Minimising the Effect of Mirror Perturbations on Quantum Decoherence

LIGO SURF 2020

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Outline

Introduction

Motivation

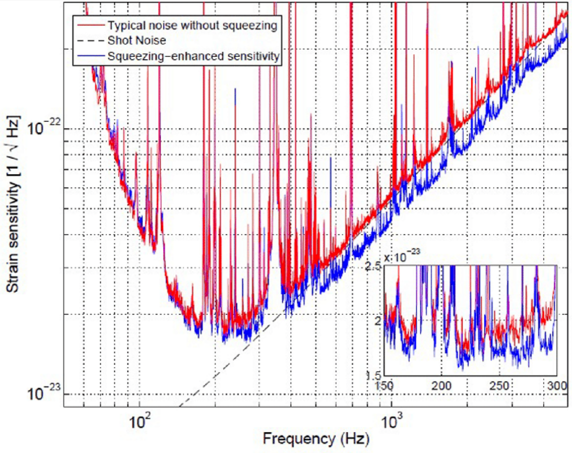

Figure 1: The strain sensitivity of the LIGO detectors is currently limited above approximately 200 Hz by quantum noise.[1]

[1] Publicly available image at https://www.ligo.caltech.edu/

Squeezed states of light: our saviour

$\Delta A \, \Delta\phi \geq \frac{\hbar}{2} \rightarrow$ Unevenly distributed

3 dB $\uparrow \implies 3 \times$ Event Rate!

Limitations

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What causes these mode-mismatches?

Perturbation in apparatus (R = 5.932418 m?!)

The Project

Can we find an optimal set of design parameters such that the interferometer becomes minimally sensitive to design perturbations?

Perturbations $\rightarrow$ curvatures and positions of the optical elements

Background

Laser beam $\rightarrow$ Spatial intensity distribution

$$E(t,x,y,z) = \sum_{j}\sum_{n,m} a_{jnm}\, u_{nm}(x,y,z)\, exp(i(\omega_j t - k_j z)) \qquad (1)$$

$u_{nm}(x,y,z) \rightarrow$ set of Hermite-Gauss (HG) or Laguerre-Gauss (LG) polynomials

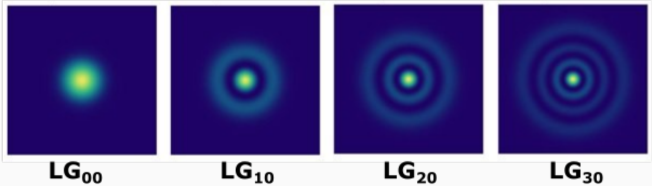


Figure 2: u_{nm} spatial distributions in the LG basis

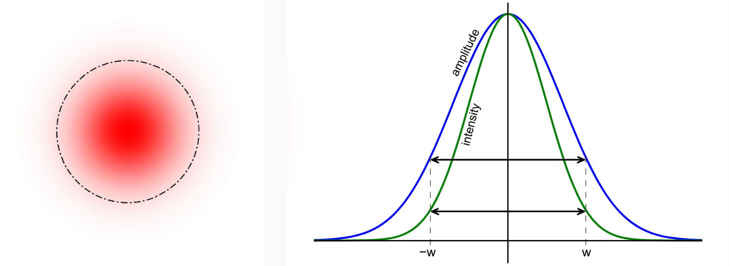


Figure 3: Intensity pattern of a Gaussian beam (left) and the intensity and amplitude distributions of a normalised Gaussian beam (right).[3]

Cavity Eigenmode and Mode-Mismatch

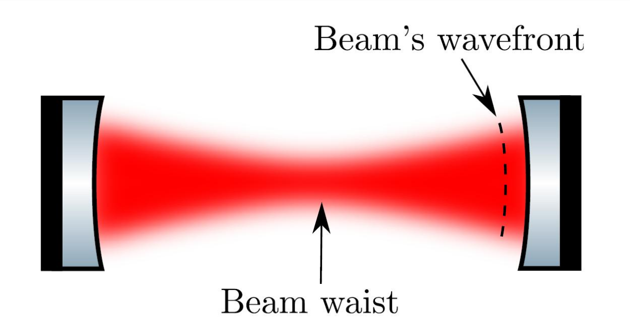


Figure 4: Cavity eigenmode: The beam curvature must be equal to the curvature of the mirrors at the mirror positions.[3]

Goals

Goals

- Algorithm for numerically optimising any optical setup
- Implement algorithm to optimise the aLIGO design
- Analytic formalism to calculate loss

Analytic Formalism

Aim

To develop an analytic formalism to calculate the total mode-matching loss in a complex optical system as a function of small perturbations of the optic positions and curvatures.

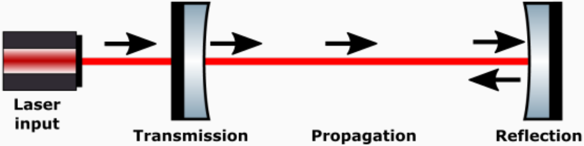
Optical Field Vector

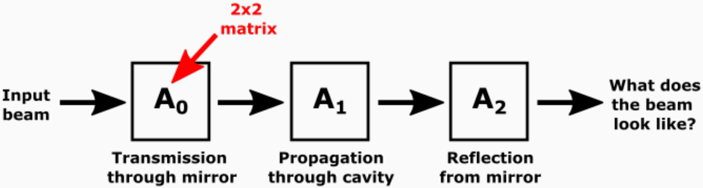
Small curvature and position perturbations: $LG_{00} \leftrightarrow LG_{10}$[4, 1]

Electric field at any point

$$|\Psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha\,|LG_{00}\rangle + \beta\,|LG_{10}\rangle \tag{2}$$

Mode-Mixing Matrices





$$|\Psi_{\mathrm{out}}\rangle = A_2 \times A_1 \times A_0 \times |\Psi_{\mathrm{in}}\rangle \quad (3)$$

Formalism and Simulations: Do They Agree?

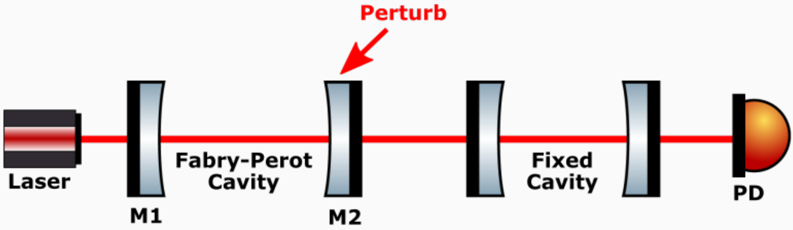


Figure 5: Fabry-Perot cavity before the OMC.

Formalism and Simulations: Do They Agree?

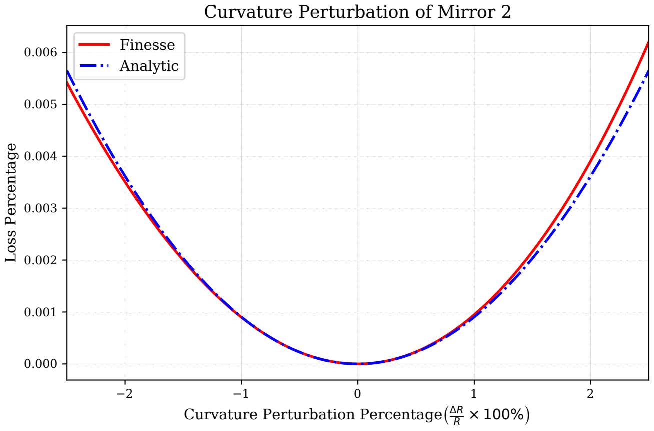


Figure 6: Analytic and simulation results for power loss percentage as a function of curvature perturbation percentage at M2.

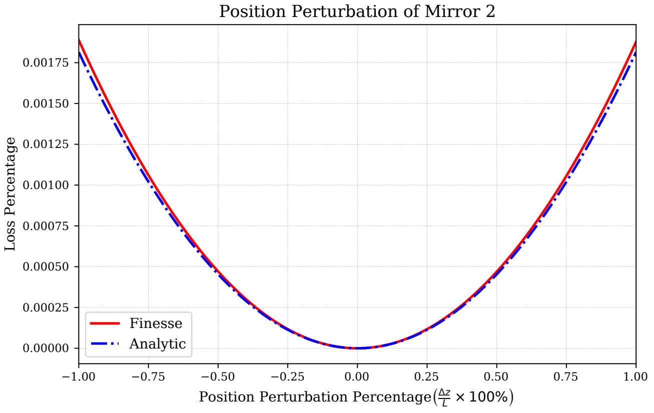


Figure 7: Analytic and simulation results for power loss percentage as a function of position perturbation percentage at M2.

Numerical Optimisation of the aLIGO System

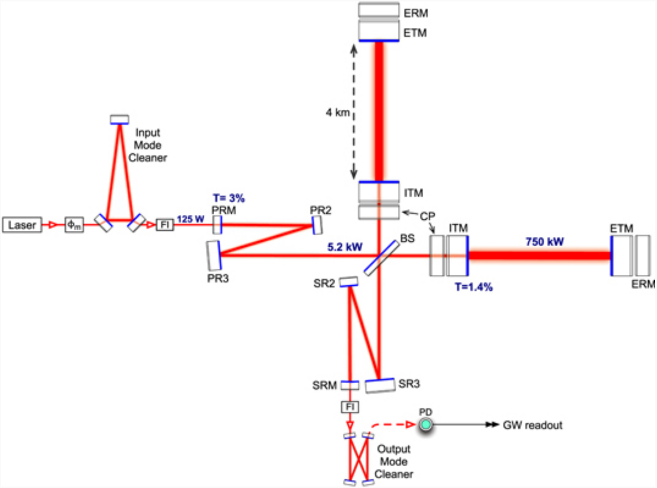


Figure 8: aLIGO setup

Optimising the Signal Recycling Cavity

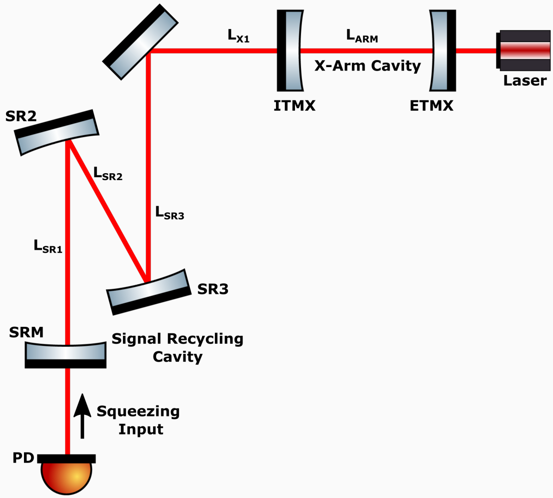


Figure 9: aLIGO X-arm cavity and Signal Recycling Cavity (SRC)

Optimisation Routine - Particle Swarm Optimisation

Ensure the setup is mode-matched

$\downarrow$

Ensure beam-size remains small ($<$ 1cm) and total length of the SRC remains fixed

$\downarrow$

Perturb curvature and position of SRC mirrors and observe degradation in squeezing level[2]

$\downarrow$

Minimise this degradation

[2] Laser and squeezer inputs remain mode-matched to the unperturbed cavity

Monte Carlo Analysis for the Expected Squeezing Level

- $\Delta R \sim N(\mu = 0, \sigma = 0.01R)$ and $\Delta z \sim N(\mu = 0, \sigma = 3mm)$, where $\sim N$ indicates a normal distribution.
- Repeat 1,000 times
- Plot probability distribution of the squeezing level
- Compute the 85^{th} percentile

Expected Squeezing Level for aLIGO SRC

Effective squeezing level of unperturbed cavity = 9.8dB

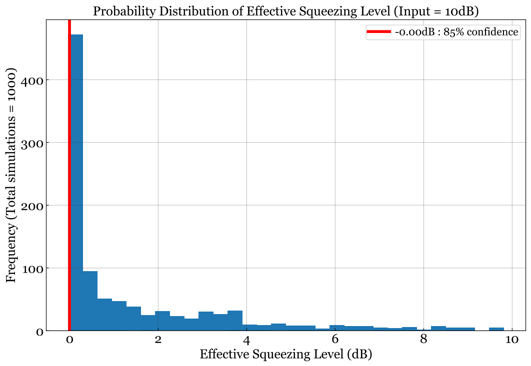


Figure 10: Monte Carlo analysis for perturbed cavity

	R_{SR3}	R_{SR2}	R_{SRM}	L_{SR3}	L_{SR2}	L_{SR1}
aLIGO (m)	35.97	-6.41	-5.69	19.37	15.44	15.76

Pertrubation Analysis for R_{SRC}

Effective squeezing level of unperturbed cavity = 9.8dB

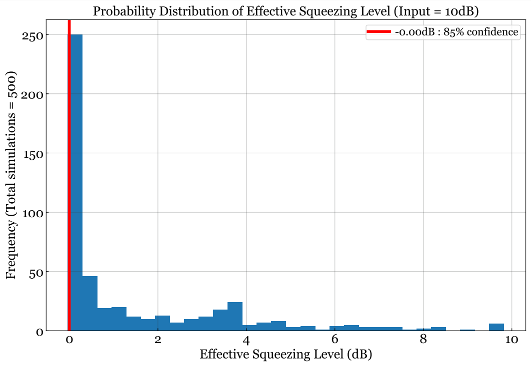


Figure 11: Perturbing only the R_{SRC}

	R_{SR3}	R_{SR2}	R_{SRM}	L_{SR3}	L_{SR2}	L_{SR1}
aLIGO (m)	35.97	-6.41	-5.69	19.37	15.44	15.76

Expected Squeezing Level for Optimised SRC

Effective squeezing level of unperturbed cavity = 9.7dB

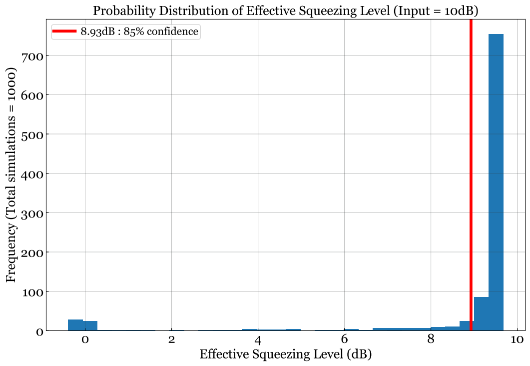


Figure 12: Monte Carlo analysis for perturbed cavity

	R_{SR3}	R_{SR2}	R_{SRM}	L_{SR3}	L_{SR2}	L_{SR1}
Optimised (m)	59.66	-233.60	-3.53	27.69	18.98	9.87

Conclusion

Conclusion

- Developed algorithm to minimise sensitivity of any optical setup.
- Optimised the aLIGO Signal Recycling Cavity
- Quantified the improvement.
- Developed analytic formalism.
- Checked agreement between formalism and simulations.

Next Steps

Further Work

The next steps for this project are:

- Apply to the entire aLIGO setup.
- Take into account the LIGO thermal compensation system.

Thank You

My sincere thanks and gratitude to my mentors and to LIGO Laboratory for giving me this opportunity.

Questions?

Reference Slides

Helmholtz equation:

$$\nabla^2 \mathsf{E} - \frac{\ddot{\mathsf{E}}}{c^2} = 0 \tag{4}$$

The fundamental Gaussian mode at a position z:

$$u_{00}(r,z) = \sqrt{\frac{2}{\pi}} \frac{1}{\omega(z)} \exp(\iota\psi(z)) \exp\left[-r^2 \left(\frac{1}{w^2(z)} + \iota \frac{\pi}{\lambda R(z)}\right)\right] \tag{5}$$

Reflection Matrix

Reflection from a mirror having amplitude reflectivity coefficient r

$$\mathbf{r} = \begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix} \tag{6}$$

Modified reflection matrix for a small curvature perturbation δR in a mirror of curvature R

$$\mathbf{r}' = \mathbf{a}.\mathbf{r} = \begin{pmatrix} r\sqrt{1-a^2} & -\iota r a \\ -\iota r a & r\sqrt{1-a^2} \end{pmatrix}, \quad a = \frac{\pi\omega^2(z_m)}{2\lambda R^2}\delta R \tag{7}$$

where $\omega(z_m)$ is the beam size at the mirror position z_m

Transmission Matrix

The scattering matrix for transmission of a beam through a mirror having amplitude transmissivity coefficient t is given by

$$\mathsf{t} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} \tag{8}$$

Propagation Matrix

If one the one-way propagation of a beam across a distance or cavity accumulates a phase ϕ_0 in the LG_{00} mode and ϕ_1 in the LG_{10} mode, this phase accumulation during propagation can be represented by the scattering matrix

$$\phi = \begin{pmatrix} e^{\iota\phi_0} & 0 \\ 0 & e^{\iota\phi_1} \end{pmatrix} \tag{9}$$

The modified propagation matrix for a small position perturbation δz in a mirror of radius of curvature R

$$\phi' = \mathrm{b}.\phi = \begin{pmatrix} \sqrt{1-b^2}e^{\iota\phi_0} & -be^{\iota\phi_1} \\ -be^{\iota\phi_0} & \sqrt{1-b^2}e^{\iota\phi_1} \end{pmatrix}, \quad b = \frac{\delta z}{R} \tag{10}$$

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