An Investigation on the Effects of Non-Gaussian Noise Transients and Their Mitigations to Tests of General Relativity

Jack Y. L. Kwok*
Department of Physics, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong

Mentors: Alan J. Weinstein, Rico K. L. Lo
LIGO, California Institute of Technology, Pasadena, California 91125, USA
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I. INTRODUCTION

Current gravitational-wave detectors, such as Advanced LIGO [1] and Advanced Virgo [2], are interferometers with multi-kilometer-long arms. A beam-splitter is used to split incident laser beam into two equal parts, directing them into the two arms [3]. The splitted beams are made to travel several hundred round-trips within each arm in Fabry-Pérot cavities composed of highly reflective mirrors [3], increasing the average total distance traveled by photons in each arm to about 1000 km on average, optimal for the detection of gravitational waves generated by stellar-mass coalescing binaries [4].

Each mirror is suspended by a system of pendulums mounted on a seismic isolation platform [5]. A gravitational wave passing through the detector is expected to change the separation of mirrors in the two arms to different extents (in the detector frame), resulting in a phase difference between the two beams; this phase difference characterizes the light intensity of the recombined beam, which could be detected by a photodiode [3].

Aside from gravitational waves, the detected change in light intensity can be attributed to many independent sources of random noise [3]. If these noise sources are also *stationary*, i.e. their probability distributions do not change over time, the central limit theorem states that the total noise tends to be *Gaussian* when the number of noise sources is large [6]. As such, noise in gravitational-wave detectors is typically modeled to be Gaussian during data analysis [7, 8]. When signals are absent, the light intensity x received by the photodiode at each instance of time follows the Gaussian probability density

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2} \ . \tag{1}$$

Without loss of generality, the *mean* of the distribution is set to zero. Gaussian noise is thus solely characterized by the *variance* σ^2 of the distribution. Given a discrete time series $x(t_i)$ consisting of N equally-spaced samples $x(t_0), ..., x(t_{N-1})$ sampled at rate f_s, the variance can be measured in the frequency domain by computing the discrete Fourier Transform $\tilde{x}(f_i)$ of $x(t_i)$ and invoking Parseval's Theorem [9]:

$$\sigma^2 = \frac{1}{N} \sum_{i=0}^{N-1} x^2(t_i) = \sum_{k=0}^{N/2} \frac{2}{N^2} |\tilde{x}(f_k)|^2 \ . \tag{2}$$

Re-expressing Eq. (2) with the *frequency resolution* $\Delta f = f_s/N$ of the Discrete Fourier Transform [6],

$$\sigma^2 = \sum_{k=0}^{N/2} \left(\frac{2}{N f_s} |\tilde{x}(f_k)|^2 \right) \Delta f \ . \tag{3}$$

The bracketed quantity in Eq. (3) is called the *power spectral density* (PSD). In practice, noise characteristics of a data segment of interest is estimated using the PSD of adjacent data segments [8].

The underlying assumption of stationary noise cannot account for transient, non-Gaussian noise features in gravitational-wave detectors, commonly-known as *glitches* [10–12]. Three classes of commonly-seen glitches are shown in Figure 1.

Although rare, it is possible for glitches to be present in data segments containing gravitational-wave signals; this happened during the event GW170817, in which a glitch was found to overlap the signal in the LIGO-Livingston detector [15]. Furthermore, certain noise sources such as Schumann atmospheric resonances [16, 17] can theoretically produce glitches in multiple detectors almost simultaneously [12], making such glitches more difficult to identify.

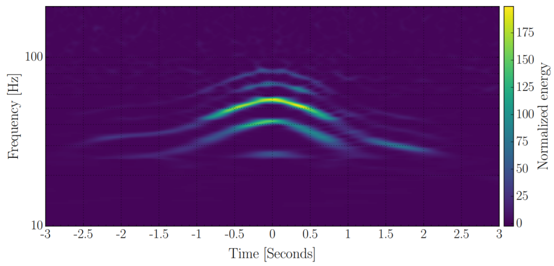
If the presence of glitches were not accounted for, one may infer from the detected waveform that a deviation from General Relativity (GR) has occurred. The extent to which glitches mimic the effects of a deviation of GR certainly deserve a systematic study.

II. OBJECTIVE AND METHODOLOGY

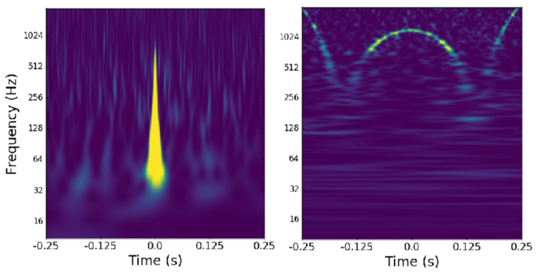
Our goal is to investigate the extent to which glitches mimic the effects of a deviation of GR, and evaluate the effectiveness of common glitch-related mitigation measures on tests of GR.

To this end, we first prepare a collection of data samples by injecting simulated signals generated by a GR waveform approximant onto data stretches containing

* Email: jackkwok@link.cuhk.edu.hk



(a) A light scattering glitch [13]



(b) A blip glitch [11] (c) A whistle glitch [11]

FIG. 1. Three normalized spectrograms of commonly-seen glitches. The colour represents the 'loudness' of the signal at each time-frequency bin [11]. (a) A light scattering glitch has a characteristic arch shape; it is caused by slight misalignments of the laser beam and the mirrors [13]. (b) A blip glitch has a characteristic 'teardrop' shape; its noise source has not been identified [14]. (c) A whistle glitch has a characteristic 'W' or 'V' shape; it is caused by radio signals generated by the interferometer control system [10].

glitches of varying signal-to-noise ratio (SNR) and morphologies; each glitch is made to overlap with the simulated signal at various stages of coalescence, including 1) inspiral, 2) intermediate and 3) merger-ringdown. Under the assumption of Gaussian noise, we perform a test of GR on each data sample before and after applying different mitigation measures in turn; the former test simulate the result we would get if the existence of glitches were not accounted for, whereas the latter tests determine the relative effectiveness of mitigation measures under different circumstances. In the following subsections, we will propose the mitigation measures, the waveform approximation to GR and the test of GR to be employed in our study.

A. Mitigation Measures

Many efforts are made to develop algorithms that identify glitches [18–21]; these algorithms play an important role in gravitational-wave searches. Once a glitch is identified, it could be eliminated either by hand or automatically by search pipelines [22, 23] through a process called *gating*, which zeroes out the time interval containing the glitch by multiplying the time series with an *inverse* window function [22, 23]. An example of gating is illustrated in Figure 2.

A similar procedure can be done in the frequency domain: if the glitch is localized in certain intervals of frequency, zeroing out the corresponding frequency bins would eliminate the glitch. These two procedures will henceforth be denoted as gating in the time and frequency domain respectively.

A more sophisticated approach would be to subtract off a glitch model from the original time series. This procedure, called *de-glitching*, was employed for the glitch-contaminated GW170817 data [15], as illustrated in Figure 2. The de-glitching procedure can be extended beyond well-modeled glitches using the `BayesWave` framework, which introduces a method to model glitches using wavelets and infer the most probable model using Bayesian statistics [24, 25].

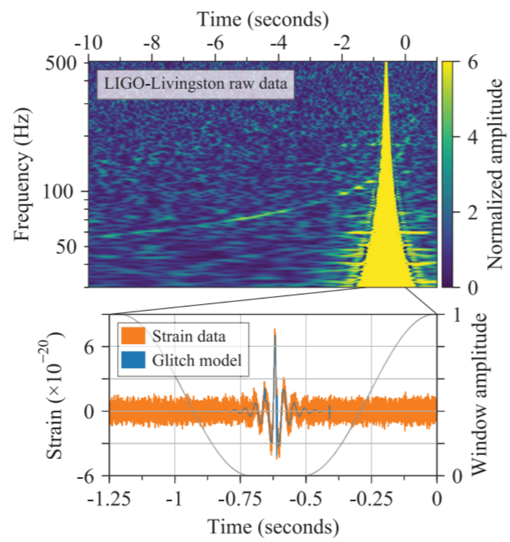


FIG. 2. The output data from the LIGO-Livingston detector during GW170817 is plotted over time in the bottom figure (orange curve). A glitch was identified around the time $t = -0.75$s to -0.5s in the figure. To determine the sky location of the event, data was gated in the time domain by multiplying the inverse window function (black curve) [15]. To infer the source properties during parameter estimation, data was de-glitched by subtracting off a glitch model (blue curve) reconstructed with `BayesWave` [15]. The upper figure shows a spectrogram of the raw LIGO-Livingston data. The figure is retrieved from Abbott et al. [15]

In our study, we will separately apply the three standard mitigation measures of 1) gating in time domain, 2) gating in frequency domain and 3) de-glitching to data samples.

B. Waveform Model

In GR, the two-body problem cannot be solved analytically, thus the orbital motion of coalescing binaries and the gravitational-wave signal they emit can only be computed *up to a precision*. Henceforth, an approximate waveform model predicted by GR is said to be GR-*consistent*.

In our study, we adopt the frequency-domain precessing inspiral-merger-ringdown waveform model `IMRPhenomPv2` [26–28], which is a "hybrid" waveform model [29] constructed by matching and combining post-Newtonian (PN) waveforms [30], applicable to early inspiral, with Numerical Relativity (NR) waveforms [31] used in merger and ringdown. We will use `IMRPhenomPv2` during injection and test of GR in virtue of its good match with Numerical Relativity waveforms [28] and low computational costs.

The phase of `IMRPhenomPv2` composes of terms with known frequency-dependence, thus can be parameterized by the coefficients of these terms; these coefficients, denoted as the *phase coefficients* p_i, can be categorized into three groups, depending on the stages of coalescence in which they predominantly assert their effect on [28, 32]: (i) *inspiral* PN coefficients $\{\varphi_0, ..., \varphi_5, \varphi_{5l}, \varphi_6, \varphi_{6l}, \varphi_7\}$ and phenomenological coefficients $\{\sigma_0, ..., \sigma_4\}$; (ii) the *intermediate* phenomenological coefficients $\{\beta_0, ..., \beta_3\}$; (iii) the *merger-ringdown* phenomenological and black hole perturbation theory coefficients $\{\alpha_0, ..., \alpha_5\}$.

C. Testing GR

In this project, we will focus on a parameterized test of GR, named as the Test Infrastructure for GEneral Relativity (TIGER) [33]. This test infrastructure is chosen in our study as it does not require an alternative theory of gravity to compare against; moreover, it hinges on the measurement of parameterizable deviations from a GR-consistent waveform model [33]. In our study, we will measure the *fractional* deviations δp_i of the `IMRPhenomPv2` phase coefficients p_i from their prescribed value of p_i^{GR} [34]:

$$p_i = p_i^{\mathrm{GR}}[1 + \delta p_i]\ . \tag{4}$$

In practice, we do not allow some of the `IMRPhenomPv2` phase coefficients to deviate from their prescribed value as they have large uncertainties or are degenerate with with other coefficients or physical parameters [32]. We therefore measure the fractional deviations δp_i , also known as the *dephasing coefficients*, of the remaining 13 phase coefficients [32]:

$$\begin{aligned}\{\delta p_i\} = \{&\delta\varphi_0, ..., \delta\varphi_4, \delta\varphi_{5l}, \delta\varphi_6, \delta\varphi_{6l}, \delta\varphi_7,\\ &\delta\beta_2, \delta\beta_3, \delta\alpha_2, \delta\alpha_3, \delta\alpha_4\}\ .\end{aligned} \tag{5}$$

To quantify a deviation from GR in the context of parameterized tests of GR, we can compare GR against a modified theory of gravity through hypothesis testing: We denote $\mathcal{H}_{\mathrm{GR}}$ as the hypothesis that the gravitational-wave signal has the exact functional form predicted by a GR-consistent waveform model with $\delta p_i = 0$; to test against this hypothesis, we denote $\mathcal{H}_{\mathrm{modGR}}$ as the hypothesis that $\boldsymbol{h}$ has the functional form predicted by a GR-consistent waveform but with at least one $\delta p_i \neq 0$.

It is evident that the two hypotheses $\mathcal{H}_{\mathrm{GR}}$ and $\mathcal{H}_{\mathrm{modGR}}$ are mutually exclusive. Given data $\boldsymbol{d}$ and prior information I, we prefer the hypothesis which is relatively more probable. To quantify this statement, we can define the *odds ratio*

$$O_{\mathrm{GR}}^{\mathrm{modGR}} \equiv \frac{P(\mathcal{H}_{\mathrm{modGR}}|\boldsymbol{d}, I)}{P(\mathcal{H}_{\mathrm{GR}}|\boldsymbol{d}, I)}\ . \tag{6}$$

If the odds ratio is much greater than one, we prefer the hypothesis $\mathcal{H}_{\mathrm{modGR}}$; if it is much less than one, we prefer $\mathcal{H}_{\mathrm{GR}}$. If the odds ratio is close to 1, then the current data is inconclusive [35]. By invoking Bayes' Theorem, the odds ratio can be rewritten as

$$O_{\mathrm{GR}}^{\mathrm{modGR}} = \frac{P(\mathcal{H}_{\mathrm{modGR}}|I)}{P(\mathcal{H}_{\mathrm{GR}}|I)} \times \frac{P(\boldsymbol{d}|\mathcal{H}_{\mathrm{modGR}}, I)}{P(\boldsymbol{d}|\mathcal{H}_{\mathrm{GR}}, I)}\ . \tag{7}$$

The second term on the R.H.S. of Eq. (7) is called the *Bayes factor*, which could be readily computed with the support of Bayesian inference libraries [7, 36].

III. TIMELINE

16 / 06 - 28 / 06:	Run TIGER on simulated data (without glitch)
29 / 06 - 12 / 07:	Study data **First Interim Report** Run TIGER on glitched data
13 / 07 - 26 / 07:	Visualize and summarize first batch of results
27 / 07 - 09 / 08:	**Second Interim Report** Visualize and summarize all results
10 / 08 - 21 / 08:	**Presentation, Final Report**

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