Note on the radius of curvature of compression fit mirror and a common solution with piezo mirror for SAMS

T2000183

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The calculations are based on document E1900370 (https://dcc.ligo.org/E1900370)

A. Definition of variables:

P_{design} ≡ Mirror's nominal defocus at optical design = $2/R_{design}$ [mD]

P_{manuf} ≡ Manufactured optical power = $2/R_{manuf}$ [mD]

$P_{preload,p}$ ≡ Change of optical power through preloading of the piezo actuator [mD]

$P_{actuation,p}$ ≡ Full range of optical power change through piezo actuation [mD]

$P_{preload,t}$ ≡ Change of optical power by compression fit at ambient temperature (20°C) [mD]

$P_{actuation,t}$ ≡ Full range of optical power change by heating of the thermal mirror [mD]

T_{max} ≡ Maximum operating temperature [°C]

ΔT_{max} ≡ Maximum actuation temperature range [K]

T_{assem} = CFM assembly temperature [°C]

δr = Radial interference [μm]

Sign convention: '+' for concave mirror, '-' for convex mirror

B. Radius of curvature of mirror for CFM mirrors:

a. Optimal actuation range

Two main limits that will affect the choice of radius of curvature for the CFM:

1. The maximum operating temperature: this affects actuation range, assembly temperature and thus preload defocus ($P_{preload,t}$). Ideally, we want the assembly temperature to be at least 20% higher than that of maximum operating temperature.
2. The maximum allowed preload on piezo mirror: This is important to find a common solution for both piezo and thermal mirrors. The higher $P_{preload,p}$ is, the higher $P_{preload,t}$ can be, thus a higher actuation range for thermal mirror can be achieved. However,

we want to make sure that the stress in piezo is within the acceptable range (including some safety factor)

We assume a maximum operating temperature of 70°C

T_{max} [°C]	ΔT_{max} [K]	T_{assem} [°C]	$2\delta r$ [μm]*	$P_{preload,t}$ [mD] **	$P_{actuation,t}$ [mD] **
70	50	84	73	-420	+330

(*) *The diametric interference is computed from the following:*

Let:

$OD_{m,amb}$ ≡ Mirror's nominal diameter at ambient temperature (20°C) = 50.0 mm.

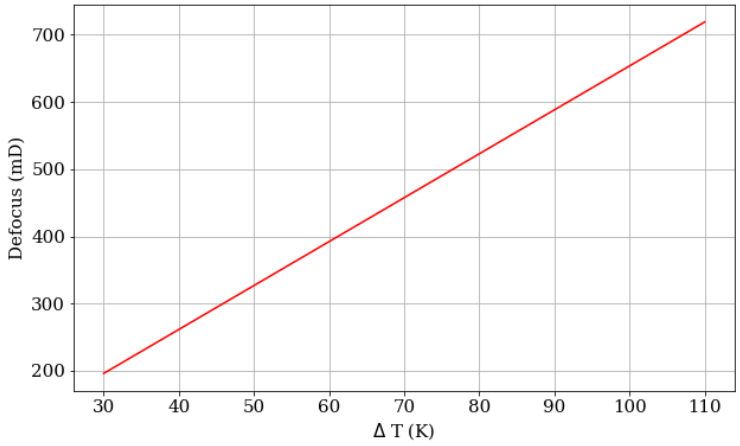
$ID_{r,amb}$ ≡ Mirror's nominal diameter at ambient temperature (20°C) = $OD_{m,amb}$ -2δr

α_m, α_r = Coefficient of thermal expansion of mirror and ring (0.55E-6 and 23.4E-6 respectively)

$$OD_{m,amb}(1 + \alpha_m \Delta T_{assem}) = (OD_{m,amb} - 2\delta r)(1 + \alpha_r \Delta T_{assem})$$

$$\therefore 2\delta r = OD_{m,amb}\left(1 - \frac{1 + \alpha_m \Delta T_{assem}}{1 + \alpha_r \Delta T_{assem}}\right)$$

(**) *Estimation of preload defocus and actuation range from FEA model of the CFM:*



X-axis is change in actuation temperature, y-axis is corresponding change in defocus due to temperature change.

P_{manuf} is expressed as followings in the case of CFM:

$$P_{manuf} = P_{design} + |P_{preload,t}| - \frac{|P_{actuation,t}|}{2}$$

then

$$P_{manuf} = P_{design} + 255\ [mD]$$

According to the current design of the piezo mirror (T2000101), P_{manuf} is determined by:

$$P_{manuf} = P_{design} + |P_{preload,p}| + \frac{|P_{actuation,p}|}{2} = P_{design} + 150[mD]$$

The manufactured mirror need to be more concave for the thermal mirror.

Question: Can we change $P_{actuation,p}$ such that there is a common solution?

$P_{preload,p}$ = -155[mD]

Recent measurement of piezo mirror RoC with ZYGO suggests that $P_{preload,p}$ is -160 [mD] for 80in-lbs preloading → *it looks feasible? What's the stress in flexures at this preload? Is it still in acceptable range?*

Assuming the new preload, the manufactured RoC of the mirrors are related to the design RoC as followings:

$$R_{manuf} = \frac{R_{design}}{1 + 0.1275 R_{design}}$$

With these new relations, the following manufactured RoC's are proposed:

Maximum operating temperature: 70°C, maximum actuation range: 330 [mD]

Optics	R_{design} [m]	P_{design} [D]	R_{manuf} [m]	P_{manuf} [D]
ZM2	0.85	2.35	0.77	2.61
ZM5	3.4	0.59	2.37	0.84
ZM4	-13.3	-0.15	19.1	0.10
OM2-1(nom)	1.7	1.18	1.40	1.43
OM2-2	1.5	1.33	1.26	1.59
OM2-3	1.94	1.03	1.56	1.29

b. Absolute conservative design:

In the conservative design, the aims are to:

1. Lowest maximum operating temperature.
2. 200 mD actuation range.
3. Axially shift the position of the ring

200 mD actuation range requires ΔT=31K (51°C absolute temperature). Assembly temperature is 61°C (diametric interference of 47 microns). $P_{preload,t}$ is then -268 [mD].

$\left|P_{preload,t}\right| - \frac{|P_{actuation,t}|}{2} = 168\ [mD]$>150[mD].

→**2 solutions**:

1. Increase piezo preload offset slightly

$$P_{preload,p} = -68\ mD$$

2. Increase nominal operating temperature of the CFM:

$$\left|P_{preload,t}\right| - |P_{nom}| = 150\ [mD]$$

$$\therefore |P_{nom}| = 118[mD]$$

i.e. Nominal operating temperature is 38°C and actuation range is -118 mD/ +82 mD.

If **solution 2** is selected, there is no need to change the RoC of manufactured mirrors as specified in T2000101.

If **solution 1** is selected, R_{manuf} and R_{design} are related by:

$$R_{manuf} = \frac{R_{design}}{1 + 0.086R_{design}}$$

Optics	R_{design} [m]	P_{design} [D]	R_{manuf} [m]	P_{manuf} [D]
ZM2	0.85	2.35	0.79	2.52
ZM5	3.4	0.59	2.63	0.76
ZM4	-13.3	-0.26	92.5	0.02
OM2-1(nom)	1.7	1.18	1.48	1.35
OM2-2	1.5	1.33	1.33	1.51
OM2-3	1.94	1.03	1.66	1.20

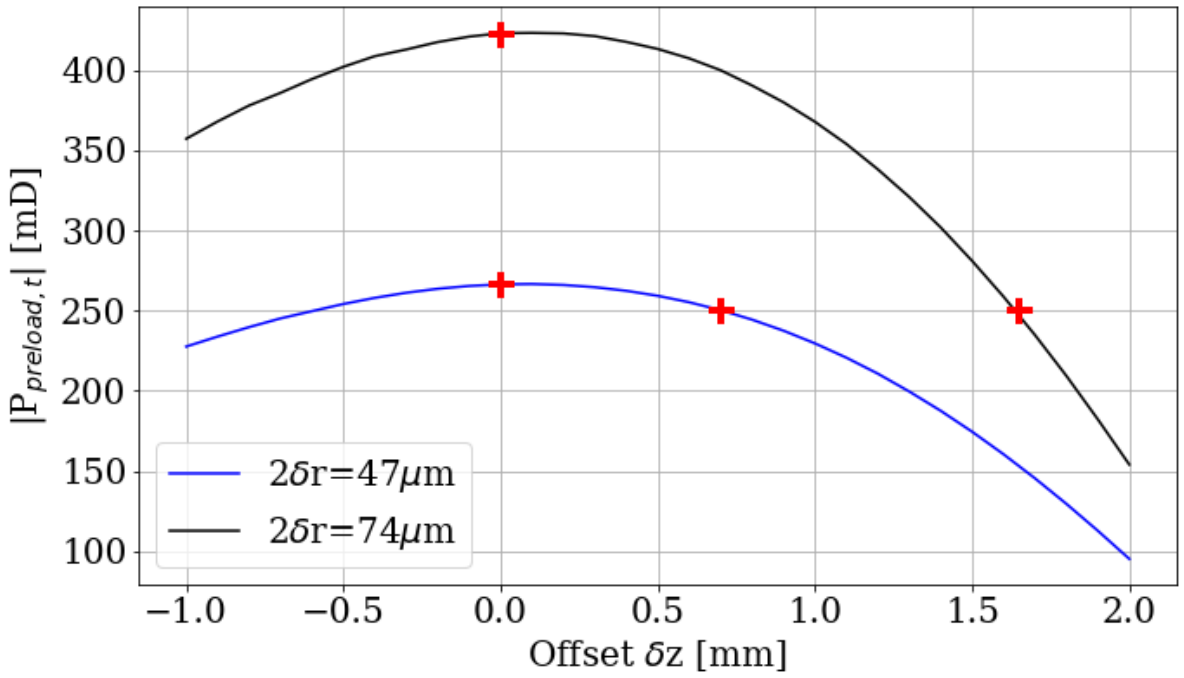
If **solution 3** is selected, we will move the actuating ring up axially by 0.7 [mm]. The bending moment is now smaller for the CFM. $P_{preload,t}$ will now drop down to 250 [mD] (see the the blue curve in Figure below).

If this solution is selected, no change in R_{manuf} is required

The mirror can still actuate symmetrically with +100[mD]/-100[mD] with a haft way range actuation at ΔT = 16.5K (36.5°C).

At ambient temperature, the mirror would also see more compressive stress than tensile stress.

The trade off is a higher maximum operating temperature require to obtain $P_{actuation}$ of 200 mD (ΔT = 33K instead of 31K).



If the actuation range is found to be too limited during testing/commissioning, we can shift the ring back to 0 to enable greater actuation range.

This principle also applies for the 73 micron interference configuration in part a. In this case, we will need to shift the ring up by 1.65 mm to achieve the common solution with the PDM. 200 [mD] with then be achieved with $\Delta T = 51K$ (71^oC absolute temperature – potentially too high). However, if needed to increase actuation range, we can drop the ring back and easily gain up to 420 [mD] actuation.

Changing the offset of the ring is a good method to achieve the change in preload defocus without requiring fabrication of new ring. Only a differ ring setter is required. The tolerance on the axial offset is also not as stringent as that of the interference, therefore more easily machined.

Comments:

To improve the cooling thermal time constant, we suggest the minimum operating temperature to be higher than that of the assumed temperature. This will result in higher maximum operating temperature.