

# Studying Gravitational Waves from Exceptional Binary Black Hole Merger Events

## LIGO SURF 2020 First Interim Report

Jennifer Sanchez

*Physics Department, California State University, Fullerton*

Mentors: Dr. Alan Weinstein and Dr. Colm Talbot

*LIGO Laboratory, Physics Department, California Institute of Technology*

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Advanced LIGO and Advanced Virgo have announced 66 candidate gravitational wave signals. Each observation contains encoded information about the physical properties of the binary system. As the detectors continue to improve their sensitivity, these developments will allow us to detect rarer systems and make more confident statements regarding their source properties. In order to fully characterize the gravitational wave observations, we rely on numerical and analytical models that approximate the signal waveforms from the emitted source as specified by the source parameters (masses, spins, sky location, etc). The dominant emission frequency of gravitational waves from compact binary coalescences is at twice the orbital frequency; however, recently published events has demonstrated subdominant higher order harmonic contributions. The primary focus of this summer project is to explore higher order modes in gravitational wave signals with newly improved signal models.

### I. INTRODUCTION

Black hole binaries, along with other compact binaries, are responsible for the gravitational waves that have been detected by the LIGO Scientific Collaboration [1] and Virgo [2]. Since 2015, the year in which the first gravitational wave was detected, there have been nine additionally confirmed gravitational waves from binary black hole mergers, 56 candidate detections of gravitational wave signals, and one confirmed detection from a binary neutron star merger [3]. The current operational detectors are only able to observe the closest and loudest sources that exist in the local universe. Now that gravitational waves can be detected and analyzed, this is only the beginning for gravitational wave science.

During the third observing run (O3) of Advanced LIGO and Advanced Virgo, all three interferometers operated at unprecedented sensitivity [4]. With the observations of 66 gravitational wave signals, the need for models of the emitted sources is essential [5]. Observations made during the third observing run have provided the first evidence for binaries with unequal mass and misaligned spins [6].

Parameters describing the merging binaries are inferred using Bayesian inference by comparing waveform templates ("approximants") to the data. Earlier parameter waveform families did not include higher order modes; however, the recent observations require waveform models including higher-order emission modes. Not only will these models provide us with information regarding the inspiral, merger, and ringdown of the source, but will also allow tests of general relativity [7], measurements of the underlying population of compact objects [8] and the expansion history of the Universe [9].

In this report, we will be discussing the properties of the events called GW190412 and GW190814 which was observed by both Advanced LIGO and Advanced Virgo detectors [6] [10]. Their signal waveform included its signature dominant quadrupole radiation; however, it contained *detectable* higher harmonics which will provide us with a greater insight to the dynamics of coalescing binary black holes. The focus of this study this summer will be on events like GW190412 and GW190814.

### II. METHODS

The first three weeks were spent on learning how to use LIGO and Virgo strain data along with their affiliated software libraries. The programming exercises provided by the GW Open Data Workshop #3 [11] have allowed me to utilize the gravitational wave data as well as their software tools. Upon completing all of the tutorials, I began to learn how to use Bilby [12], a Bayesian inference library for gravitational-wave astronomy, through their tutorials. Through the Bilby tutorials, I was able to run parameter estimations on different astrophysical systems. My last week was spent learning the dynamics of surrogate modeling, and how to use the newest surrogate model, NRSur7dq4 (Numerical Relativity Surrogate with 7-dimensions  $q \leq 4$  where  $q$  represents the mass ratio).

#### A. Source Properties

The source properties we have been studying prior to O3 included: masses ( $m_1$  and  $m_2$ ), spins of the black

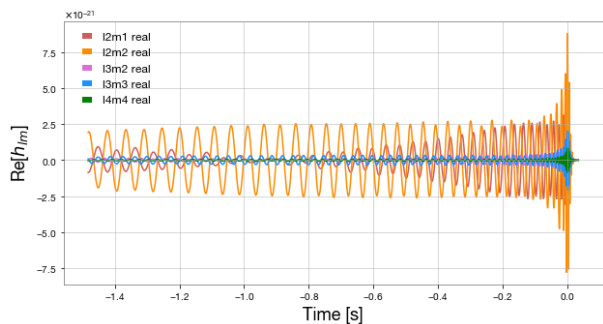


FIG. 1. The real part of the (2,1), (2,2), (3,2), (3,3), and (4,4) modes of the NRSur7dq4 surrogate model waveform. To produce this figure, all waveforms used a  $q=4$ ,  $\chi_1=[-0.2, 0.4, 0.1]$ , and  $\chi_2=[-0.5, 0.2, -0.4]$ . Each waveform has been time shifted so its total amplitude appears at  $t = 0$ .

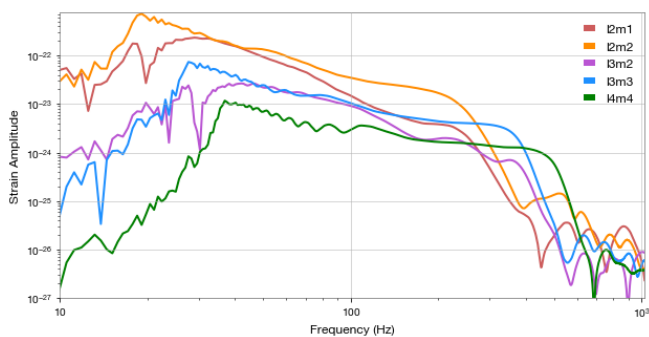


FIG. 2. The (2,1), (2,2), (3,2), (3,3), and (4,4) modes of the NRSur7dq4 waveform in the frequency domain.

holes in the binary ( $\vec{S}_1$  and  $\vec{S}_2$ ), the sky location, luminosity distance ( $D_L$ ), binary orbital orientation, time ( $t_c$ ) and phase ( $\phi_c$ ) of coalescence. All of these properties comprise to a total of 15 parameters [13]. Specifying these parameters allows one to predict the signal waveform that may be present in the detector network data.

Gravitational waves are transverse waves that have two independent polarization states denoted as  $h_+$  and  $h_\times$ , where  $h_\times$  has its principle axis rotated  $45^\circ$  with respect to  $h_+$  [14]. With these two polarizations at hand, we can write the strain of the gravitational wave as the complex quantity  $h = h_+ - ih_\times$  [15]. According to the works of pioneering scientists, another fundamental property of gravitational waves from compact binaries is that they are quadrupolar. While gravitational radiation often includes the lowest order radiation term, the quadrupole radiation, there are predictions that include higher multipoles [16]. However, higher multipoles, which are terms above the dominant, quadrupolar term, are particularly complex to infer from a gravitational wave signal produced by near equal mass binaries [6].

Recently, signal waveform families have been extended to include higher order modes. The dominant mode is  $(l, m) = (2, 2)$  described by a spin-weighted spherical harmonic,  ${}_{-2}Y_{lm}(\theta, \phi)$ , where  $\theta$  and  $\phi$  are the emission

angles toward the observer relative to the orbital angular momentum,  $\ell$ . We can describe the multipolar decomposition as:

$$h_+ - ih_\times = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} \frac{h(t, \lambda)}{D_L} {}_{-2}Y_{\ell m}(\theta, \phi) \quad (1)$$

where  $t$  denotes the time,  $\lambda$  represents the intrinsic parameters such as the black hole's masses and spins, and  $D_L$  represents the luminosity distance from the observer [3]. Lastly, we recognize that,  $h = h_+ - ih_\times$  is in the form of a complex sinusoidal,  $\cos(\omega t) - i\sin(\omega t)$  or similarly  $e^{-i\omega t}$ .

The event's mass ratio,  $q$ , is an important factor to consider since it affects the geometry of the source. General relativity predicts that as asymmetric systems increase, so does the importance of higher multipoles.

Another contributor to higher multipoles is the orientation of the source. Higher order multipoles are more prominent when  $\theta$  is divergent from 0 or  $\pi$ , also said to be "face on." As a result to the degeneracy between  $\theta$  and  $D_L$ , higher multipoles allow us to break this degeneracy

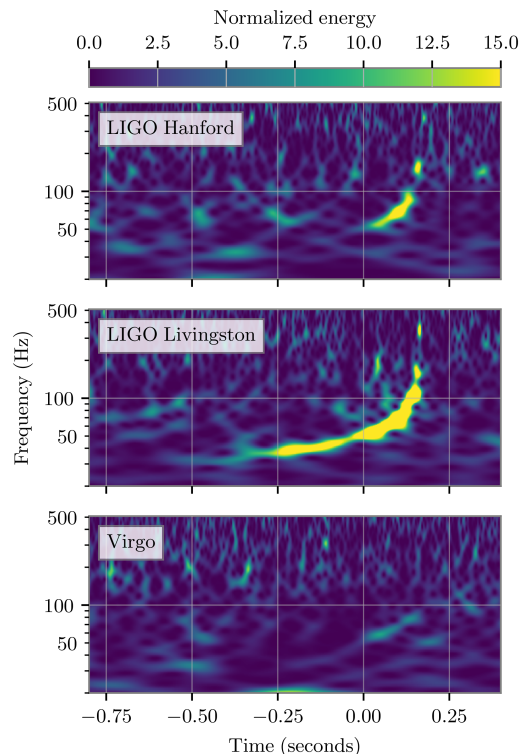


FIG. 3. Spectrogram of GW190412 in LIGO Hanford (top), LIGO Livingston (middle), and Virgo (bottom). The x-axis represents time while the y-axis represents the frequency of the gravitational wave signal. In this time-frequency representation, we show the energy given a specific frequency at a specific time. The increase in frequency and energy over time is a product of the inspiraling black holes followed by their merger.

which allows us to tighten constraints on the two [6].

## B. Higher multipoles

It is critical to have theoretical models of a gravitational wave signal agree with the measurements made from Advanced LIGO and Virgo. Typically, these models include the dominant  $(l, m) = (2, 2)$  mode, however this is not always the case. When the sources have a mass ratio close to one, these models are sufficient to analyze the source parameters of the system, such as distance and inclination. However, when the black hole binary contains an unequal mass ratio, subdominant multipole models may be remarkably more accurate.

Shown in Fig. 3, we can see the strain data, the fractional change in displacement between two nearby masses due to the gravitational wave, taken by the three detectors. The time-frequency representation of GW190412 shows the loudness of the event in each detector. Despite the fact that each detector is thousands of kilometers away from one another, we can definitively see that the signal is present in all three which implies that the source has an astrophysical origin rather than local detector noise [6].

What makes this event so unique is the notable asymmetry of the black hole masses, that is – one of the black hole’s mass is roughly three times heavier than the other [6]. This makes GW190412 a favorable system for identifying the presence of higher order modes. Previously, nearly all detected binary systems were consistent with having net aligned spin; however we found that GW190412 has significant net positive aligned spin

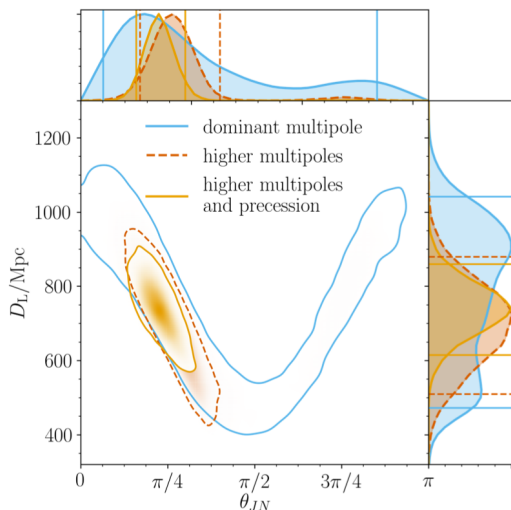


FIG. 4. Posterior distribution of GW190412 luminosity distance ( $D_L$ ) and inclination ( $\theta$ ). We see that by using models that include the dominant multipole, higher multipoles, and higher multipoles and precession, we are able to constrain  $D_L$  and  $\theta$ . Degeneracy breaks when higher order modes are included which allows us to improve our accuracy.

$\chi_{eff}$  [6]. On account of the unequal mass ratio, we can apply a constraint on the spin of the larger black hole. Making use of higher order modes in the model waveforms will allow us to make more precise measurements of all the parameters including the distance and inclination of the system, also shown in Fig. 4. General relativity predictions are precise predictions for the content and strength of higher order multipoles. This summer, I will be investigating the degree to which the constraint predictions do not accurately describe the data.

This is the first time that waveform models incorporating higher order modes from numerical relativity have been applied to an observed gravitational wave signal [6]. This summer, I will be applying these improved signal models to other events observed in O3.

## C. Gravitational waveform modelling

The three waveform families currently used to accurately model gravitational waves for binary systems are the phenomenological waveforms, effective-one-body waveforms, and numerical relativity surrogates. The phenomenological and effective-one-body waveforms model the inspiral, merger, and ringdown of the gravitational wave signal [17]; however, the numerical relativity surrogates are typically limited before the merger and ringdown [13].

## D. Phenomenological models

The phenomenological waveforms focus on modeling the inspiral, merger, and ringdown (IMR) of a gravitational wave signal without providing equations of motions for the black hole dynamics [17]. The phenomenological waveform model is typically created in the frequency domain which makes it computationally quick to evaluate for data analysis applications [17]. The newest edition of the phenomenological waveform models are IMRPhenomPv3HM and IMRPhenomXPHM. IMRPhenomPv3HM and IMRPhenomXPHM tracks precession and includes higher modes [17].

## E. Effective-one-body

Effective-one-body (EOB) waveform models have calibrated to numerical relativity simulations as well as waveforms from black hole perturbation theory [17]. These waveform models have been utilized to construct frequency domain, phenomenological models for the inspiral, merger and ringdown of a binary black hole coalescence. The newest EOB model, SEOBNRv4PHM, includes both spin precession and higher modes [17]. The numerical relativity waveforms which have been used to calibrate EOB waveforms included mass ratios  $q = 1 - 4$  and dimensionless  $\chi_{1,2} = 0.3 - 0.9$  [17].

## F. Numerical relativity

Numerical relativity simulations are able to accurately model the complex morphology of binary black hole mergers; however, the simulations are computationally expensive. Numerical relativity based surrogates are an alternative approach as they are substitute models for the outputs of these simulations. These models use numerical relativity simulated waveforms to reconstruct underlying phenomenology [13]. The newest numerical relativity surrogate mode, NRSur7dq4 begins  $\sim 20$  orbits before merger and includes all  $\ell \leq 4$  spin-weighted spherical harmonic modes [13]. The seven parameters in the precessing binary black hole system are the mass ratio ( $q = m_1/m_2$ ) where  $m_1 \leq m_2$  and the two spin vectors  $\chi_1$  and  $\chi_2$  where  $\chi_1$  represents the heavier black hole [13]. To generate a waveform in the time domain using the NRSur7dq4 model, parameters are required to calculate the waveform. These inputs include, the system's mass ratio, dimensionless spin vector of the two black holes, total mass of the total black hole, maximum  $\ell$  index for modes to use, reference frequency, initial frequency, distance, and step size.

## G. Phenomenology of Higher-Order Modes

To explore the detectability of higher order modes in varying parameter space, in Fig.1, I show the real part of the  $(\ell, m) = (2,1), (2,2), (3,2), (3,3),$  and  $(4,4)$  mode in the time-domain. Fig. 1 was generated using the NRSur7dq4 model with parameters located in Table. I. We see that in both the time and frequency-domain, the  $(2,2)$  mode has the highest amplitude while the features in the  $(3,3)$  mode occur at 1.5 times the frequency as for the  $(2,2)$  mode and the  $(4,4)$  mode had the lowest amplitude.

To analyze gravitational wave signals, it is often helpful to convert the waveform to the frequency-domain. This can be accomplished using a fast Fourier transform. In order to mitigate numerical artefacts, I apply a Tukey window function to the time-domain data before performing the fast Fourier transform. This windowing function reduces the effects of spectral leakage and Gibbs phenomena [18].

TABLE I. Parameters used to evaluate all waveforms

| $q$ | $\chi_1$         | $\chi_2$ ,        | $M_\odot$ | $d$ (Mpc) | $\ell_{max}$ |
|-----|------------------|-------------------|-----------|-----------|--------------|
| 4   | [-0.2, 0.4, 0.1] | [-0.5, 0.2, -0.4] | 70        | 100       | 4            |

Fig.2 shows that as the mode of the waveform increases, the amplitude decreases. The ‘wiggles’ located at lower and higher frequencies are likely an artefact of signal processing and Gibbs phenomenon which could

be further reduced with a more careful choice of window function. Looking at the orange line representing the  $(2,2)$  mode, the inspiral occurs between the minimum simulated frequency, 20 to  $\sim 200$ Hz. The plunge and merger then appears at 200-250Hz where the curve flattens. Lastly, the steep decrease in amplitude is the ringdown. We also note that the frequency behaves accordingly to  $m$  which means  $m = 4$  is one and a half the frequency of  $m = 3$  and twice the frequency of  $m = 2$ . Taking a closer look at the modes reveals that the  $(2,1)$  and  $(2,2)$  mode have some equivalent frequencies, and the same goes for the  $(3,2)$  and  $(3,3)$  mode. This is due to the precession of the orbital plane which is caused by the spins misalignment with the orbital angular momentum. The occasional matching frequencies for the modes can also be seen in the time-domain (Fig. 1).

## H. Comparison of higher-mode models

Fig. 5 shows a comparison of the GW190814 waveforms computed using SEOBNRv4PHM, IMRPhenomPv3HM, and IMRPhenomXPHM. All three surrogate models used show a reasonable agreement, especially IMRPhenomPv3HM and IMRPhenomXPHM during the merger and ringdown. SEOBNRv4PHM has a noticeably greater deviation from the other two surrogate models. The models in Fig. 5 and Fig. 6 use all the available modes for SEOBNRv4PHM, IMRPhenomPv3HM, and IMRPhenomXPHM. SEOBNRv4PHM and IMRPhenomXPHM models all modes  $\ell \leq 5$  while IMRPhenomPv3HM models all modes  $\ell \leq 4$  [13][17]. An attempt to evaluate the GW190814 waveform using NRSur7dq4 was attempted; however, the domain error suggests that NRSur7dq4 may not be used due to its mass ratio parameter restriction.

Computing the waveform for GW190412 using SEOBNRv4PHM, IMRPhenomPv3HM, and IMRPhenomXPHM shows and even stronger agreement in all three waveforms (see Fig. 6). IMRPhenomPv3HM and IMRPhenomXPHM are nearly identical throughout the inspiral, merger, and ringdown.

## III. FUTURE WORK

I will continue to use NRSur7dq4 to investigate the characteristics of higher order modes using distinct regions of parameter space. Once I have familiarized myself with NRSur7dq4, I will be applying this model to other unique events in O3. I also plan to use Bilby to analyze simulated signals to compare the performance of different waveform models.

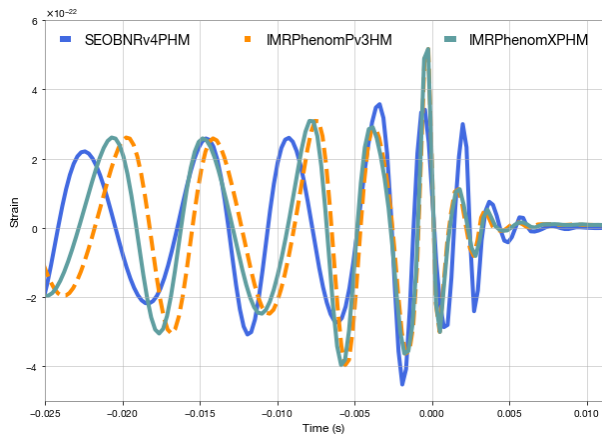


FIG. 5. Time-domain comparison of GW190814 using SEOBNRv4PHM, IMRPhenomPv3HM, and IMRPhenomXPHM with mass ratio  $q = 0.11$ , total mass  $M = 25.6M_{\odot}$ , distance  $d = 196\text{Mpc}$ , and  $f_{start} = 30\text{Hz}$ .

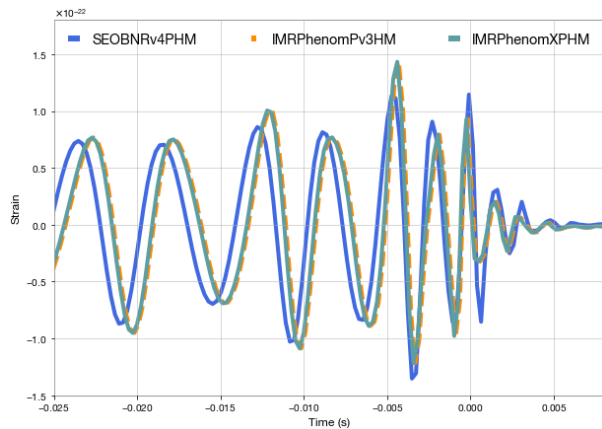


FIG. 6. Time-domain comparison of GW190412 using SEOBNRv4PHM, IMRPhenomPv3HM, and IMRPhenomXPHM with mass ratio  $q = 0.21$ , total mass  $M = 33.1M_{\odot}$ , distance  $d = 736\text{Mpc}$ , and  $f_{start} = 30\text{Hz}$ .

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