Do Binary Black Hole Merger Events Observed by LIGO and Virgo in their Third Observing Run Agree with Waveforms from General Relativity? A Study of Residuals.

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We present a study of the fitness of General Relativity-predicted waveform models to binary black hole signal data from LIGO and Virgo's third observing run (O3). The data series observed by LIGO is composed of a gravitational wave signal superimposed on instrumental noise. Our hypothesis is that, should a waveform template predicted by General Relativity be fitted to the merger signal and then subtracted from the data series, what should be left over is a residual made up of pure instrumental noise. Thus, our objective is to determine if GR waveform templates accurately model O3 binary black hole signal data by creating a digital signal processing script that will construct a best fit waveform template from event parameters, subtract the template from the event data series, and then run a variety of statistical tests to check the consistency between the residuals and instrumental noises.

I. INTRODUCTION AND BACKGROUND

First predicted by Albert Einstein in the year 1916, gravitational waves are the squeezing and stretching, or strain, of space over time. According to Einstein's Theory of General Relativity (GR), gravitational waves (GWs) are created through the acceleration of any object in space. However, only the acceleration of incredibly massive compact objects create gravitational waves strong enough to be detected by LIGO. The intense movement of these objects disturb space-time to such an extent that it ripples outward in all directions at the speed of light [1].

Though much theoretical work was done after 1916 to further elucidate the properties of gravitational waves such as Schwartzchild's solution to Einstein's field equations in 1916 (which describes black holes) [2] and Kerr's 1963 generalization of Schwartschild's solution to rotating black holes [3], gravitational waves were only experimentally discovered for the first time on September 14th, 2015 by LIGO detectors in Livingston, LA and Hanford, WA. [4]

The significant event GW150914, detected by Advanced LIGO in September 2015, was discovered to be a binary black hole merger from a distant galaxy. Thus, the spacetime "ripples" picked up by the LIGO detectors at Livingston and Hanford appeared to originate from the merging two stellar-mass black holes. Eventually, after investigating the GW150914 signal data by (1) using its parameters to match it to a General Relativity-derived waveform, (2) subtracting the GR waveform from the signal data, and (3) analyzing the residual waveform using a cross correlation technique across the Livingston, LA data and the Hanford, WA to ensure the residual waveform consisted of just noise, it was concluded that the data from the GW150914 event was consistent with gravitational wave behavior predicted by Einstein's theory of General Relativity [4].

Since this event, LIGO has detected over 60 new compact binary coalescence events, each emitting their own, unique gravitational waves. Our job is to now test how General Relativity holds up for not just a singular significant event, but many (with varying masses and spins). Doing so will give us important insight as to whether Einstein's predictions were entirely correct. If not, this may imply that GR is not the correct theory of gravity in the strong-field, highly dynamical regime explored by GWs from these events.

II. PROJECT OBJECTIVES

The purpose of this summer research was to test that the data gathered from binary black hole merger events are consistent with General Relativity. To test that these significant merger events behave as expected from General Relativity, actual merger data was compared to GR-predicted waveforms. This was done under the assumption that Einstein's Theory of General Relativity correctly describes the behavior of gravitational waves from environments of strong field, highly dynamic gravity. Thus, the GR waveforms were expected to be accurate models of the signal from the compact binary merger in the LIGO data [5].

This research will be successful if we are able to develop data analysis software capable of correctly distinguishing whether the data are consistent with a GR based model [5]. As a result of such success, we will be able to address two main outcomes. The first outcome would be that the waveforms predicted by General Relativity are accurate models of the data gathered from both the Livingston, LA and Hanford, WA LIGO detectors, thus implying that general relativity is the correct theory to describe the behavior of gravitational waves from environments of strong field, highly dynamic gravity. Or, instead, the GR-predicted waveforms are not accurate models for the GW data gathered from the merger events, which would suggest that General Relativity does not completely describe the behavior of the source of these gravitational waves. This could potentially be evidence for physics and astrophysics beyond General Relativity [5].

III. APPROACH AND METHODS

A Python digital signal processing software was written with the intent to subtract a best fit, parameterized GR waveform model from noisy data from the LIGO detectors. This will result in what is known as a residual. If these parametrized GR based binary black hole waveform models are an accurate representation of the signal in the data, then it is expected that the residuals will be consistent with detector noise only. This implies that the binary black hole merger event behaves as predicted by General Relativity [5]. A graphic detailing the components that go into creating the residual timeseries is shown below in Figure 1 for representative O3b event S200129m:

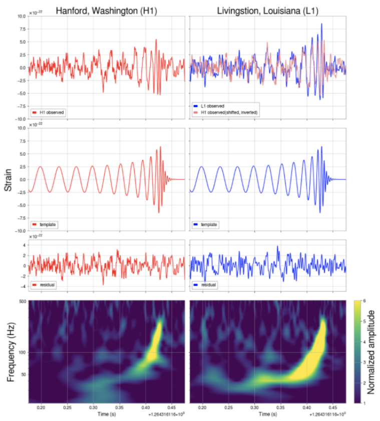

FIG. 1: Composition of residual components for representative event S200129m. The whitened and bandpassed signal strain is shown on the first row. The second row then displays the GR-derived template waveform created from parameter estimation samples for event S200129m. The third row then graphs the resulting residual, which should consist of instrumental noise if signal subtraction was successful. The last row then displays a q-transform of the event within the frequency domain at merger.

The GR waveform templates that were subtracted from the event data were created using parameter estimation (PE) samples (shown in Figure) that were generated using the IMRPhenomPv2 waveform family.[6] A function was then developed to create a best fit waveform using 8 event parameters: the masses of both black holes, the distance of the event from LIGO, the phase, the inclination, and the xyz spins of each black hole. The content of the residual waveform was then be analyzed for correlations to Gaussian noise and any signal remaining in the data.

A. Gaussianity Test

The strain data time series observed by Advanced LIGO can be assumed to consist of both a signal and instrumental noise. In the absence of transient detector disturbances (glitches), however, this noise is approximated to be Gaussian in nature. Thus, the purpose of the Gaussianity Test is to ensure that the residual is statistically similar to other parts of the data that do not contain a signal and consist of pure noise.

To prepare the data for statistical analysis, both the strain and the residual were normalized and whitened using a simple bandpass filter. The strain, residual, signal, and noise were then all time-sliced from the dataseries as shown in. the strain, residual, signal, and noise were then all fitted to a Gaussian function and compared. Such is seen below with significant O3b binary black hole event S200129m:

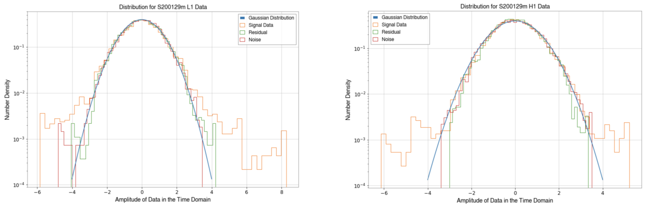

FIG. 2: Histograms of the data, residual, and noise time series for the event S200129m. A Gaussian distribution (with mean = 0 and standard deviation = 1, normalized to the data at the peak) is superimposed on these histograms to guide the eye to non-Gaussian outliers. The whitened event data is represented by the x-axis (in units of standard deviation of the detector noise), and the y-axis is logarithmic (so that the Gaussian does not have the familiar 'bell curve' shape). As expected for a loud BBH event, the signal is represented as outliers from the central Gaussian peak. However, the residual and noise from the event do not exhibit outliers in either detector. Thus it can be assumed that there is not remaining signal left in the residual of either detector, and therefore the GR-derived waveform template is a good description of the signal in the data.

As demonstrated in Figure 1, a signal is able to be seen clearly in the histogram of the data time series from S200129m. Event S200129m has the second loudest signal-to-noise ratio (SNR) of any BBH event observed by aLIGO. Since this event's signal is so prominent, we can use this to determine if any signal is left within the residual. Unlike the S200129m signal shown in orange in Figure 1, which has significant amounts of data outside of the Gaussian curve due to the sudden increase of power from the BBH merger, neither the residual (red) nor the noise (green) of the S200129m data stream lie outside of the Gaussian curve. Thus, we can assume that the residual does not contain any remaining signal.

Though the Gaussianity Test is decent for determining if event residuals consist of only noise, this method is truly only useful for events with loud signals. For events that may not be incredibly loud - which includes the majority of them - it becomes more difficult to determine how clean the event residual is, as some signal may be buried underneath the Gaussian curve. This is seen in a lower SNR O3 event S191109d in Fig. 2:

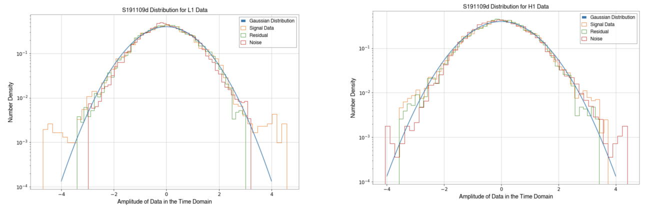

FIG. 3: Gaussian distribution of event S191109d in both Livington and Hanford detectors. Unlike event S20019m, event S191109d is much quieter, making it harder to differentiate the signal from the residual and the noise.

Thus, this led to the use of the Cross-Correlation test to analyze the residuals of quiet events such as S191109d.

B. Cross-Correlation Test

The purpose of the Cross-Correlation test is to ensure that no signal is left in an event residual by correlating the residuals from both the Hanford and Livingston detectors. This becomes incredibly important for testing the integrity of residuals from quieter events where excess signal can not easily be seen using a Gaussianity Test, or when the event data contains non-Gaussian fluctuations (glitches).

To determine whether or not the event residuals between the Livingston and Hanford detectors are correlated, a correlation coefficient was generated through use of the Pearson Cross Correlation statistic proposed by Liu and Jackson [7], shown below:

$$C(\tau; t, \omega) = \int_{t}^{t+\omega} \frac{H(t'+\tau)}{\sigma_H} \frac{L(t')}{\sigma_L} \mathrm{d}t'$$

FIG. 4: Pearson Correlation Coefficient statistic proposed by Liu and Jackson.[7]

Should the residuals from the Livingston and the Hanford detectors have a correlation coefficient that tends to 0, the residuals will be considered uncorrelated and only containing instrumental noise. However, if the noise is uncorrelated , there may still be a signal still left in the residual, implying that there may have been an error in creating the residual.

Additionally, due to the Hanford and Livingston detectors being located 3000 kilometers away from each other, there is a time delay in signal observation between the two detectors. Thus, this time delay was accounted for in the digital processing code by continuously shifting the data from Hanford while holding the Livingston data constant until the signal matched up in detection time in both detectors.

Cross Correlation tests for both S191109d and S200129m are shown below in Figures 4 and 5, respectively:

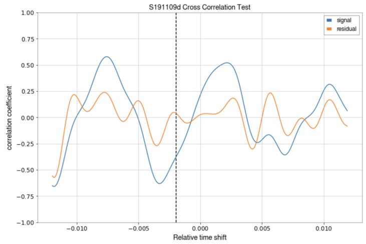

FIG. 5: Correlation test for event S191109d. Before correlation, the event data between Livingston and Hanford had a correlation coefficient of -0.45, implying the presence of a signal. After correlating the residuals of the event from both detectors, the correlation coefficient for event S191109d became very close to 0, implying that a majority of the signal had been accurately subtracted by the best-fit waveform template.

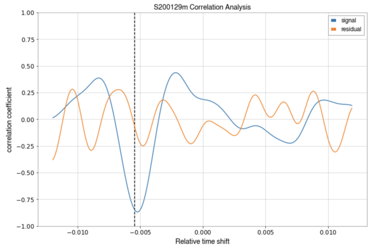

FIG. 6: Correlation test for event S200129m. Before correlation, the event data between Livingston and Hanford had a correlation coefficient of -0.8. After correlating the residuals of the event from both detectors, the correlation coefficient for event S200129m became much closer to 0.

Through cross-correlation, we were able to infer that our digital processing code successfully created a 'clean' residual of these two events.

IV. CONCLUSION

The focus of this past summer's research was to determine whether or not the signals observed by Advanced LIGO are consistent with the waveforms derived from General Relativity. To accomplish this, a digital processing program was written in order to subtract a GR-derived best-fit waveform from events to create residuals.

The residuals were then statistically analyzed using both a Gaussianity Test and a Cross Correlation Test to ensure no signal was left in the residual.

At the moment, two O3b event residuals have been successfully created and analyzed. However, there still remains over 60 events within LIGO's third observing run that still need to undergo residual analysis. Short term goals for this project will be editing the digital processing code so that it can analyze multiple events in one script, thus greatly shortening the residual analysis process. After this is completed, residual analysis will then be conducted on all 80 observed LIGO events from observing runs GWTC-1, GWTC-2, and GWTC-3. This extended study may find discrepancies requiring further investigation. We may even find evidence for template waveforms that lack relevant physics or even failure of GR in the strong-field, highly dynamical regime that LIGO is probing. Additionally, future runs with higher detector sensitivity are expected to detect events with high SNRs greater than 20 (and possibly ranging up to even 100 or more). Studying the residuals for these events will allow us to test the accuracy of our GR-derived waveform templates, and thereby test General Relativity to even higher precision.

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