interpreting the circuit
- Tweaking the shape and parameters in FEA to get the desired output

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Thank You