LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
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CALIFORNIA INSTITUTE OF TECHNOLOGY
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Technical Note **LIGO-T2000513–v3** 2020/10/07

Cryogenic High-Q Mechanical Resonator

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Abstract

The aim of the project is to design a metamaterial which can mechanically isolate a silicon disk from the environment around its resonant frequency, by acting as a bandstop filter for 1 to 10kHz. Successfully isolating the disk would allow performing high-Q measurements at cryogenic temperature. The need for using a metamaterial, as compared to other isolation techniques (such as gentle nodal suspension) is that it provides temperature control through conduction. To design the metamaterial, first electrical analogues such as EM transmission line were studied, and a bandstop filter with a transmission-line-like topology was designed using inductors and capacitors. The mechanical model for the metamaterial, containing local resonators, is currently being studied using FEA techniques, and the parameters are being optimised. After the design is completed, it would be fabricated and tested. Such a metamaterial could also be useful in isolating seismic noise.

1 Introduction

In 1916, Albert Einstein predicted the existence of gravitational waves. On September 14, 2015 the two detectors of the Laser Interferometer Gravitational Wave Observatory (LIGO) at Hanford, WA and Livingston, LA reported the first direct detection of gravitational waves and the first direct observation of a binary black hole system merging to form a single black hole, confirming the predictions of general relativity for the nonlinear dynamics of highly disturbed black holes.

An Advanced LIGO detector is a modified Michelson interferometer that measures gravitational wave strain as a difference in length of its orthogonal arms. Each arm is 4 km long and contains a resonant optical cavity, with two test mass mirrors, that multiplies the effect of a gravitational wave on the light phase by a factor of 300. Second, a partially transmissive power-recycling mirror at the input provides additional resonant buildup of the laser light in the interferometer as a whole: 20W of laser input is increased to 700W incident on the beam splitter, which is further increased to 100 kW circulating in each arm cavity. Third, a partially transmissive signal-recycling mirror at the output optimizes he gravitational-wave signal extraction by broadening the bandwidth of the arm cavities.[1]

Advanced LIGO has strain sensitivity of the order of $10^{-23}/\sqrt{Hz}$ between 70 and 1000 Hz and is susceptible to extremely small sources of noise. There is significant displacement noise from seismic motion below 10 Hz. Thermal noises arise from losses in mechanical systems and cause displacement noise. There is shot noise and radiation pressure noise, limited by the fundamental uncertainty in the phase and amplitude of the laser. Quantum noise is due to fluctuations of the optical vacuum field. Residual gas exchanges momentum with the test masses and scatters photons causing both displacement and sensing noise. There is also non uniform, time dependent charge on the test masses and noise from electronic circuits in photodetectors, actuators, ADC, DAC and RF oscillator. There are pointing fluctuations (beam jitter) due to angular and longitudinal motion of the steering mirrors. Also, the vacuum chambers and the arm tubes are not isolated from the seismic or acoustic noises, and this motion can couple to the gravitational wave channel through scattered light.[3]

The sensitivity of interferometric gravitational wave observatories is limited in part by thermally induced vibrations. These can be reduced through a combination of - lowering the mirrors' temperatures to a cryogenic regime, selecting mirror materials with low mechanical loss, and using larger mirror diameters. Development of coatings with low thermal noise is also important. Silicon is used because its mechanical loss decreases with temperature; and has very high thermal conductivity at cryogenic temperatures which reduces thermal lensing and makes it easier to get the absorbed heat out. The temperature is chosen to be 124K because silicon's thermal expansion coefficient crosses zero here, which eliminates the thermoelastic component of the thermal noise and also minimizes radius of curvature changes induced by temperature gradients.[2]

2 Project Overview

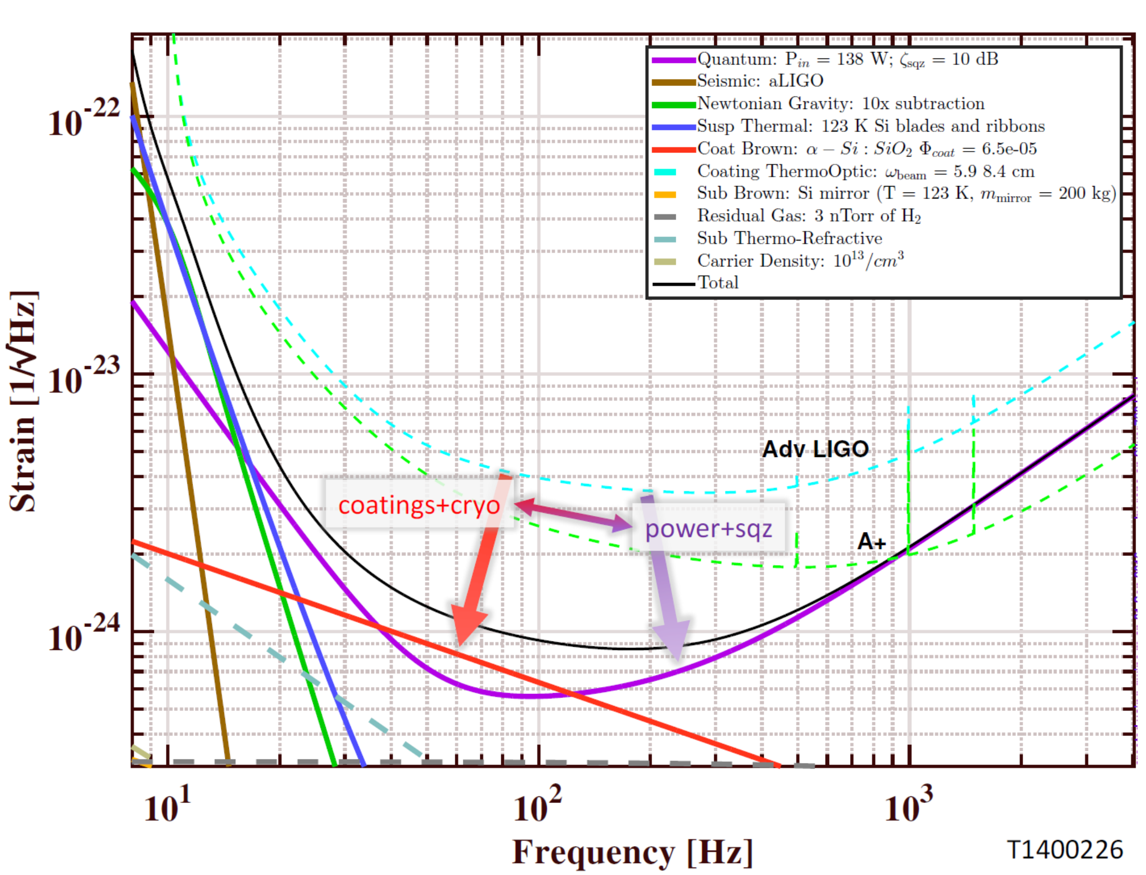

Figure 1: Proposed Noise Budget of LIGO Voyager[6]

2.1 The Cryo-Q experiment

The cryo-Q experiment is aimed at measuring the quality factor of thin film coating materials at 124K. We test for silicon disks coated with the film and compare it with properties of bare silicon which are fairly well known. A thin silicon disk (with or without the desired coating material) is suspended/balanced at 124K in vacuum and excited using an electrostatic actuator. The quality factor (Q factor) is measured by measuring the driving amplitude required to maintain oscillations at constant amplitude in the disk. The driving frequency is controlled by a PLL and VCO and the amplitude is controlled by an ALL. Ideally, the Q factor measured should only be due to the internal losses of the material, which is only possible if we eliminate all other sources of losses. Once the Q factor of bare silicon is known, it can be coated with materials to estimate their Q factor.

2.2 Problem with the current design

In the current design, gentle nodal suspension (GeNS)[4] is used, in which the disk is placed on top of a smooth convex surface. It has the advantage that since it has a single point of contact between the disk and the environment, it provides maximal mechanical isolation for modes with a node at the suspension point. The issue is that we want to cool these silicon disks to 124K, and be able to control the temperature. Radiative cooling is not suitable since it is unable to cool radiatively below 160K since the radiation is mainly in the microwave region for their dimensions, and GeNS provides very low thermal conduction due to very small area of contact and very low pressure.

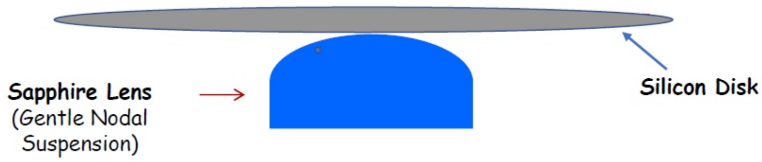

Figure 2: Gentle nodal suspension, source: [5]

2.3 Need for a metamaterial

In this project, we want to come up with a design with effective cooling and temperature control (which is only possible through conduction) without increasing the clamping loss, i.e., with adequate isolation of the mechanical energy within the sample. Increasing conduction and decreasing clamping losses are two opposite requirements, since increasing conduction requires us to increase the area of contact and the contact pressure which ends up coupling the disk to the environment, and increases clamping losses. A metamaterial is a specially designed material which has properties that normal materials don't, and our problem can be solved by using one. It provides significant conduction due to the large area of physical contact, and is designed such that it acts as a bandstop filter which doesn't allow frequencies around the resonant frequency of the disk to pass through it (1 to 10kHz). Since these frequencies are unable to pass through the material, and the disk doesn't 'respond' to other frequencies, it is isolated from the external environment. We need the higher frequencies to be able to pass through the material since they are responsible for heat transfer through conduction.

Metamaterials can be designed in two ways, either by using periodic structures, or by using locally resonant structures. In periodic structures, waves (elastic in our case) are reflected at the periodic boundaries (planes), and we get significant reflection if the reflected waves interfere constructively (Bragg reflection). This happens when the Bragg condition: $n\lambda = 2d\sin\theta, n \in \mathbb{N}$ is satisfied, where λ is the wavelength of the wave, d is the minimum separation between the periodic planes and θ is the angle that the wave makes with the planes. Basically, to be able to manipulate the wave using a periodic structure, the wavelength needs to be less than, or at least comparable to the length of the material. The speed of propagation of transverse elastic wave in silicon is $\sim$ 5800 m/s, and thus for the frequencies of our interest (1 to 10kHz), wavelength is $\sim$ 58 - 580 cm, whereas a practical length for the material (L) is only a few cm's ($\sim$ 5 cm), hence, periodic structures are not suitable.

Note that we have established that our $\lambda \gg L$, and hence there is almost no change in the phase of the wave as it travels across the material. We use structures with local resonators, which take up energy from the forward travelling wave, start oscillating, and then reproduce waves (with a particular phase difference with respect to the original wave), which for our case should be such that it cancels the original waves (in the stopband). The resonant frequency of these resonators should be in the stopband region, so that they respond to these frequencies. Also, their dominant oscillation mode should be transverse, because the waves are transverse.

3 Electrical Analogue

There is a stark similarity between electrical and mechanical systems. The differential equation describing the displacement for a driven damped oscillator is exactly similar to the one describing the voltage across the capacitor in a series L, C, R circuit, driven by an oscillating voltage source. Thus, first we will be looking at electrical analogue to the problem, and attempt to design an electrical circuit which meets our requirements: resonators on a transmission line, which acts as a bandstop filter, since it is easier to study and design in the electrical domain. A code was created to calculate the response for a circuit with transmission-line-like topology, with easily customisable 'series' and 'parallel' admittances, and then optimise the parameters to get the target response. The following circuit was found to be appropriate:

3.1 Appropriate circuit for our requirement

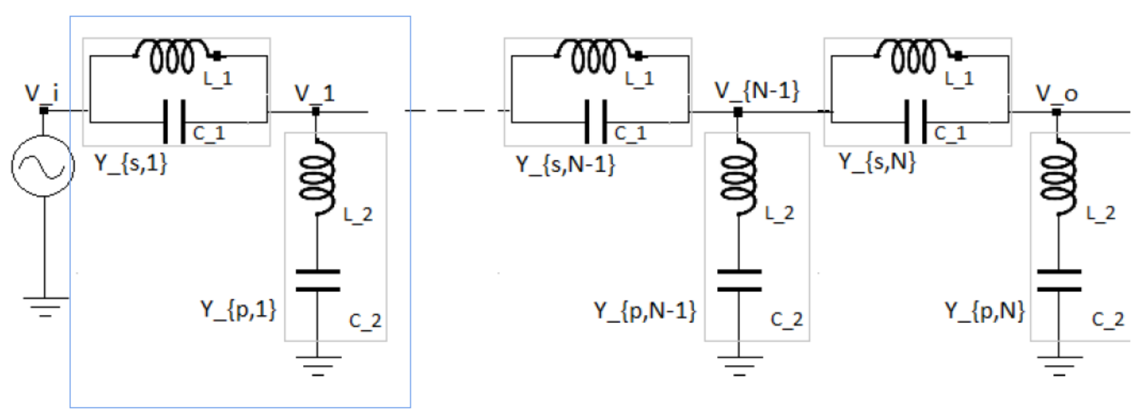

Figure 3: Appropriate circuit for our requirement

3.1.1 Transfer function for a unit cell

Since $\lambda \gg L$, we can use lumped element model to study the circuit, writing the impedance of a capacitor as $1/\imath\omega C$ (where C is it's capacitance), and that of an inductor as $\imath\omega L$ (where L is it's inductance) (note that the position doesn't play any role). For a unit cell of the circuit in fig.(3) (enclosed in the blue box), the 'series' impedance,

$$Z_s(s) = \frac{1}{Y_s} == \frac{L_1/C_1}{sL_1 + 1/sC_1} = \frac{sL_1}{s^2L_1C_1 + 1} \tag{1}$$

and the 'parallel' admittance,

$$Y_p(s) = \frac{sC_2}{s^2 L_2 C_2 + 1} \tag{2}$$

and thus the transfer function,

$$H(s) = \frac{1/Y_p}{Z_s + 1/Y_p} = \frac{1}{Z_s Y_p + 1} = \frac{(s^2 L_1 C_1 + 1)\,(s^2 L_2 C_2 + 1)}{(s^2 L_1 C_1 + 1)\,(s^2 L_2 C_2 + 1) + s^2 L_1 C_2} \tag{3}$$

i.e.,

$$\boxed{H(\imath\omega) = \frac{(-\omega^2 L_1 C_1 + 1)\,(-\omega^2 L_2 C_2 + 1)}{(-\omega^2 L_1 C_1 + 1)\,(-\omega^2 L_2 C_2 + 1) - \omega^2 L_1 C_2}} \tag{4}$$

The transfer function has $H(\omega = 0) = H(\omega \to \infty) = 1$. It has two (order 2) zeros at $\omega = \frac{1}{\sqrt{L_1 C_1}}, \frac{1}{\sqrt{L_2 C_2}}$ and two (order 2) poles, one before and one after the zeros. Thus, the circuit acts as a bandstop filter. The magnitude of the transfer function attenuates as -40 dB/dec after crossing the first pole. After crossing the first zero, the attenuation and gain are cancelled, so the magnitude remains constant. After crossing the second zero, the gain is 2*40 dB/dec due to the zeros, and -40 dB/dec due to the first pole, thus the magnitude increases as 40 dB/dec. After the second pole is crossed, it again becomes, and stays constant. Note that this discussion is for one unit cell, one can achieve higher rates of attenuation and gain in the transition band using more unit cells.

3.1.2 Nodal analysis

To find the transfer function of the whole circuit, we need to find a relation between V_i and V_o. We can do so by using 'nodal analysis', which is basically applying Kirchhoff's current law at every node and writing the equations in a matrix form.

At the first node,

$$\begin{aligned} Y_{s,1}(V_1 - V_i) + Y_{p,1}V_1 + Y_{s,2}(V_1 - V_2) = 0 \\ \implies (Y_{s,1} + Y_{p,1} + Y_{s,2})V_1 - Y_{s,2}V_2 = Y_{s,1}V_i \end{aligned} \tag{5}$$

At any intermediate node 'k', $k = 2$ to $N-1$,

$$\begin{aligned} Y_{s,k}(V_k - V_{k-1}) + Y_{p,k}V_k + Y_{s,k+1}(V_k - V_{k+1}) = 0 \\ \implies -Y_{s,k}V_{k-1} + (Y_{s,k} + Y_{p,k} + Y_{s,k+1})V_k - Y_{s,k+1}V_{k+1} = 0 \end{aligned} \tag{6}$$

And finally at the output node,

$$\begin{aligned} Y_{s,N}(V_o - V_{N-1}) + Y_{p,N}V_o = 0 \\ \implies -Y_{s,N}V_{N-1} + (Y_{s,N} + Y_{p,N})V_o = 0 \end{aligned} \tag{7}$$

Thus, the equations can be written in a matrix form,

$$Y_{mat} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_{N-1} \\ V_o \end{pmatrix} = \begin{pmatrix} Y_{s,1}V_i \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

Where,

$$Y_{mat} = \begin{pmatrix} Y_{s,1}+Y_{p,1}+Y_{s,2} & -Y_{s,2} & 0 & \cdots & 0 \\ -Y_{s,2} & Y_{s,2}+Y_{p,2}+Y_{s,3} & -Y_{s,3} & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ & & & & 0 \\ \vdots & & \ddots & \ddots & -Y_{s,N} \\ 0 & \cdots & 0 & -Y_{s,N} & Y_{s,N}+Y_{p,N} \end{pmatrix} \tag{9}$$

Thus, the system transfer function,

$$\boxed{H(s) = \frac{V_o(s)}{V_i(s)} = \{Y_{mat}^{-1}(s)\}_{N,1} Y_{s,1}(s)} \tag{10}$$

This calculation is very fast and efficient to do numerically for the whole frequency vector in one shot using three dimensional NumPy Arrays. For an $8 \times 8 \times 700$ dimensional Y_{mat} (corresponding to a 700×1 dimensional frequency vector and 8 unit cells) it took 19.1 ms $\pm$ 573 μs to calculate the response.

3.1.3 Optimisation

In particle swarm optimisation (PSO) we have 'M' particles, each having a $N \times 1$ dimensional position vector, '$\mathbf{X}^{(t)}_{i_{N\times1}}$', $i = 1, 2, \ldots, M$. The N coordinates correspond to the 'N' parameters to be optimised. Each particle also has a $N \times 1$ dimensional velocity vector '$\mathbf{V}^{(t)}_{i_{N\times1}}$', $i = 1, 2, \ldots, M$. The superscript 't' refers to the t^{th} iteration, the position and velocity vector for all M particles are updated in each iteration. There is a cost associated with each position, calculated by the cost function $C : \mathbb{R}^n \to \mathbb{R}$, and the goal is to find the 'best' position - the position at which the cost is minimised. These vectors are updated as follows:

$$\mathbf{V}^{(t+1)}_{i_{N\times1}} = w\mathbf{V}^{(t)}_{i_{N\times1}} + c_1 r_1^{(t)}(\mathbf{P}_{i_{N\times1}} - \mathbf{X}^{(t)}_{i_{N\times1}}) + c_2 r_2^{(t)}(\mathbf{G}_{N\times1} - \mathbf{X}^{(t)}_{i_{N\times1}}) \tag{11}$$

$$\mathbf{X}^{(t+1)}_{i_{N\times1}} = \mathbf{X}^{(t)}_{i_{N\times1}} + \mathbf{V}^{(t+1)}_{i_{N\times1}} \tag{12}$$

where $i = 1, 2, \ldots, M$. The first term in eq. (11) is the 'inertia component', weighted by the hyper-parameter w. The second term is the 'cognitive component', it directs the velocity towards the particle's best position $\mathbf{P}_i$ (note the index 'i'), weighted by the hyper-parameter c_1. The third term is the 'social component' which directs the velocity toward the global best position $\mathbf{G}$ (note the absence of the index 'i'), weighted by the hyper-parameter c_2. r_1 and r_2 are random numbers between 0 and 1, which provide some degree of exploration and noise, which increases the chance of finding the global minima[8].

The parameters were optimised using PSO. A target response was defined according to the requirement, and the system response calculated using nodal analysis was compared against it. Cost was defined to be sum(ELU(error)), where ELU (Exponential Linear Unit) function was used since it doesn't matter if the response is lower than the target attenuation in the stopband, or greater than the buffer in the passband, and linear variation of cost with error is fine for us and saves computation time. The optimised output is shown in fig.(4).

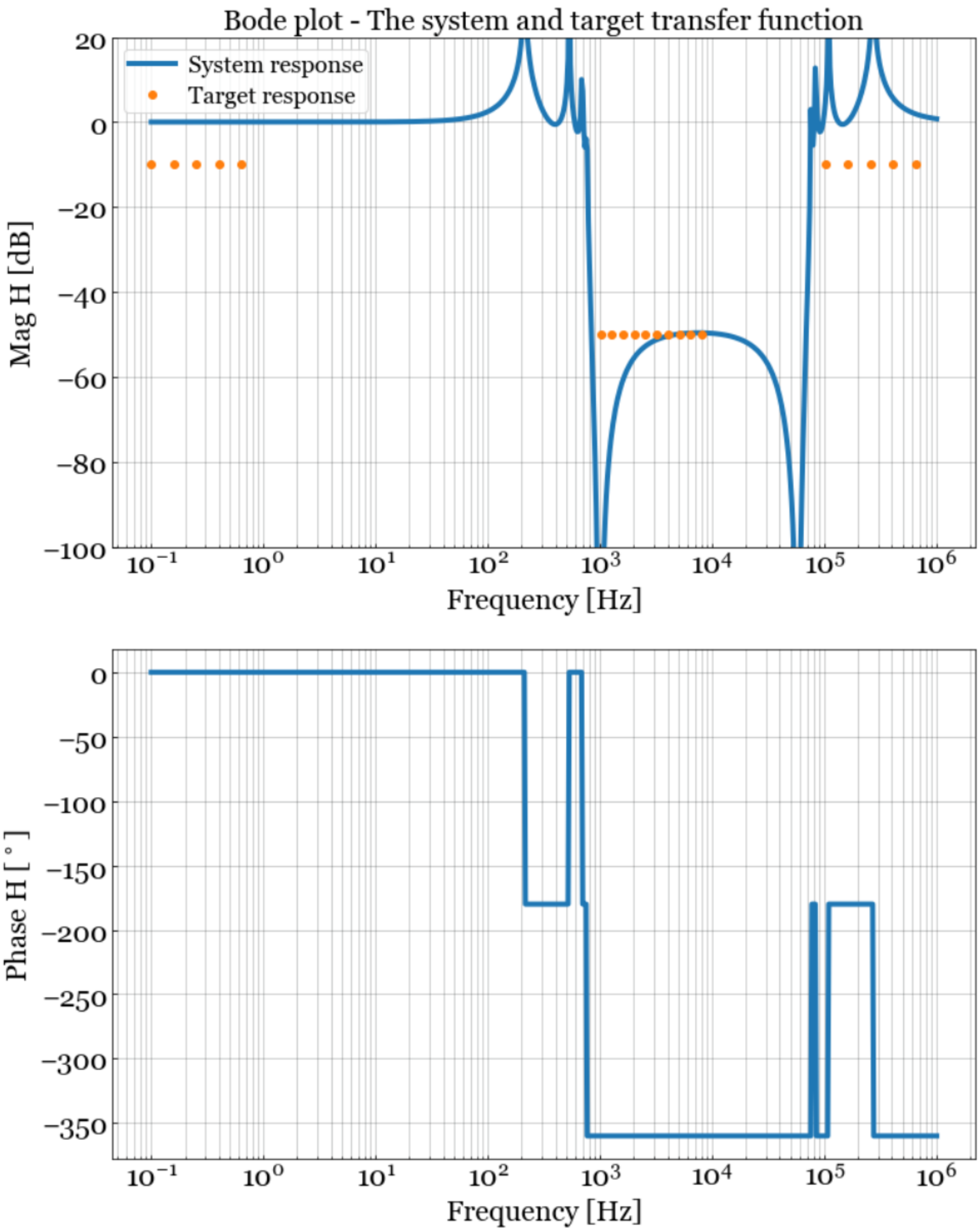

Figure 4: Bode plot of the response of the circuit with four unit cells (blue) for optimised parameters: $\frac{1}{\sqrt{L_1 C_1}} = 1.00$kHz, $\frac{1}{\sqrt{L_2 C_2}} = 10.01$kHz, $L_1 = 4.05$ mH, $C_2 = 2.75$ μF. Target magnitude response (orange)

4 Finite Element Analysis

A COMSOL model was designed with multiple spirals etched in a rectangular sheet of silicon. Say the sheet is placed in the XY plane, with length along the X axis. A frequency domain study was performed which gave the traction along 'Z' for an input driving force of different frequencies at one end. The integral of this traction at the output YZ surface is the output. This is the force that is required to be applied a the output to keep it fixed. The ratio of these output and input forces for different frequencies is the transfer function. The spiral dimensions need to be optimised which can be done using MATLAB and LiveLink for MATLAB-COMSOL. Currently it is facing some issues in meshing for different parameters, and the parameters haven't been optimised yet. Response for the current model is shown in fig. (6)

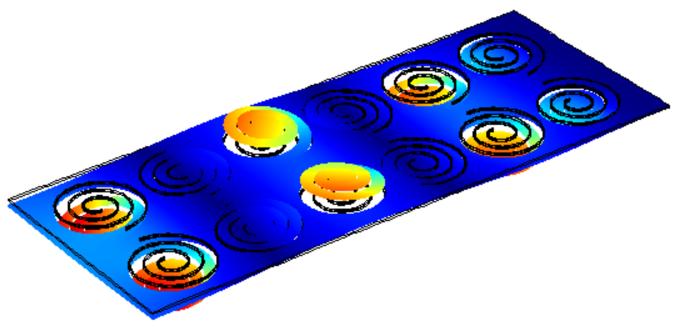

Figure 5: The COMSOL model with spirals in it. The stress at 1kHz is being displayed

5 Mechanical Dissipations in Silicon[9]

We want to isolate the disk at 124K so as to have minimal loss to the environment and be able to perform extremely low loss measurements. The loss of any elastic body is the sum of different contributions that can be generally divided into three main categories: fundamental losses ϕ_{fund} common to even perfect materials (i.e., materials with perfect crystal lattices), intrinsic losses ϕ_{int} due to material imperfections, and losses ϕ_{ext} due to coupling to the external environment. The total losses can thus be detailed as,

$$\phi_{tot} = \phi_{fund} + \phi_{int} + \phi_{ext} \tag{13}$$

5.1 Fundamental losses

These set the lower limit to ϕ_{tot}: the thermoelastic effect and the phonon-phonon interaction. The thermoelastic loss is negligible for our case since we work at $T = 124K$ for which the thermal expansion coefficient of silicon, $\alpha = 0$.

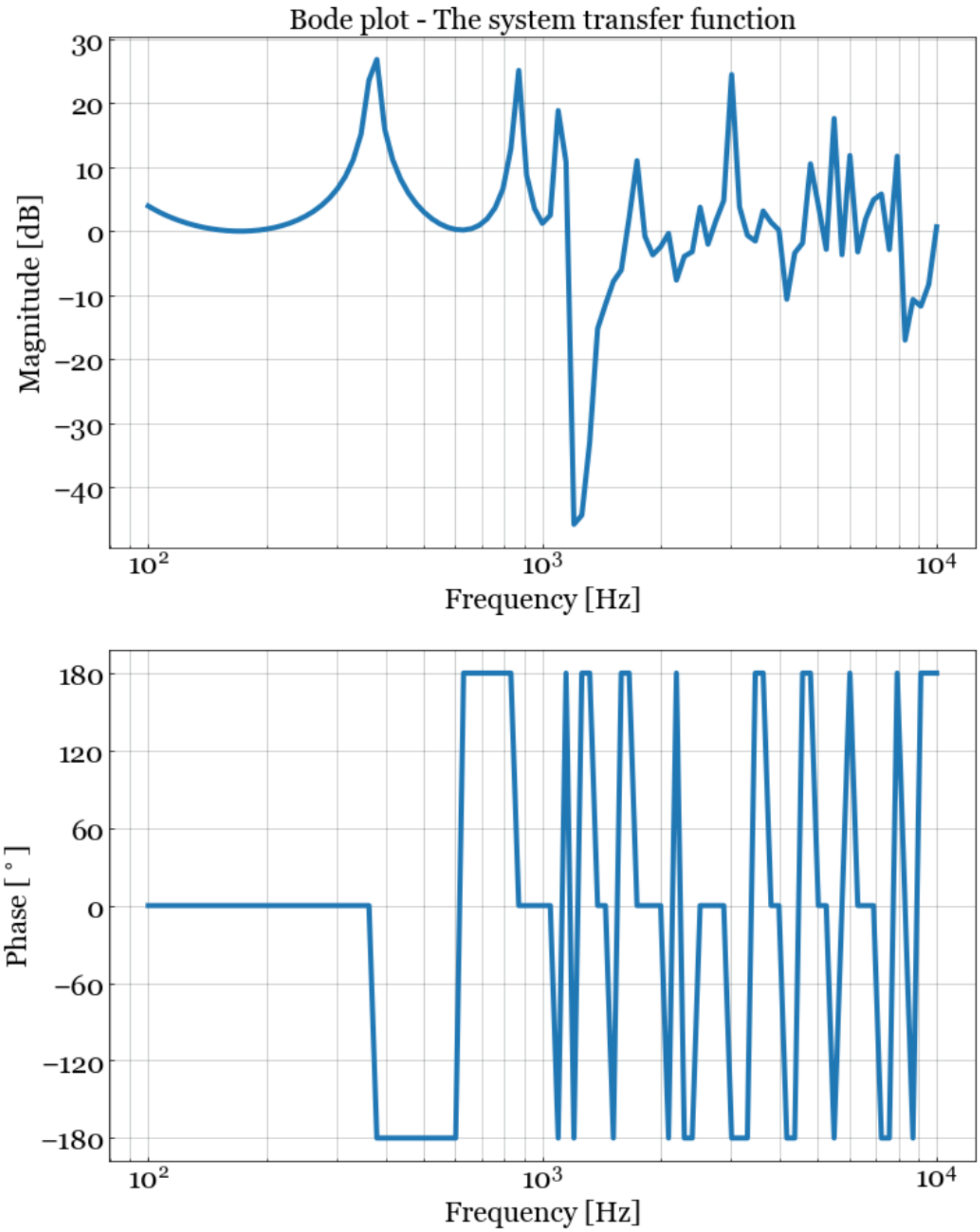

Figure 6: Bode plot of the transfer function of the COMSOL model. Beam dimensions: length = 5 cm, width = 2 cm, thickness = = 0.2 mm. Spiral dimensions: radius of outermost spiral = 0.4 cm, radial width = 1 mm, radial gap = 0.1 mm, number of turns = 3

5.2 Intrinsic losses

This is the parameter that we wish to measure. Bulk silicon is demonstrated to possess extremely low mechanical losses, though surface losses might be a limiting factor, which obstructs attaining of the maximal Q-factor.

5.3 External losses

5.3.1 Clamping losses

One of the main problems is to minimize clamping loss. It can be reduced by wisely designing the holder/suspension method. There are multiple methods to do so. The GeNS has negligible clamping loss, but as we discussed earlier, it doesn't provide any heat transfer through conduction. A metamaterial can also be placed between the disk and the external environment, designed such that it acts as a band reject filter around the natural frequency of the disk. The disk can also be clamped between two spheres, pressing against the centre of the two faces, which provides an increase in conduction due to increase in pressure.

5.3.2 Gas Damping

Loss due to the residual gas in the vacuum chamber is, $\phi_{gas} \sim \frac{L}{16\pi d}\sqrt{\frac{M_g}{M_{He}}}\frac{P_{(mbar)}}{\nu_{(Hz)}\sqrt{T_{(K)}}}$ where d is the distance between the walls and the sample and L is the typical dimension of the vibrating body.

6 Summary and Future work

An electrical circuit with transmission line like topology which acts as a bandstop filter was designed and optimised. An idea for a local resonator based metamaterial has been proposed, with spirals etched in a sheet, which oscillate transversally. The code to optimise the parameters of the model has been written but is facing some issues in meshing for various values. Future work involves optimising the current model and coming up with better designs. Then finally the model is to be fabricated and tested. Such a metamaterial, once designed, will be very useful in isolating any component from vibrations.

7 Acknowledgements

I thank my mentors Dr. Raymond Robie, Aaron Markowitz and Prof. Rana Adhikari for their guidance and suggestions. I would also like to thank Aaron for the huge help with the COMSOL model and the interesting discussions. Thank you to LIGO, LIGO SURF program and National Science Foundation for making this project possible.

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