



Improvement of Outer Boundary Conditions for Binary Black Hole Numerical Simulations

GWANW – Virtual Meeting

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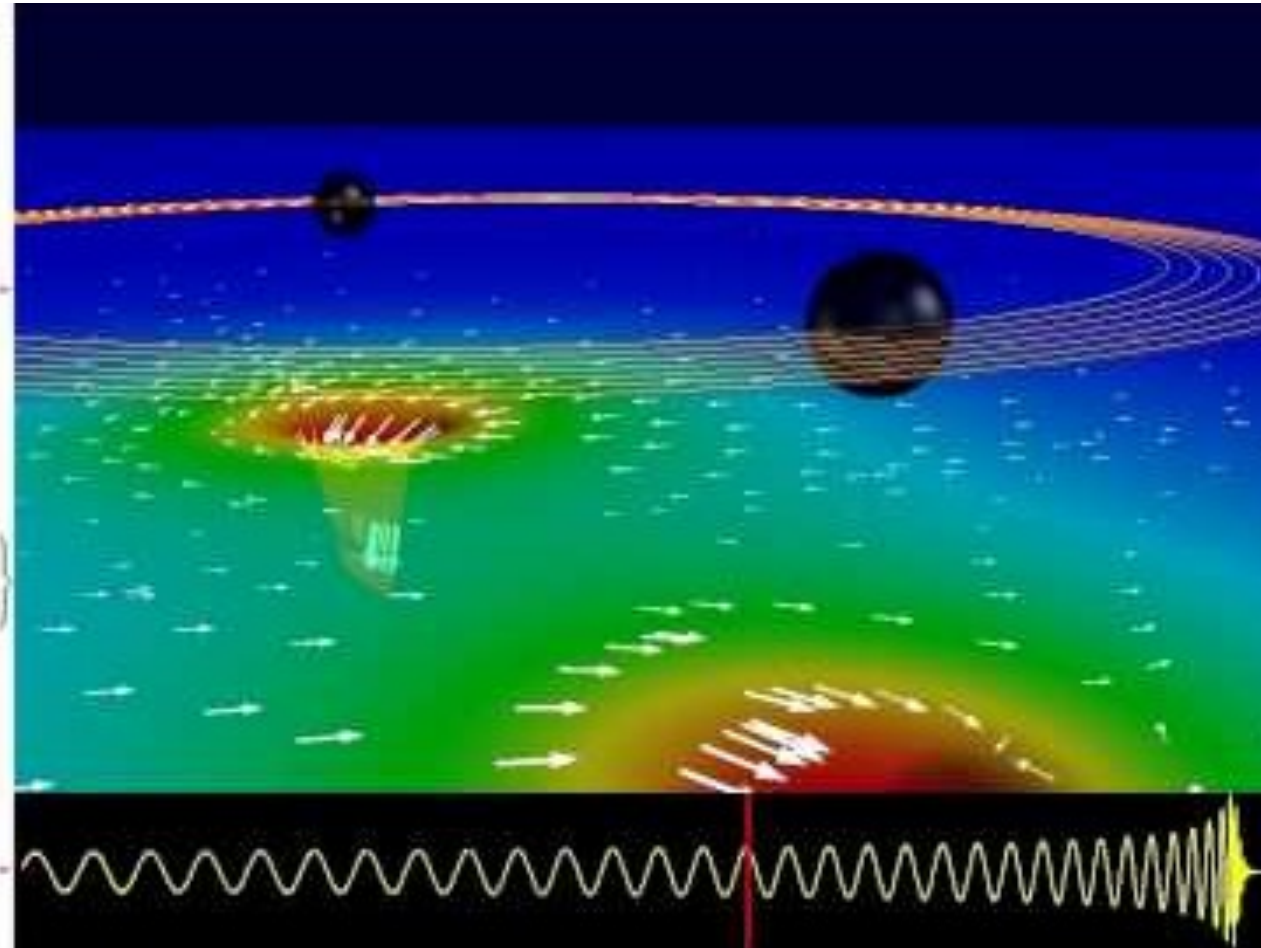
The Spectral Einstein Code - SpEC

Binary Black Hole Evolution:
Caltech/Cornell Computer Emulation

Top: 3D view of Black Holes
and Orbital Trajectory

Middle: Spacetime curvature:
Depth: Curvature of space
Colors: Rate of flow of time
Arrows: Velocity of flow of space

Bottom: Waveform
(red line shows current time)



Video from <https://www.black-holes.org/explore/movies>

Computational domain

Cauchy (3+1) formulation

- Solve EEs on spatial hypersurface
- Evolve in time (the +1)

Outer boundary: spherical domain

- Artificial time-like boundary
- Where GW are calculated ($\Psi_4^{\ell,m}$ & $h_{\ell,m}$)
- Requires boundary conditions
- Freezing $\Psi_0^{\ell,m}$

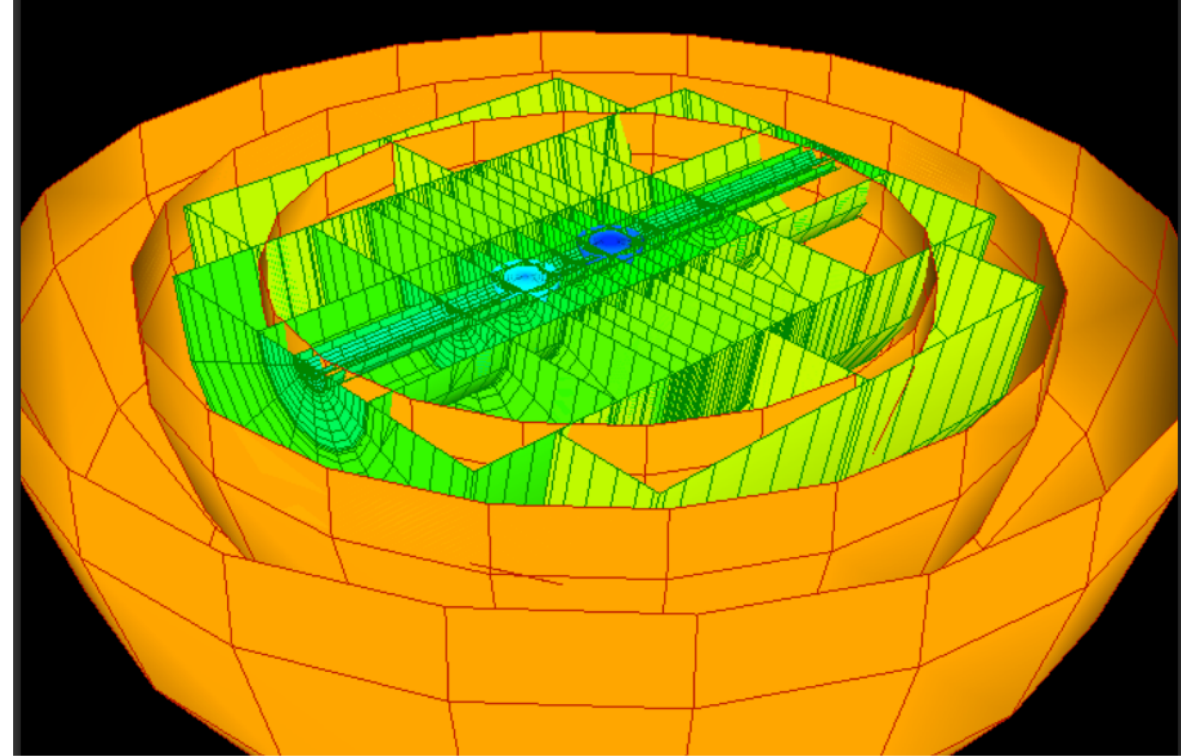


Image from the SXS Collaboration

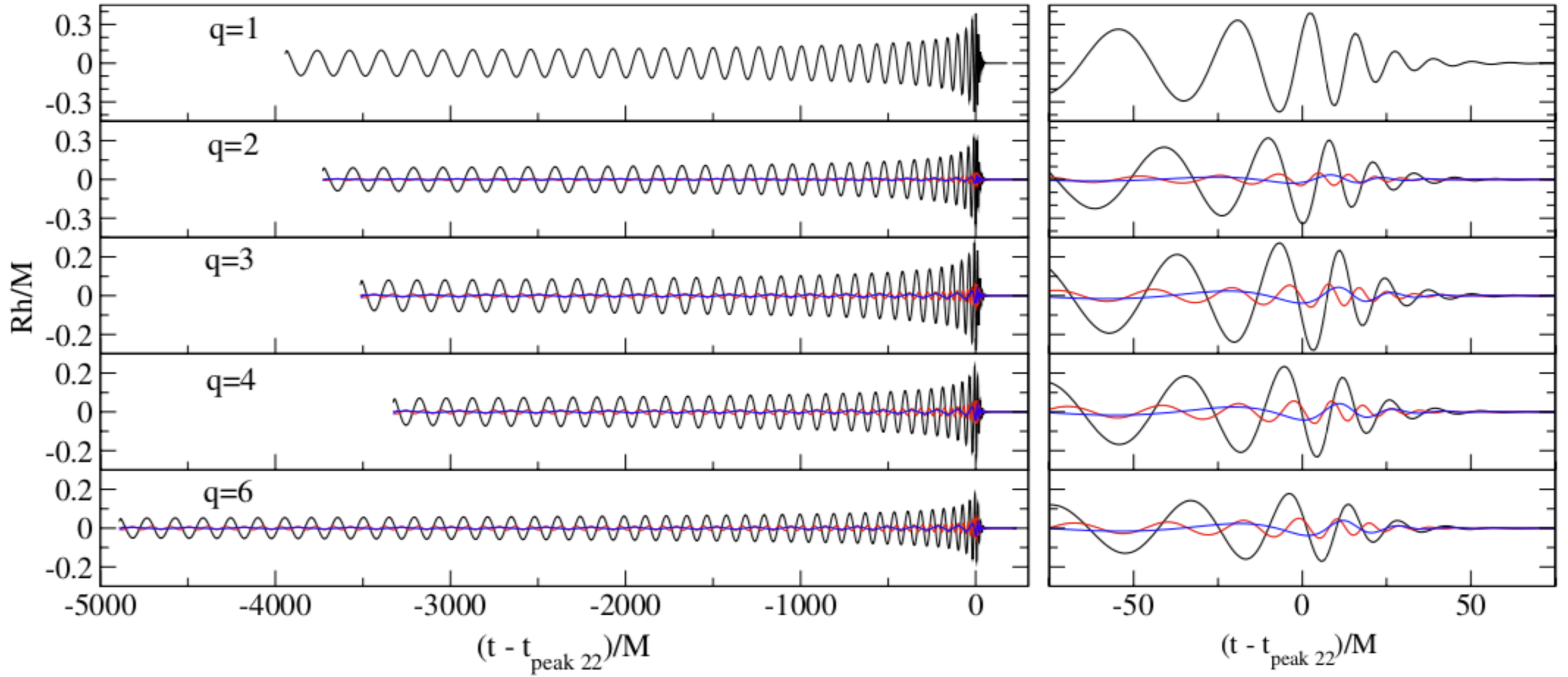
Boundary conditions (BCs) for the GWs

Angular momentum number (ℓ) of GWs

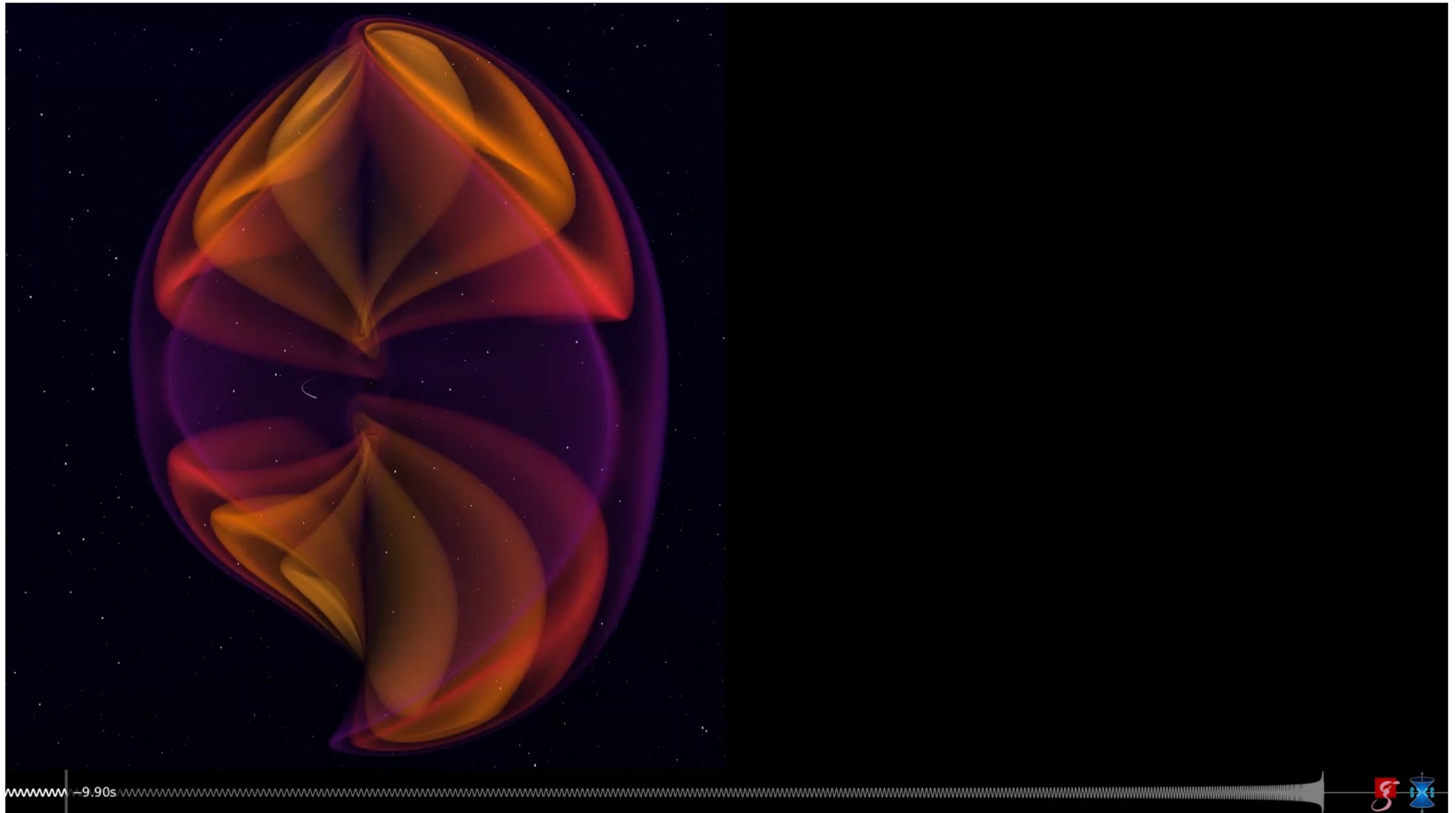
- $\ell = 2$ is the dominant mode
- $\ell = 3, 4, \dots$ are called the higher multipoles

Spurious reflections occur for higher multipolar GWs with freezing $\Psi_0^{\ell,m}$ BC

- Important for unequal mass BBHs
- GW190814



Simulation of GW190814 (SXS collaboration)



Improved Boundary conditions

Higher Order Boundary condition on $\Psi_0^{\ell,m}$ (Buchman and Sarbach 2006)

- Freezing $\Psi_0^{\ell,m}$ is the lowest order
- Higher orders reduce spurious reflections for $\ell = 3, 4, \dots$

SpEC implementation (Rinne et al 2009)

BCs reformulated:

- Now able to apply tricks developed in applied mathematics for numerical implementation

Rinne et al 2009

Successful implementation for multipolar wave initial data

- Evolved in the full SpEC code (non-linear, Einstein equations)
- This is a simple non-rotating computational grid with spherical shells

Goal now is for a successful implementation for Binary Black Hole inspiral and merger simulations

Towards BBH simulations

Adaptation of HOBCs to BBH infrastructure

- Dual frame *
- Rotating *

Repeating tests of Rinne et al in the new infrastructure

- Multipolar wave initial data ($\ell = 2, 3, 4$)
- Compare with exact solution
- Compare numerical and theoretical reflection coefficients

* Previously completed by Scheel and Buchman

BBH Simulations

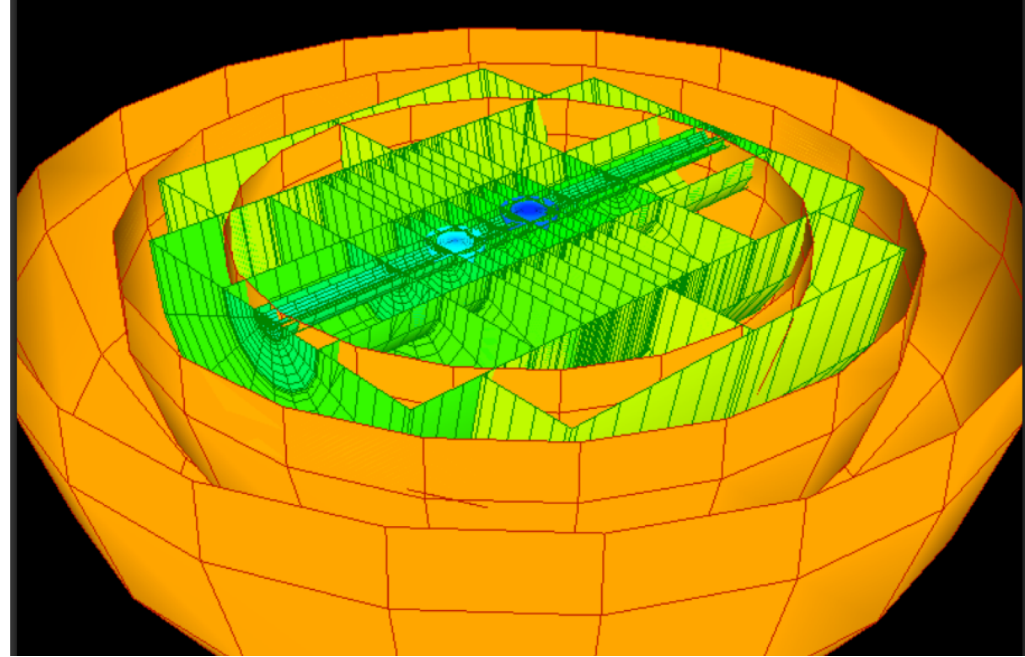
- Initial data of a BBH system
- Evolved in BBH infrastructure

Difficulties:

- Analytic solution is unknown -> no comparison can be done

Solutions:

- Calculate reflection coefficient (Buchman & Sarbach 2006)
- Compare to the numerical to the theoretical $\left(\frac{\Psi_0}{\Psi_4}\right)$



Next steps

- Testing of recent sign changes
- Multipolar wave tests (in dual frame rotating code)
- BBH coalescence tests

References

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5. Reference for SpEC code: <https://www.black-holes.org/SpEC.html>
6. Publicly available catalog of SXS waveforms available at <https://www.black-holes.org/waveforms>
7. LIGO Document G2000730-v3