GRAVITATIONAL WAVE POLARIZATION

A GENERAL RELATIVITY TEST

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LIGO-T1900468-v2

RESEARCH QUESTION

How well can we constrain small admixtures of scalar and/or vector polarizations in Gravitational Wave Transient detection, given the extended network of five ground interferometers available in the near future?

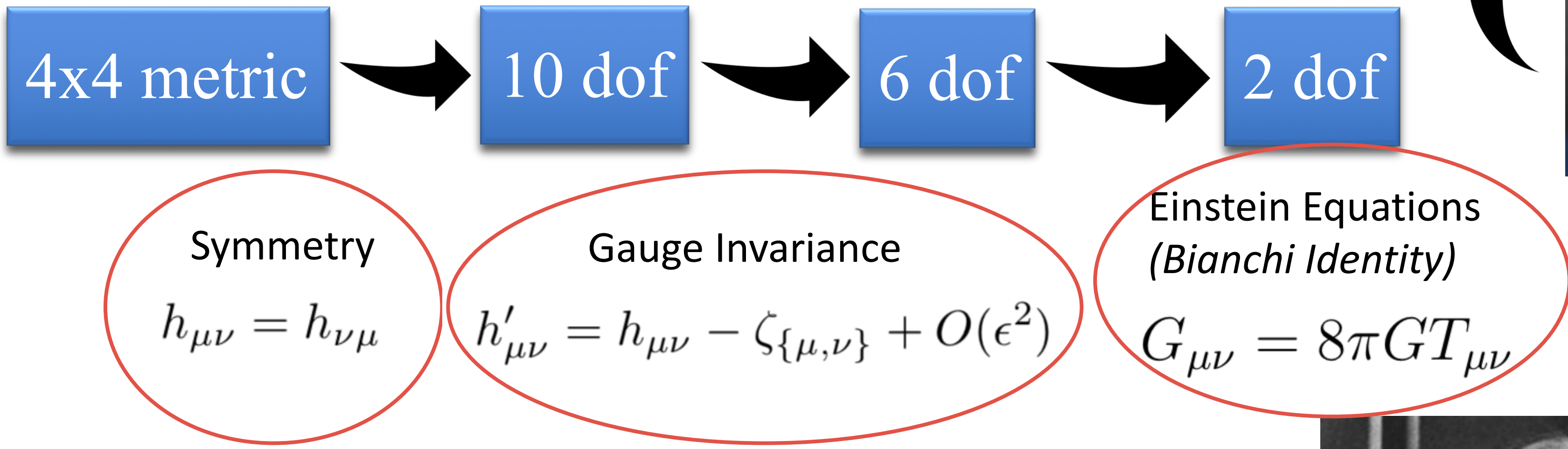
OVERVIEW

- Gravitational Waves Polarization and Detector Response
- Network of Interferometers and Overlap Factor
- Toy Model for a non-GR signal
- Nested Sampling and Bayesian Approach
- Parameter estimation
- Future work

TWO POLARIZATIONS

Linearized Theory of Gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



4x4 metric → 10 dof → 6 dof → 2 dof

Symmetry

$$h_{\mu\nu} = h_{\nu\mu}$$

Gauge Invariance

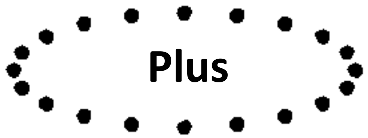
$$h'_{\mu\nu} = h_{\mu\nu} - \zeta_{\{\mu,\nu\}} + O(\epsilon^2)$$

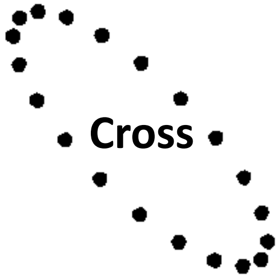
Einstein Equations

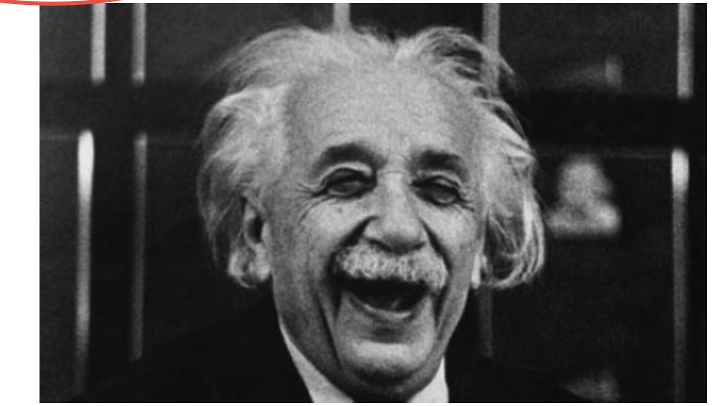
(Bianchi Identity)

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

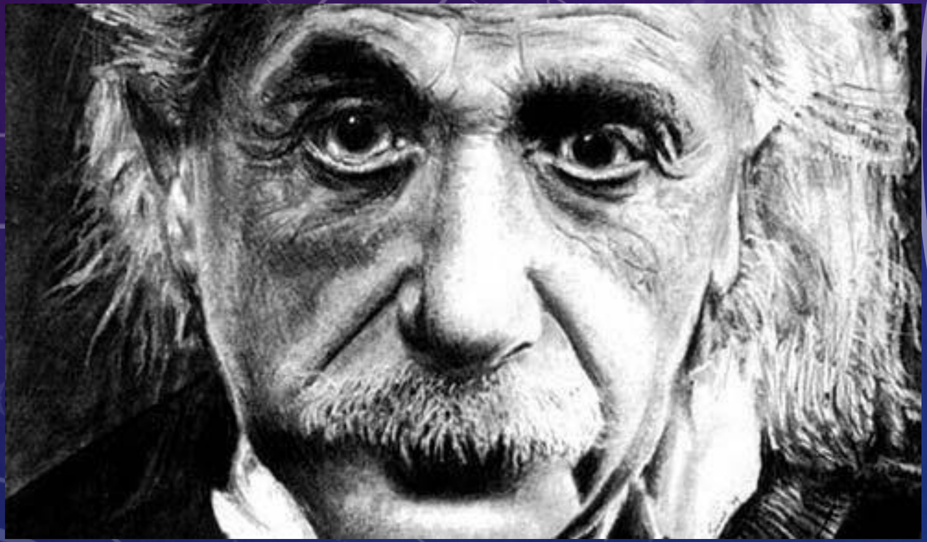
TT-Gauge

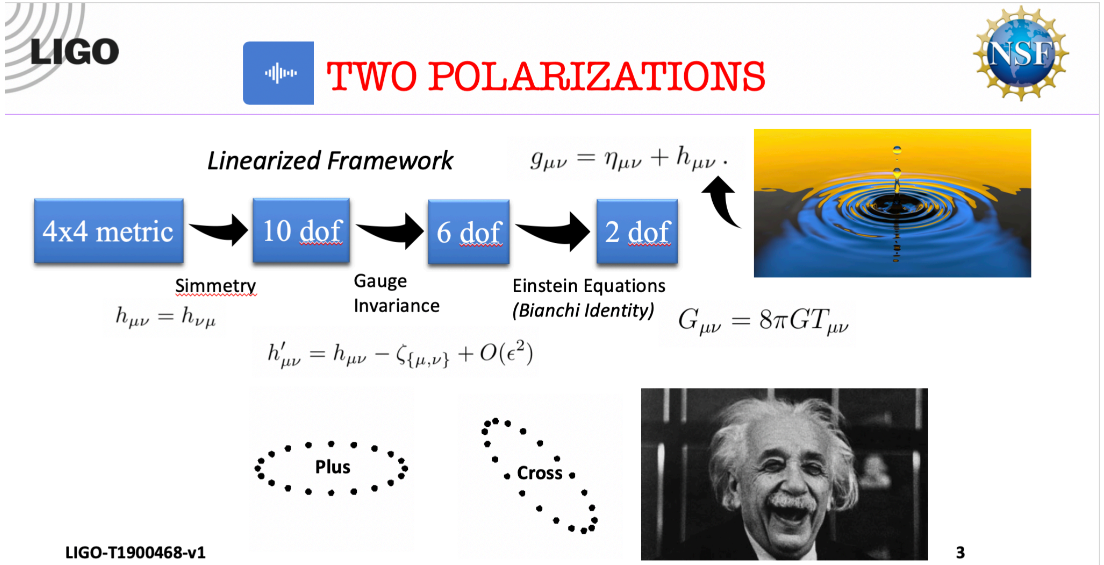






WHAT IF EINSTEIN'S GR ***IS NOT*** THE ULTIMATE THEORY OF GRAVITY ?





LIGO

TWO POLARIZATIONS

Linearized Framework

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \,.$$

4x4 metric → 10 dof → 6 dof → 2 dof

Simmetry

$$h_{\mu\nu} = h_{\nu\mu}$$

Gauge Invariance

$$h'_{\mu\nu} = h_{\mu\nu} - \zeta_{\{\mu,\nu\}} + O(\epsilon^2)$$

Einstein Equations *(Bianchi Identity)*

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

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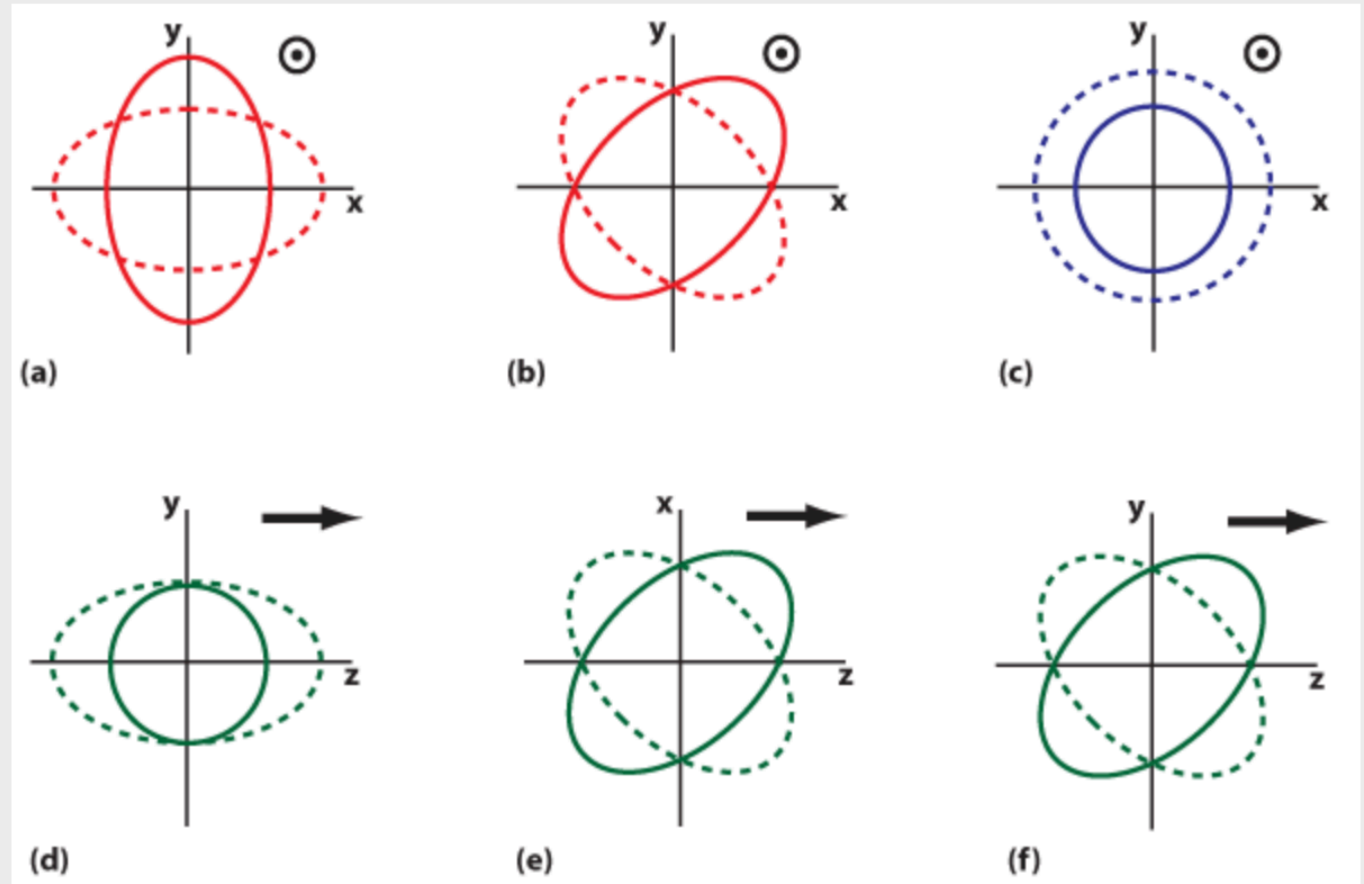
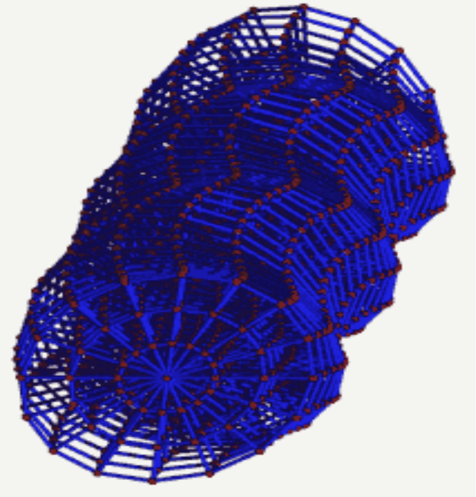
3

~~TWO~~ SIX POLARIZATIONS



Will, C.M. , arXiv



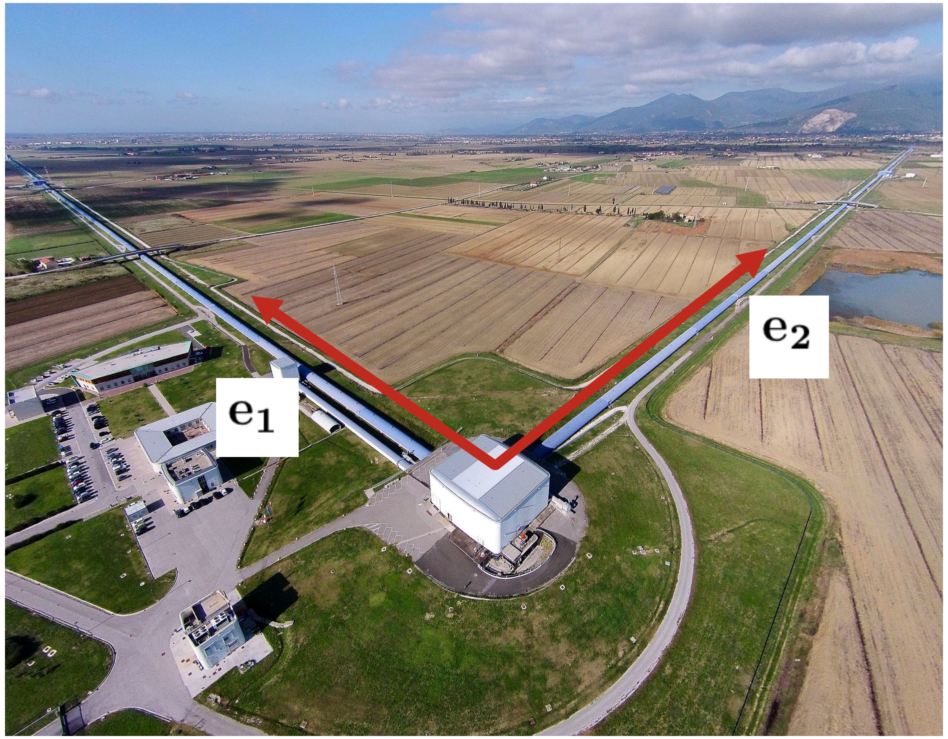
Markus Pössel, Einstein online

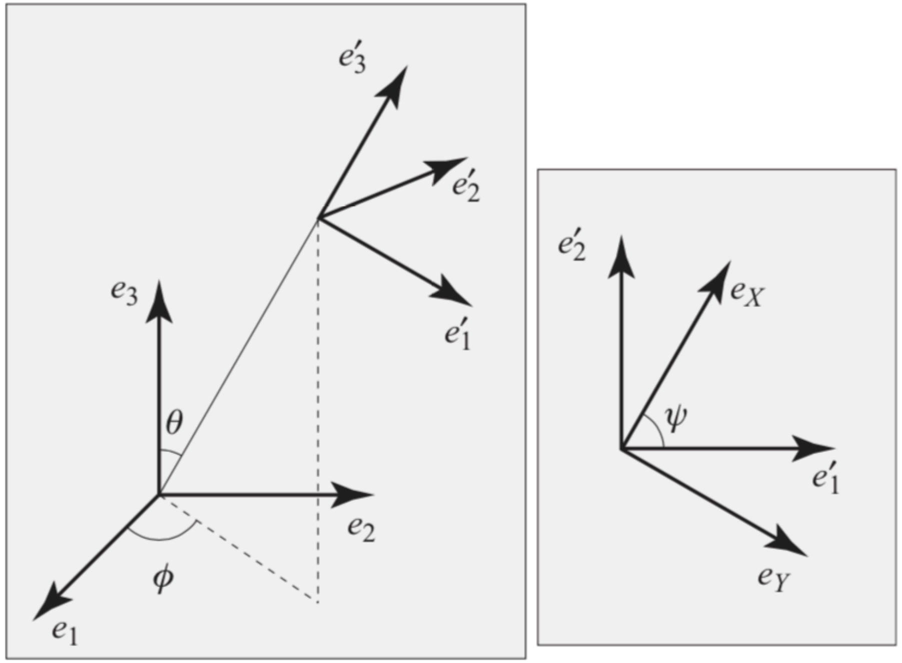
$$S_{jk} = \begin{pmatrix} A_b + \boxed{A_+} & \boxed{A_\times} & A_{Vx} \\ \boxed{A_\times} & A_b - \boxed{A_+} & A_{Vy} \\ A_{Vx} & A_{Vy} & A_l \end{pmatrix}$$

DETECTOR DARM RESPONSE

$$S(t) = \frac{1}{2}(e_1^j e_1^k - e_2^j e_2^k) S_{jk}(\tau, \mathbf{N}) = \boxed{F_P(\theta, \phi, \psi)} A_P$$

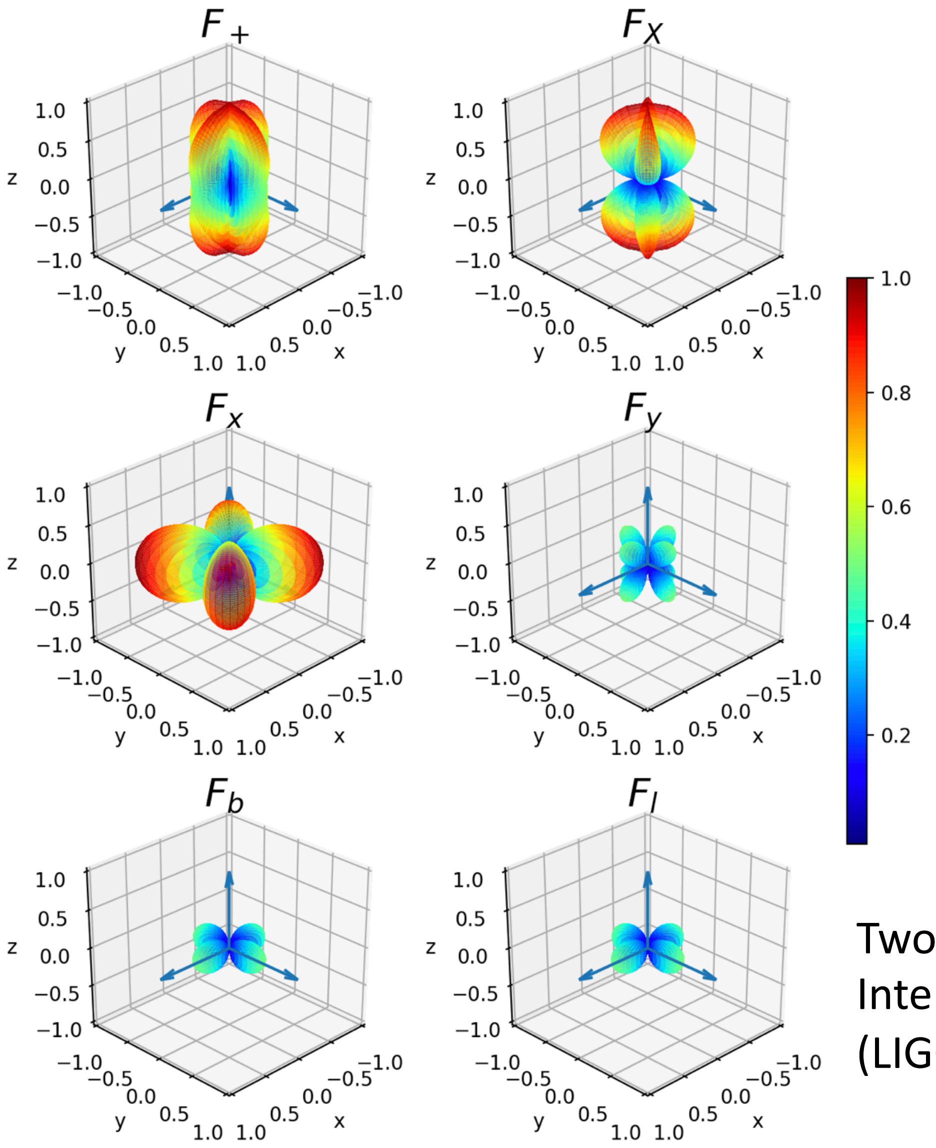
Antenna Patterns



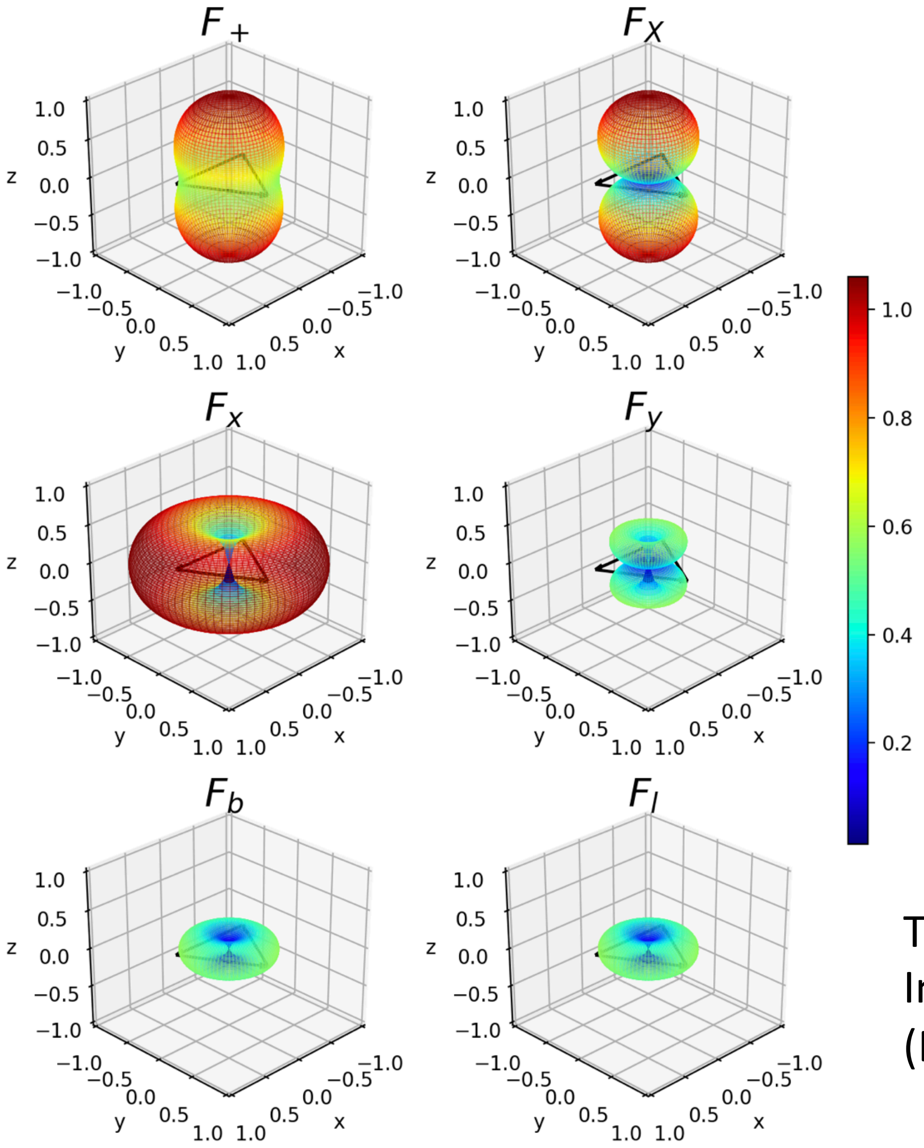


Poisson and C. Will, Gravity

ANTENNA PATTERNS

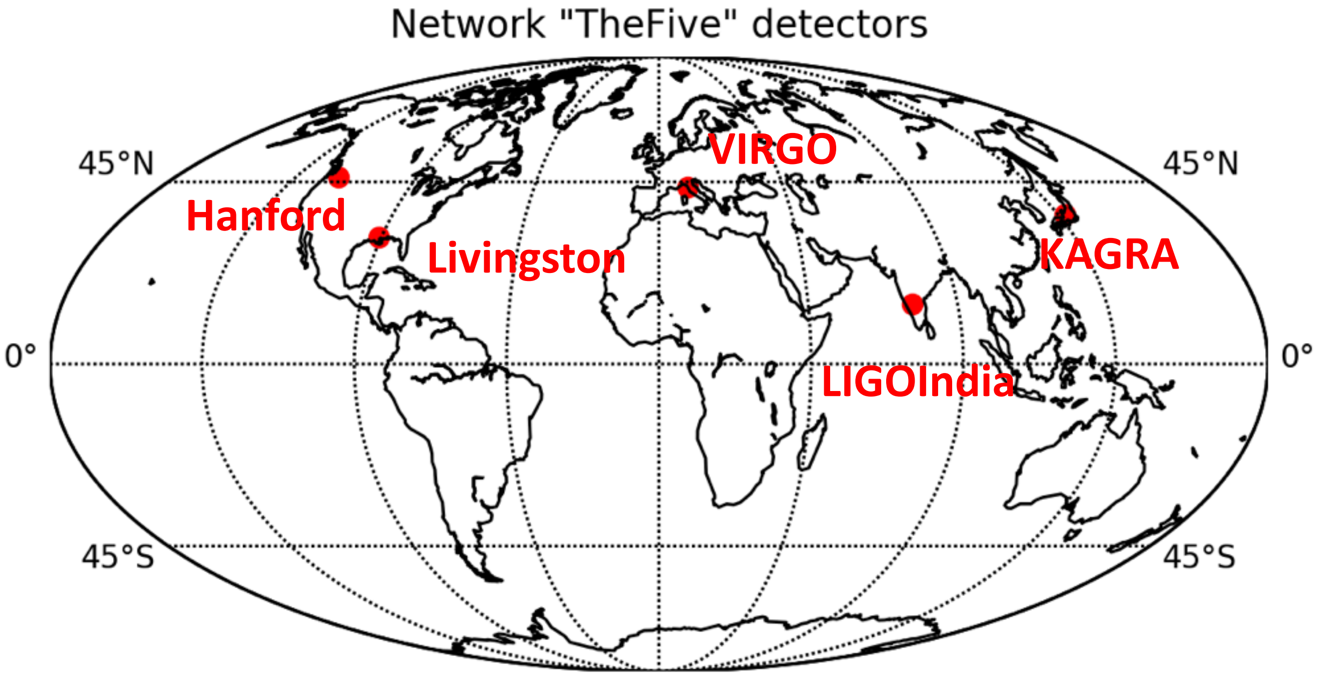


Two-Arm Interferometer (LIGO)



Three-Arm Interferometer (ET)

NETWORK SENSITIVITY



- Vector of Antenna Patterns

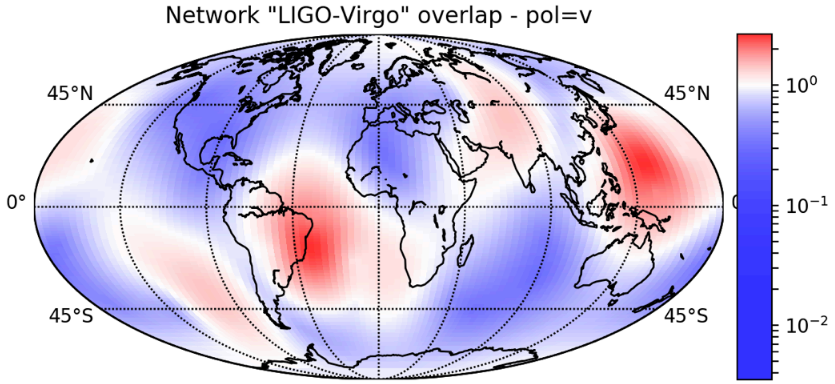
$$\vec{F}_H(\theta,\phi) := (|F_H^1(\theta,\phi)|, ..., |F_H^N(\theta,\phi)|)$$

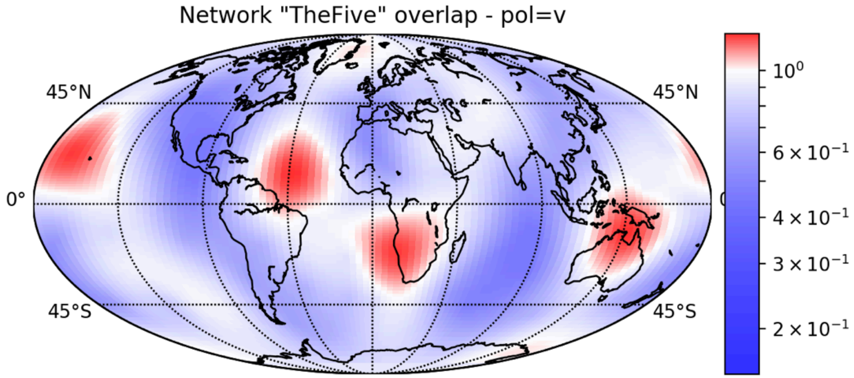
H in {s, v, t}
1 ... N = no. detector

- Overlap Factor

$$\mathcal{F}_{H/t} = \frac{\vec{F}_H(\theta,\phi)\cdot\vec{F}_t(\theta,\phi)}{\vec{F}_t(\theta,\phi)\cdot\vec{F}_t(\theta,\phi)}$$

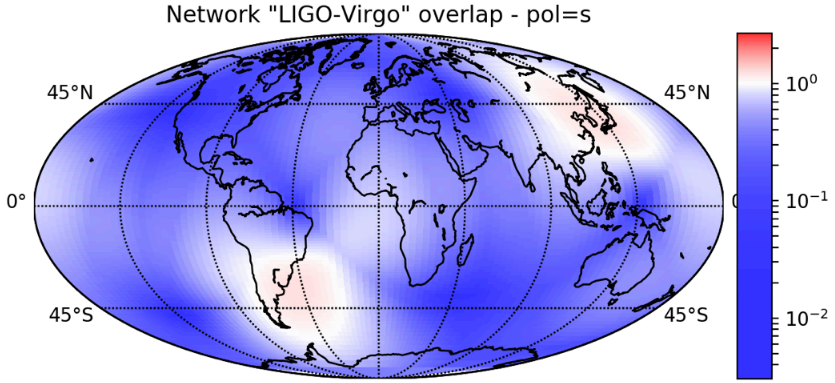
OVERLAP FACTOR

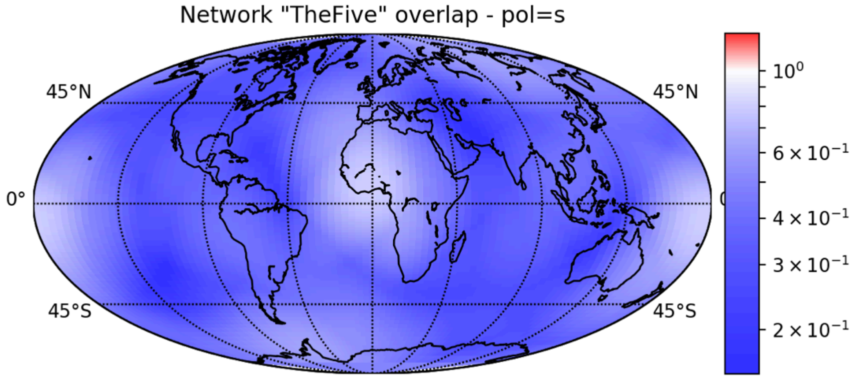

Network "LIGO-Virgo" overlap - pol=v


Network "TheFive" overlap - pol=v

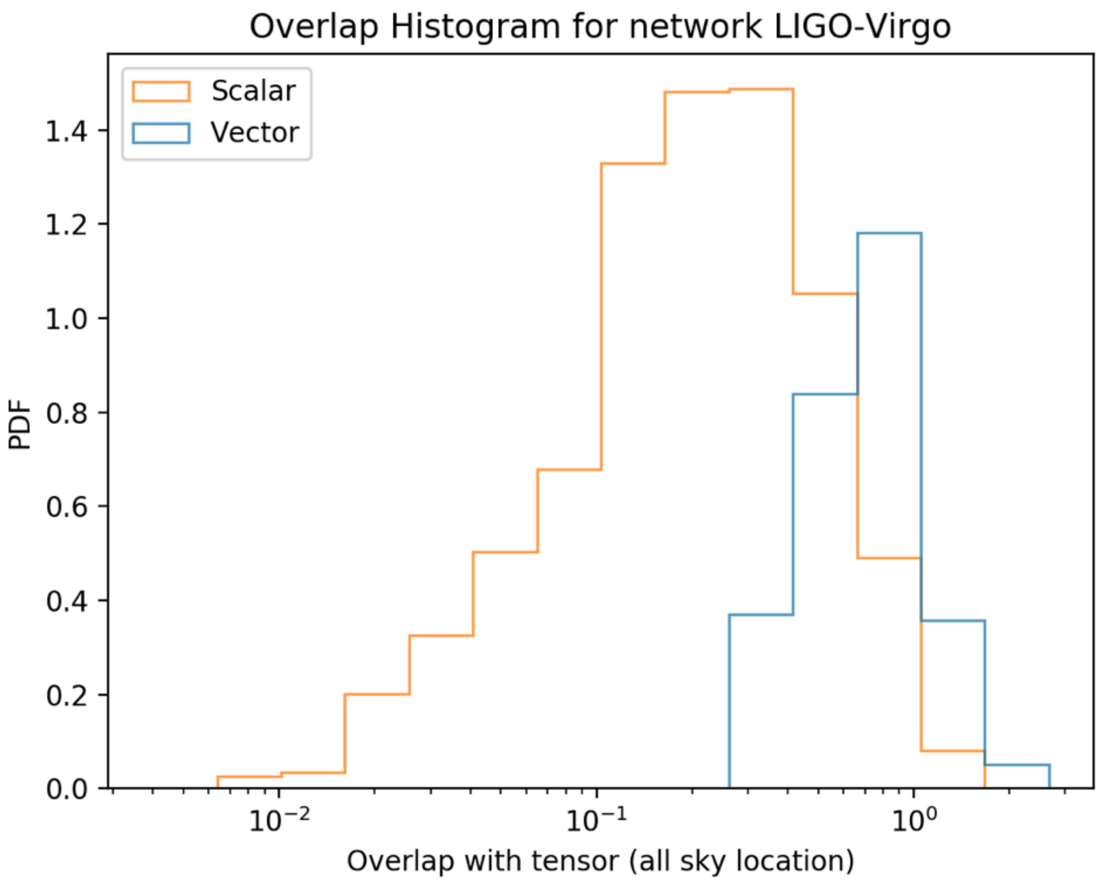
LIGO - Virgo Network

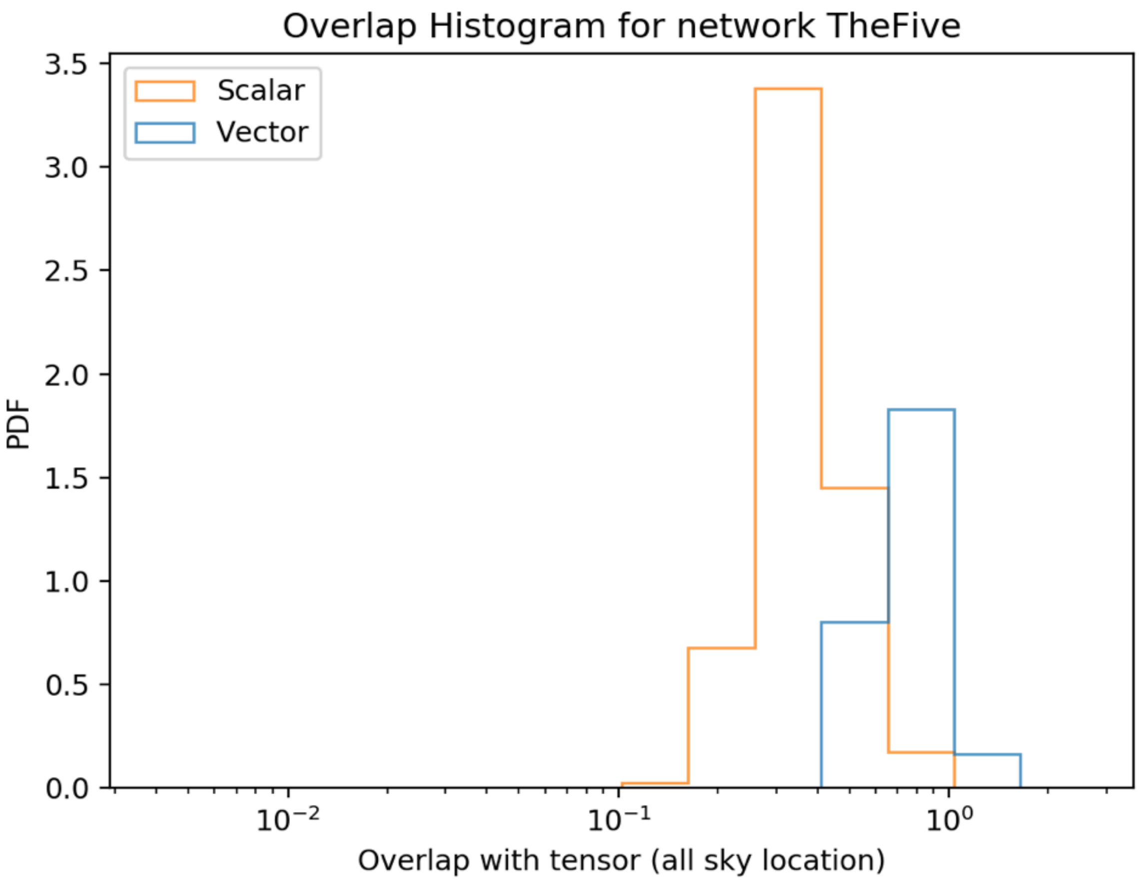
+ LIGOIndia & KAGRA


Network "LIGO-Virgo" overlap - pol=s


Network "TheFive" overlap - pol=s

OVERLAP FACTOR


Overlap Histogram for network LIGO-Virgo


Overlap Histogram for network TheFive

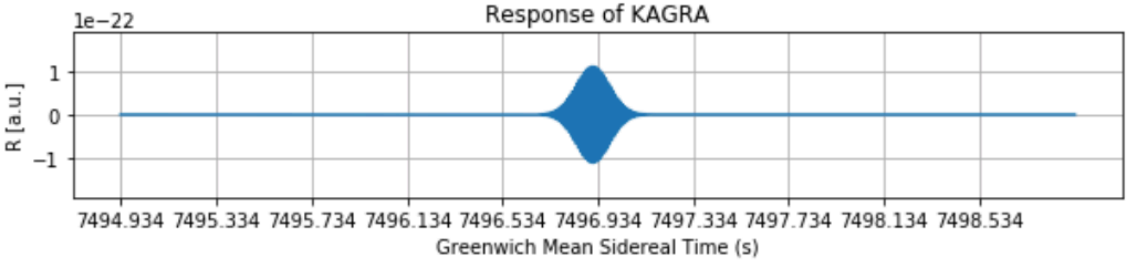
TOY MODEL OF A GW

Sine-Gaussian Waveform

$$h_I(t) = A_I \cos(\Omega t + \phi_I) \exp\left[-\frac{(t - t_0 - \delta t_I)^2}{2\tau^2} \right]$$

$$A_I = \frac{|A|}{d_L} |\tilde{A}_I|$$

$$\tilde{A}_I = \sum_{p \in \{+,\times,x,y,s\}} \epsilon_p F_p^I$$



1 s

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- Sky coordinate location: (δ,α)
- Polarization angle: $\psi = 0$
- Geocentric sidereal time of arrival: t_0 (degenerate with α)
- Luminosity distance: d_L
- Complex amplitude coefficients: $\epsilon_p = \frac{a_p}{|A|} e^{i\phi_p}$
- Overall amplitude: $|A| = \sqrt{\sum_p a_p^2}$
- Antenna Patterns: $F_p^I = F_p^I(\alpha, \delta, \psi = 0, t_0)$
- Time delay form Earth-center: δt_I
- Phase offset: $\phi_I = \arctan \frac{\mathcal{I}[\tilde{A}_I]}{\mathcal{R}[\tilde{A}_I]} - \Omega(t_0 + \delta t_I)$
- Angular frequency: $\Omega = 2\pi * (100Hz)$
- Damping time: $\tau = 0.1s$

λ HYPERPARAMETERS

- *Normalized effective strain amplitude*

$$h_t = \sqrt{|\epsilon_+|^2 + |\epsilon_\times|^2}$$

$$h_v = \sqrt{|\epsilon_x|^2 + |\epsilon_y|^2}$$

$$h_s = |\epsilon_s|$$

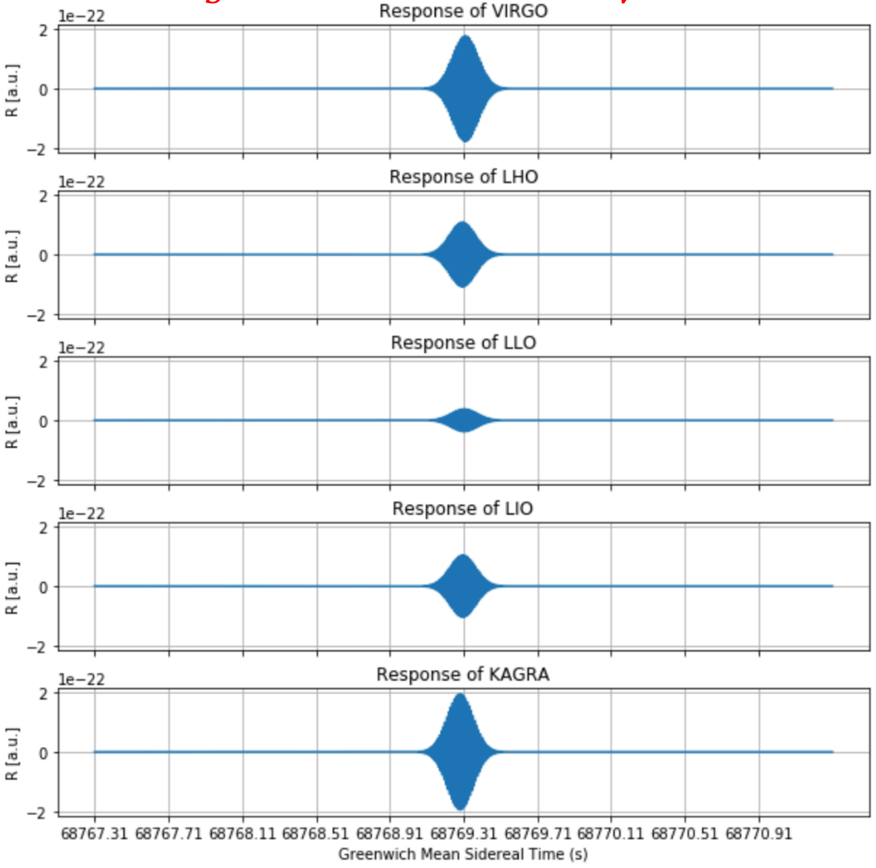
- *Hyperparameters of the model*

$$\left.\begin{aligned} h_t^2 &= 1 - \lambda_v - \lambda_s \\ h_v^2 &= \lambda_v \\ h_s^2 &= \lambda_s \end{aligned}\right]$$

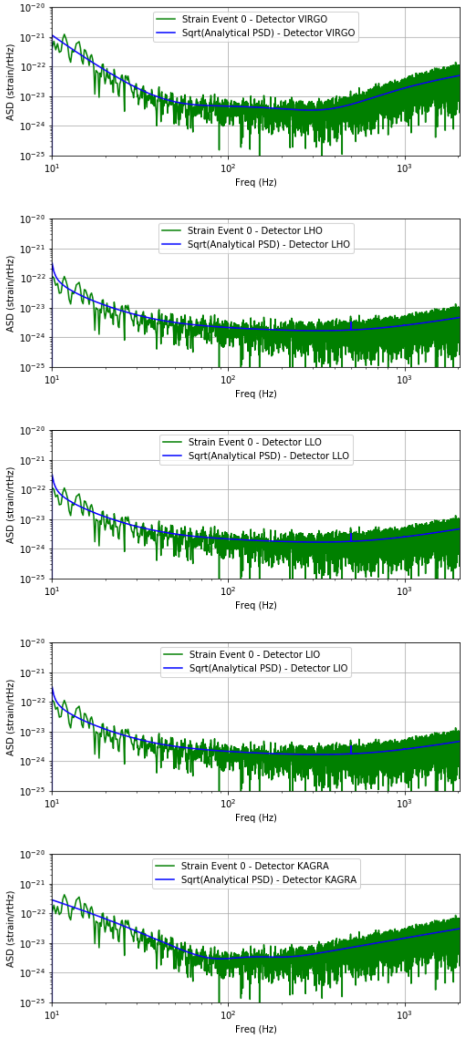
BAYES INFERENCE

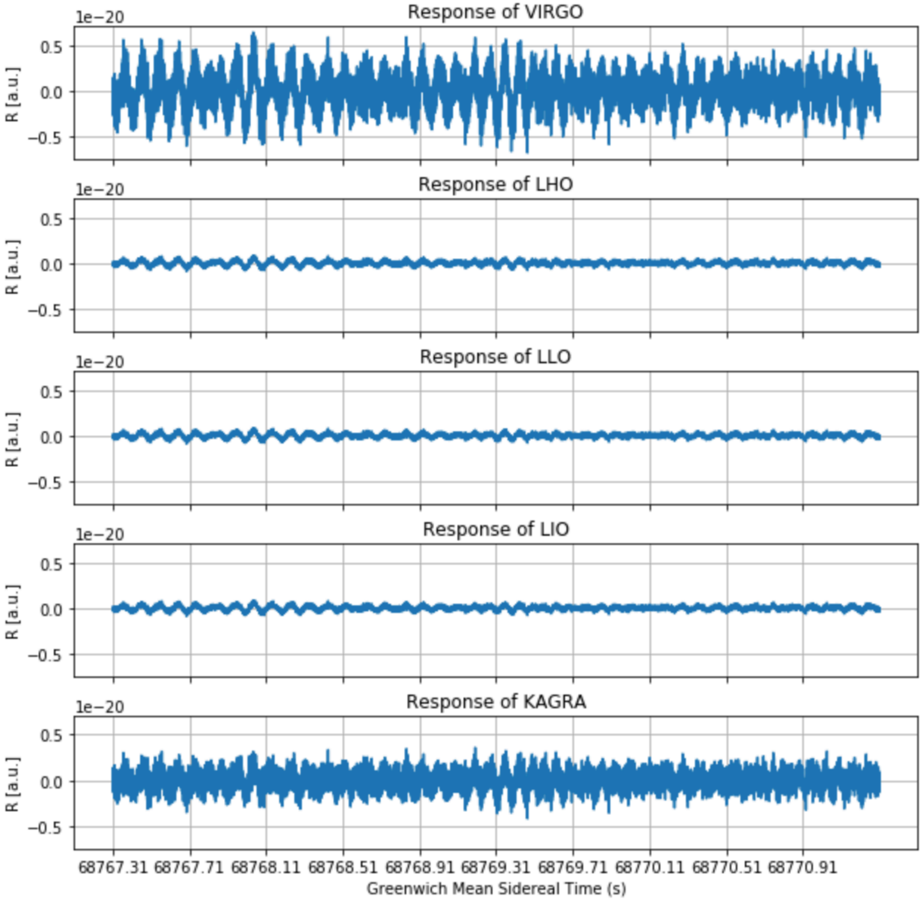
BUT FIRST…
LET’S MAKE SOME NOISE!

$\lambda_S = 0.1$ $\lambda_V = 0.1$



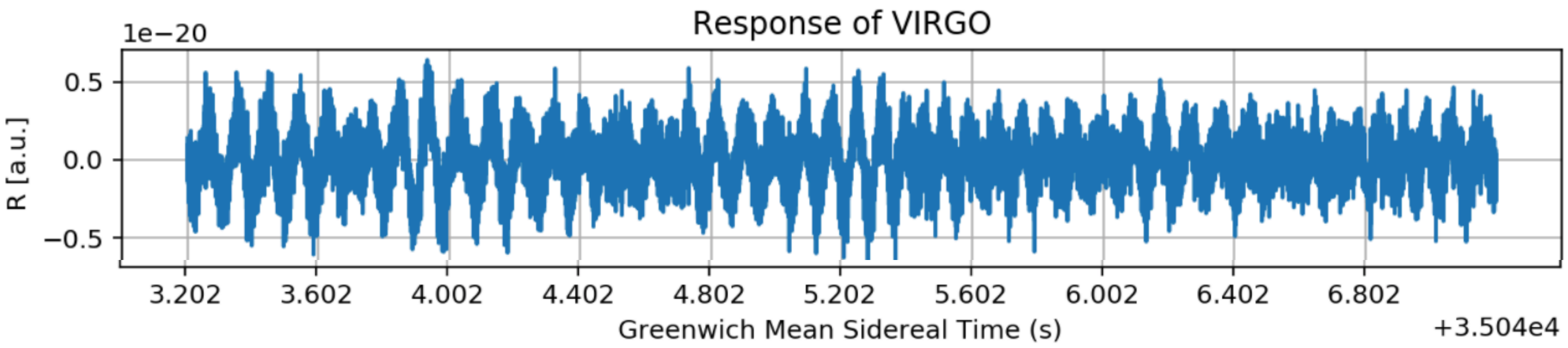
1 s





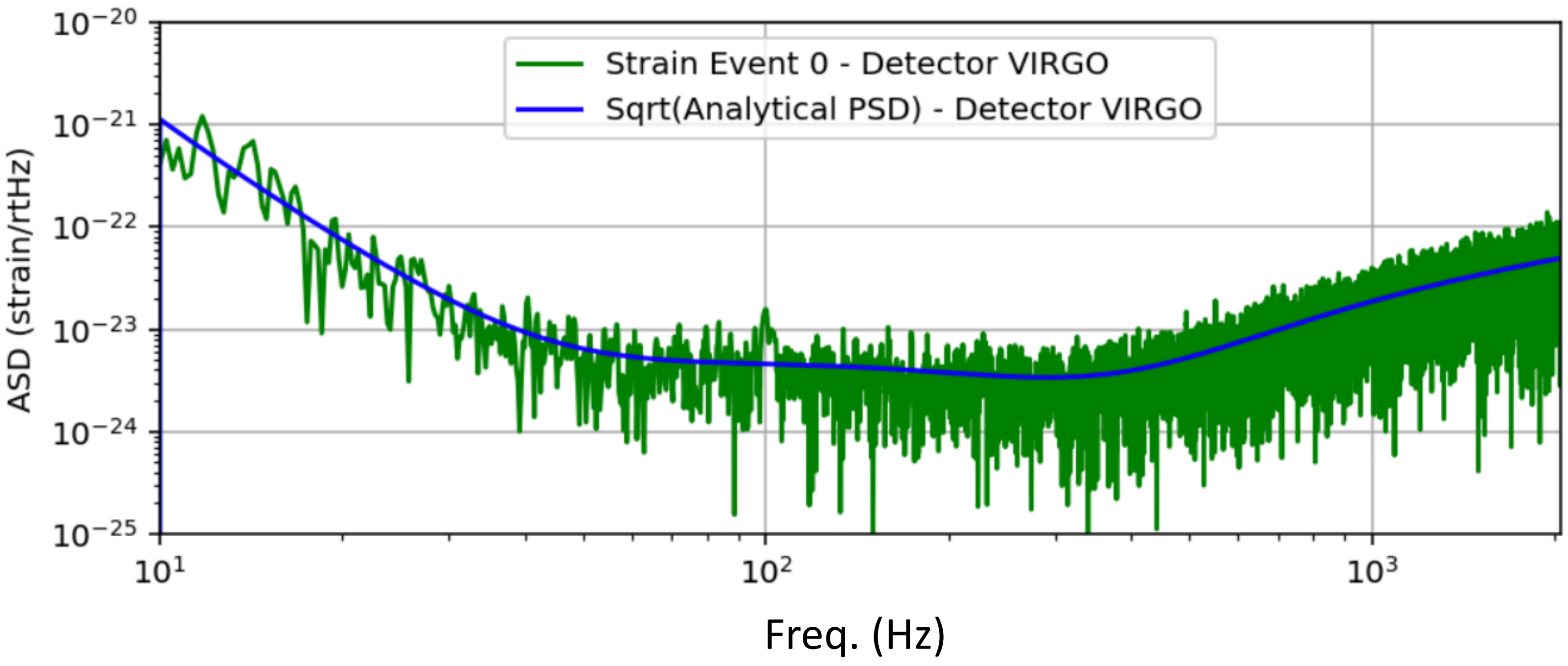
BUT FIRST…
LET'S MAKE SOME NOISE!

LOW SNR SOURCE



FFT

IFFT



BUT FIRST…
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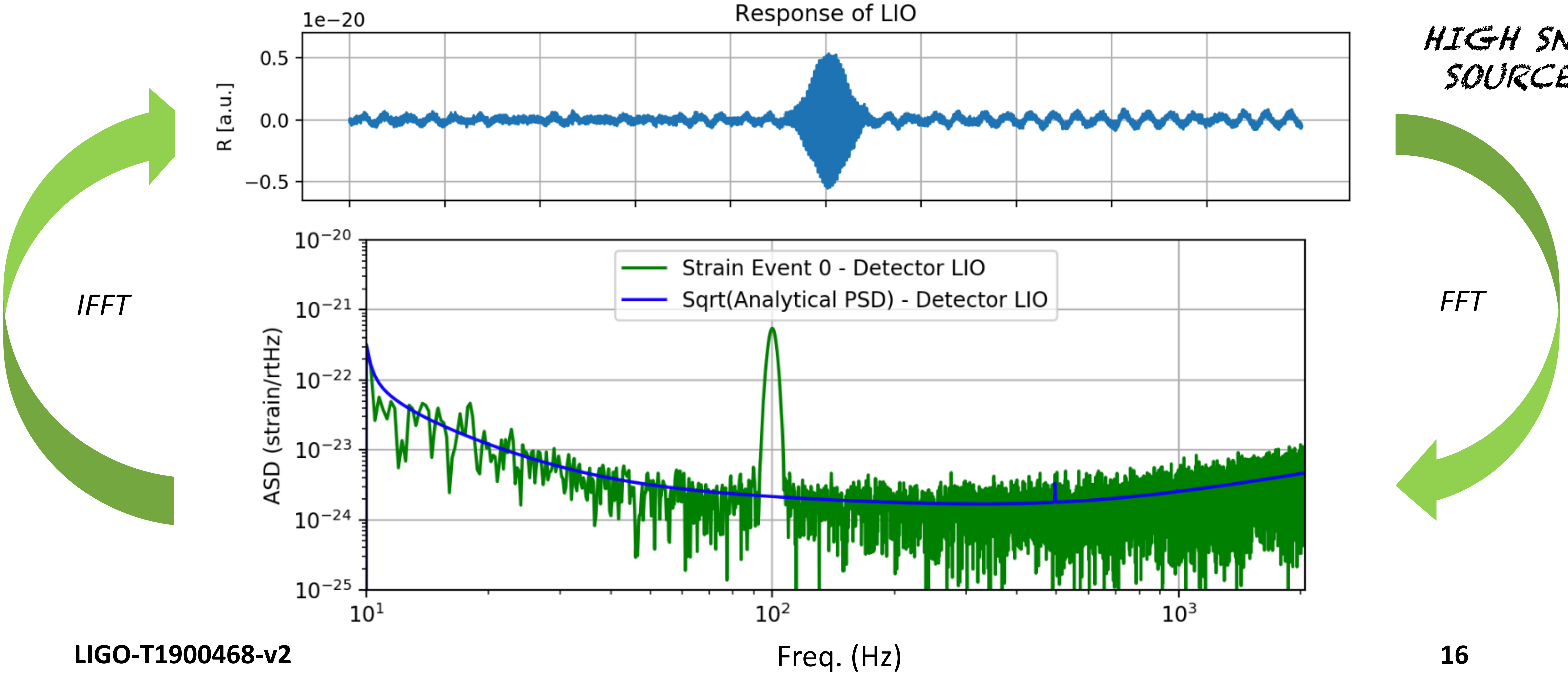


HIGH SNR SOURCE

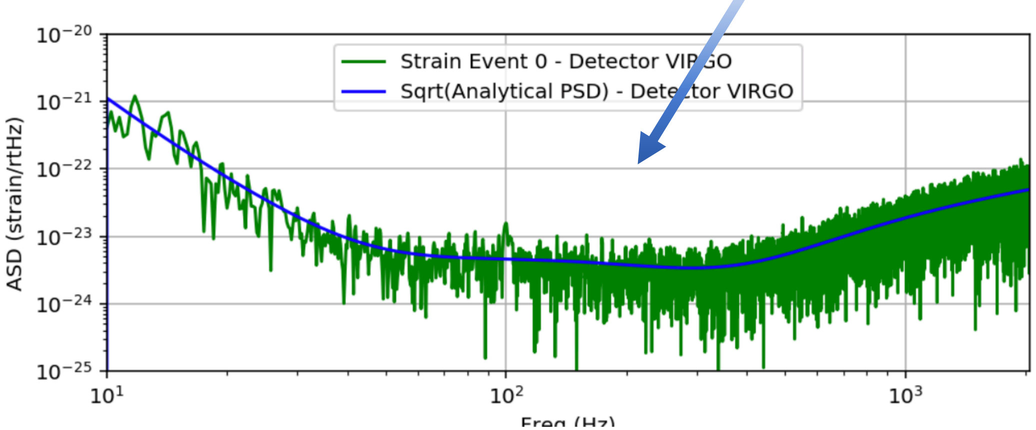
FFT

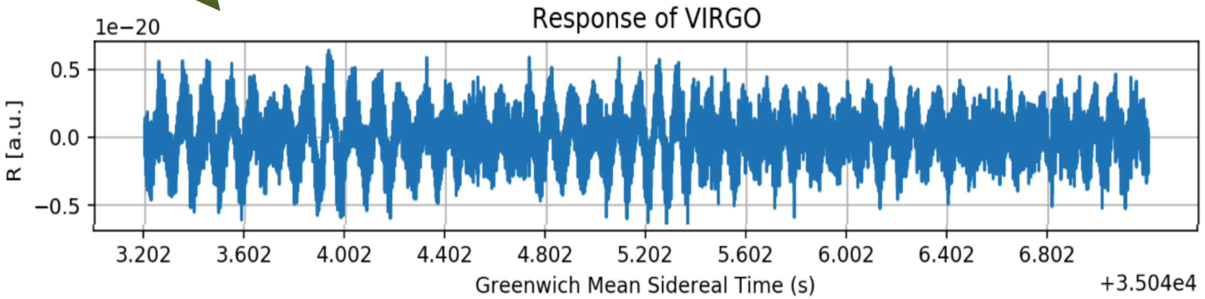
IFFT

MATCHED FILTERING

$$\langle h|s\rangle(t) = 4\Re\int_0^\infty \frac{\tilde{h}^*(f)\tilde{s}(f)e^{2\pi i f t}}{S_\mathrm{n}(f)}df$$

Wiener's Optimal Filter

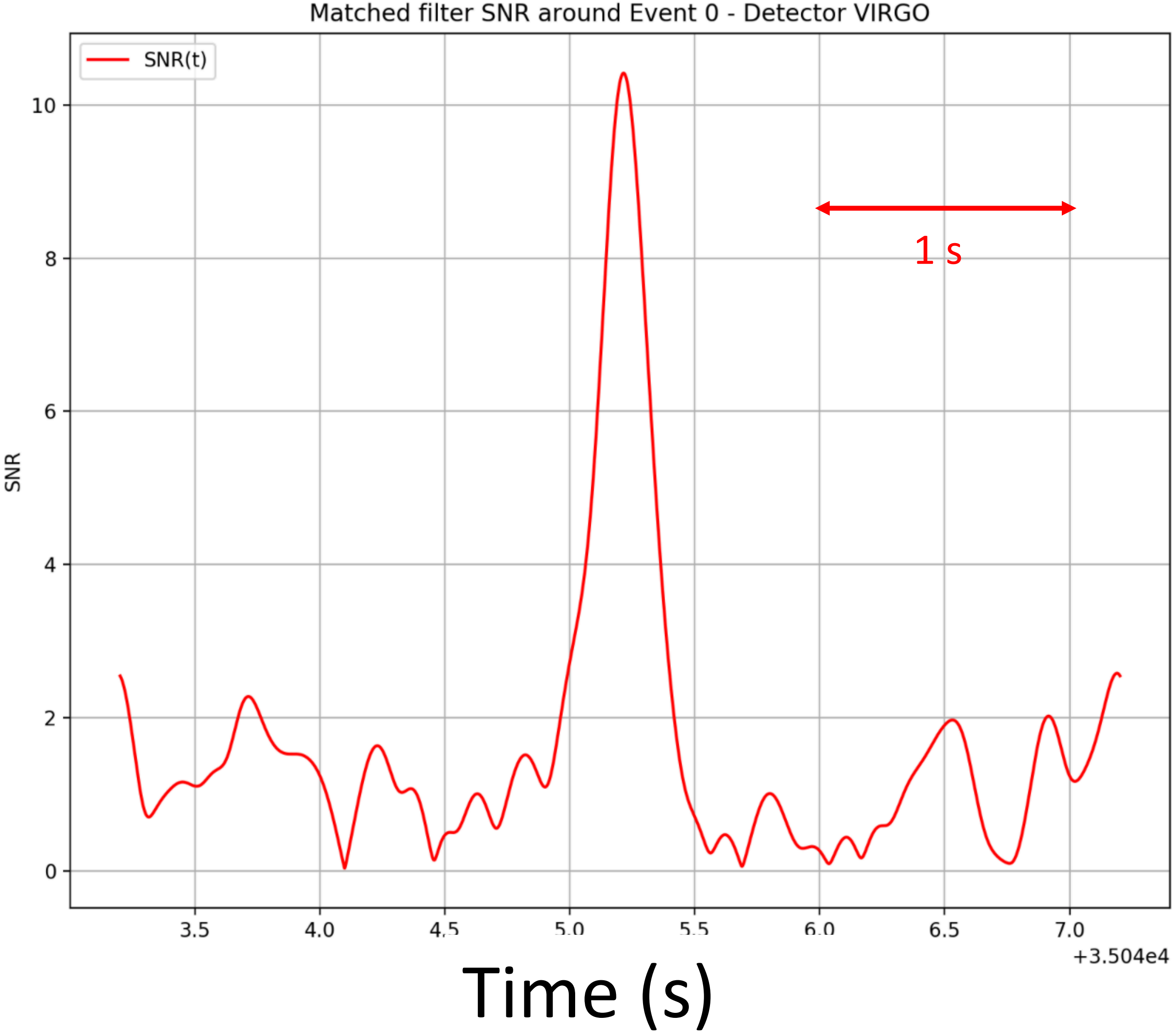




SNR

SNR (Signal-to-Noise Ratio)

$$\rho(t) = \sqrt{\frac{\langle h|s\rangle}{\langle h|h\rangle}}$$



LOW SNR SOURCE

BAYES THEOREM

Parameter Estimation

$$\underbrace{p(\vec{\theta}|\vec{d},\mathcal{H},I)}_{\text{Posterior}} = \frac{\overbrace{p(\vec{\theta}|\mathcal{H},I)}^{\text{Prior}}\,\overbrace{p(\vec{d}|\vec{\theta},\mathcal{H},I)}^{\text{Likelihood}}}{\underbrace{p(\vec{d}|\mathcal{H},I)}_{\text{Evidence}}}$$

Model Selection

$$\underbrace{O_{i,j}}_{\text{Odds}} = \underbrace{\frac{P(\mathcal{H}_i|I)}{P(\mathcal{H}_j|I)}}_{\text{Priors on the model}}\,\underbrace{\frac{P(\vec{d}|\mathcal{H}_i,I)}{P(\vec{d}|\mathcal{H}_j,I)}}_{\text{Evidence Ratio}}$$

LIKELIHOOD

- Two critical assumption at each detector for the **noise**:
  1. Gaussianity (in each frequency bin)
  2. Stationarity

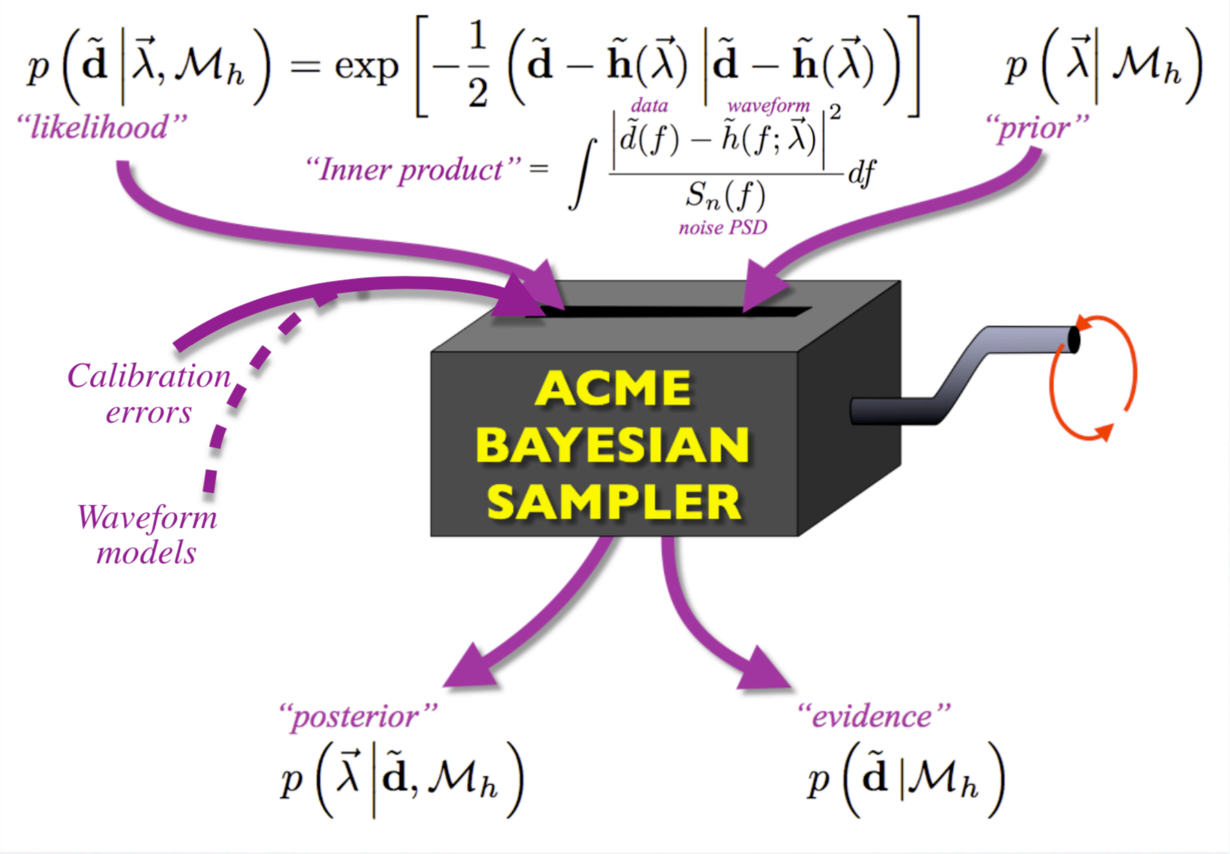
Normalization crucial for evidence computation

$$p(\mathbf{d}|H_S, S_n(f), \boldsymbol{\theta}) = \exp \sum_i \left[-\frac{2|\tilde{h}_i(\boldsymbol{\theta}) - \tilde{d}_i|^2}{T S_n(f_i)} - \frac{1}{2}\log(\pi T S_n(f_i)/2) \right]$$

Discrete Fourier Transform

$$\tilde{d}_j = \frac{T}{N} \sum_k d_k \exp(-2\pi i j k / N)$$

NESTED SAMPLING



Evidence
Numerical
Estimation

Prior

Likelihood

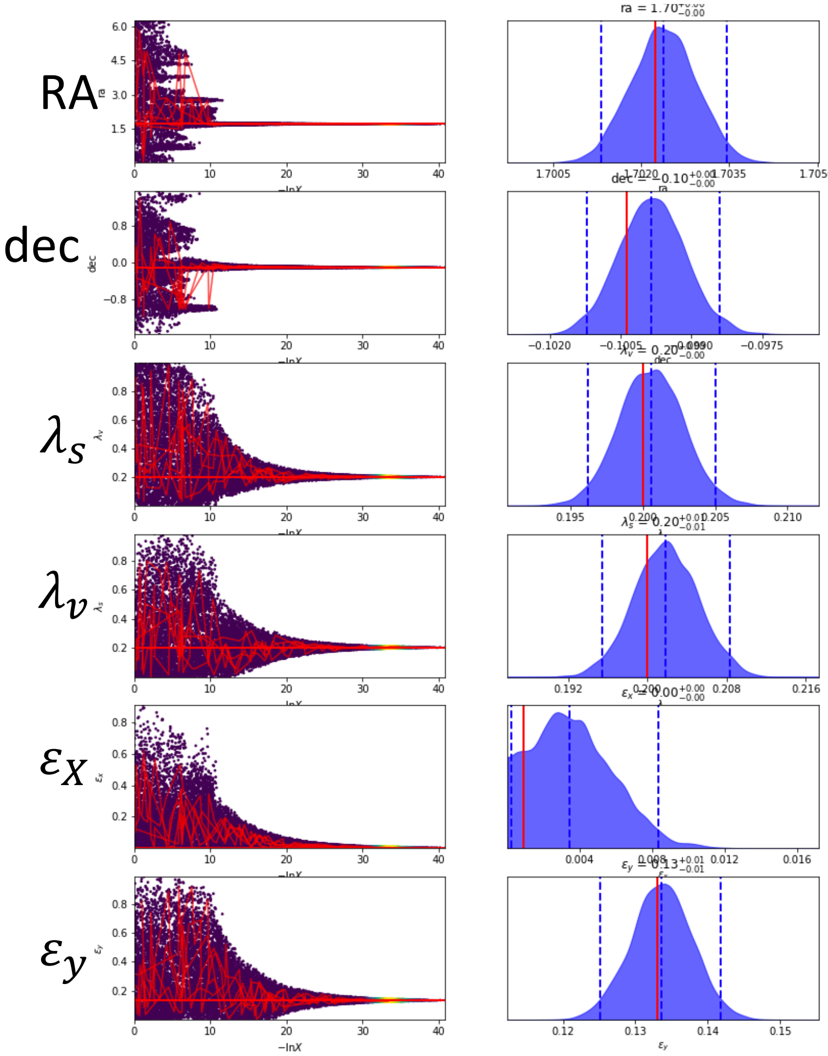
$$Z=\int_{\Theta}p(\vec{\theta}|\mathcal{H},I)\,p(\vec{d}|\mathcal{H},\vec{\theta},I)\,d\vec{\theta}$$

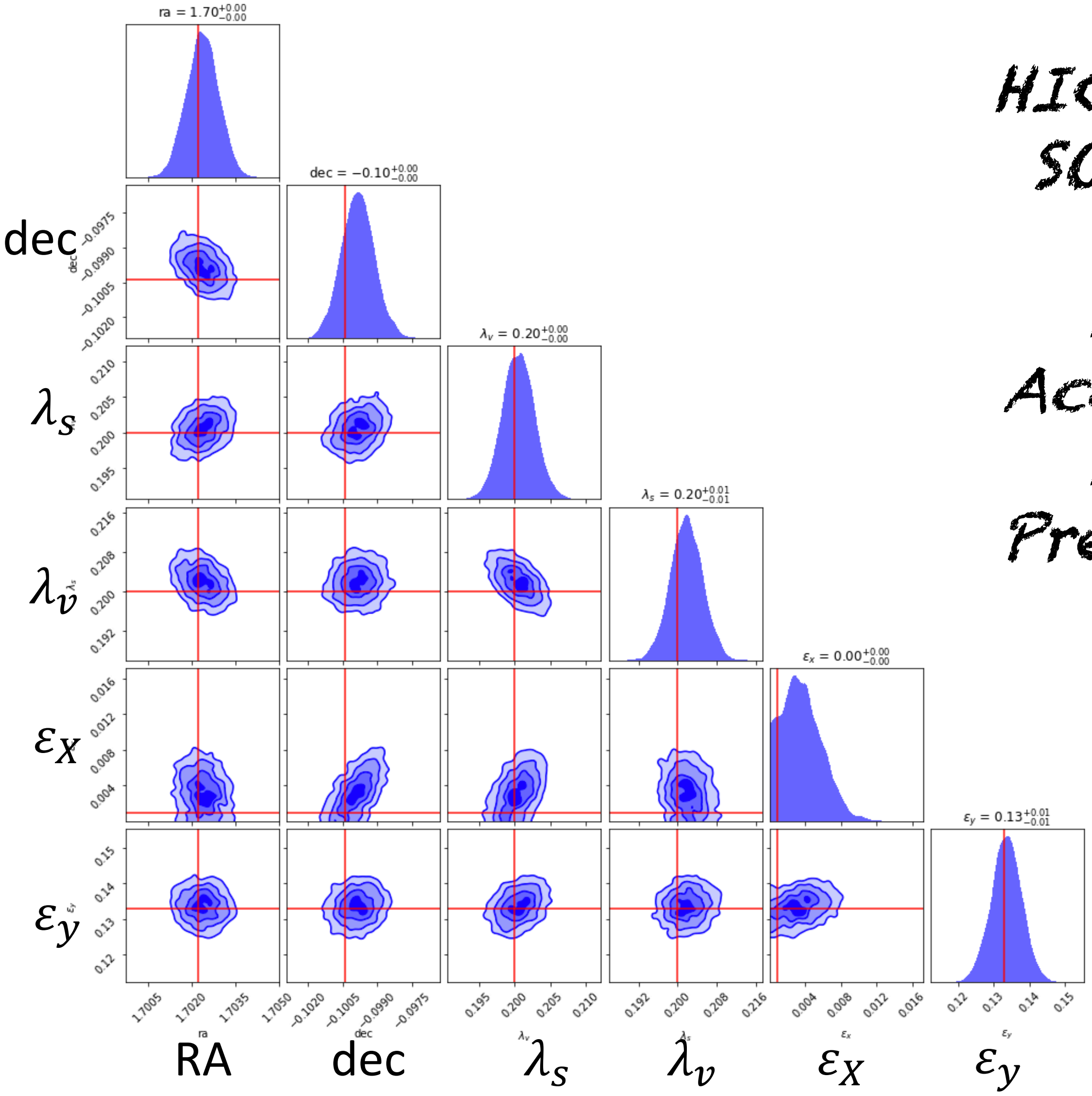
Live points in parameters space

$$\approx\sum_{i=1}^{N}L_i w_i$$

Credits: Littenberg

PARAMETER ESTIMATION

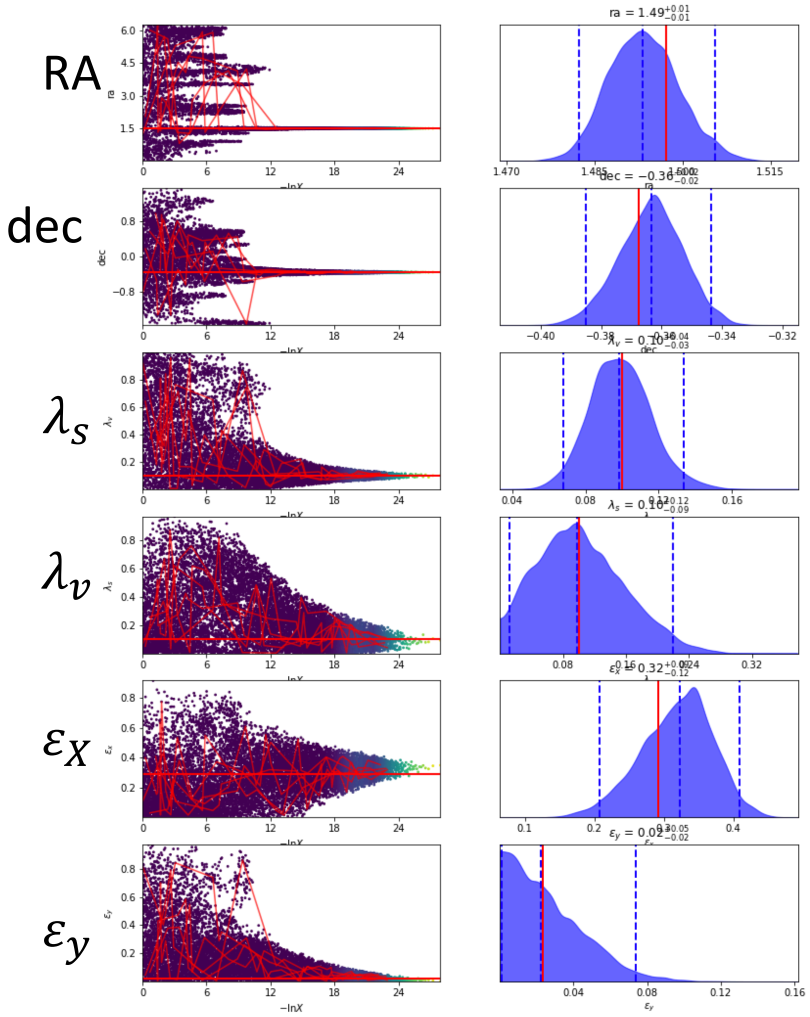


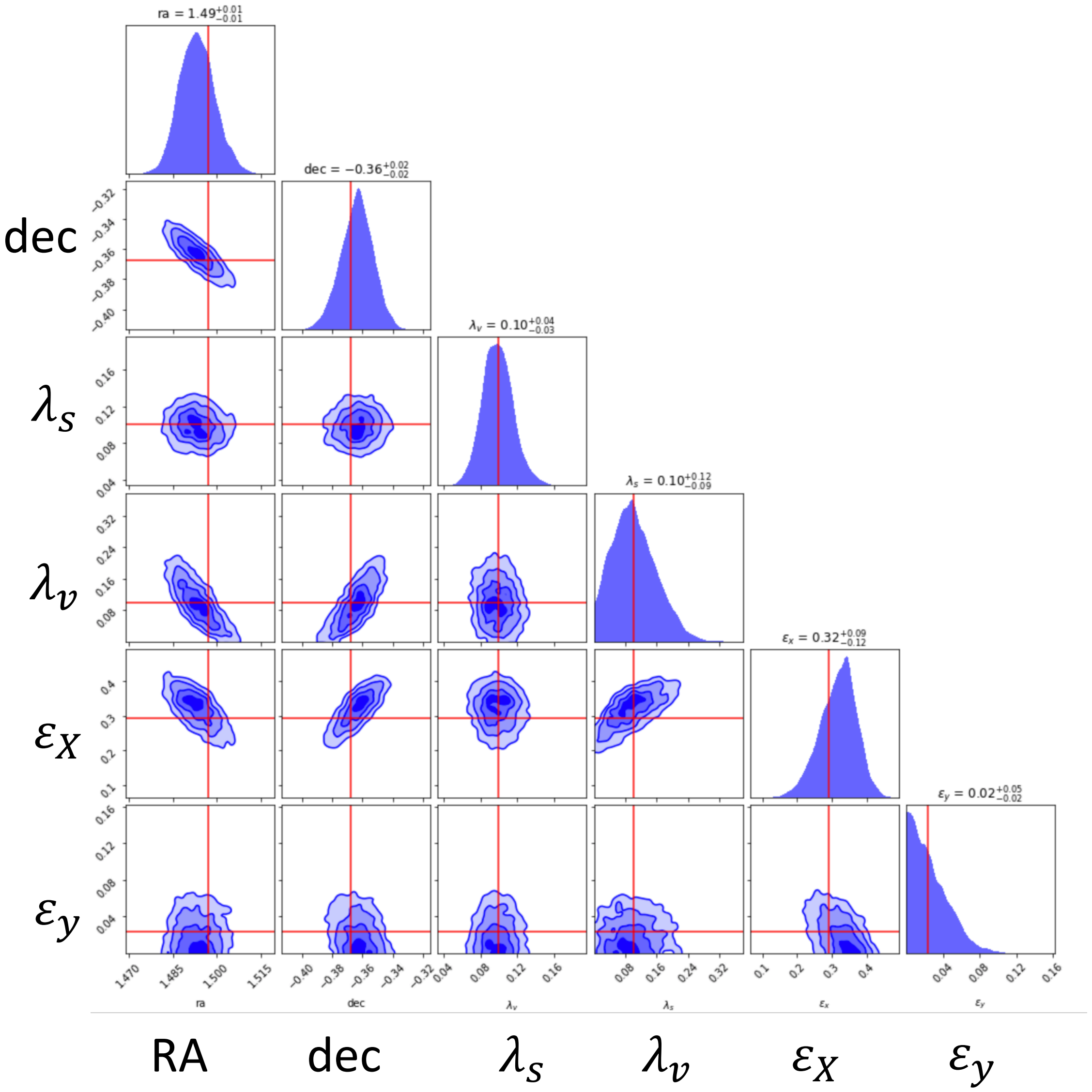


HIGH SNR SOURCE

High Accuracy
High Precision

PARAMETER ESTIMATION

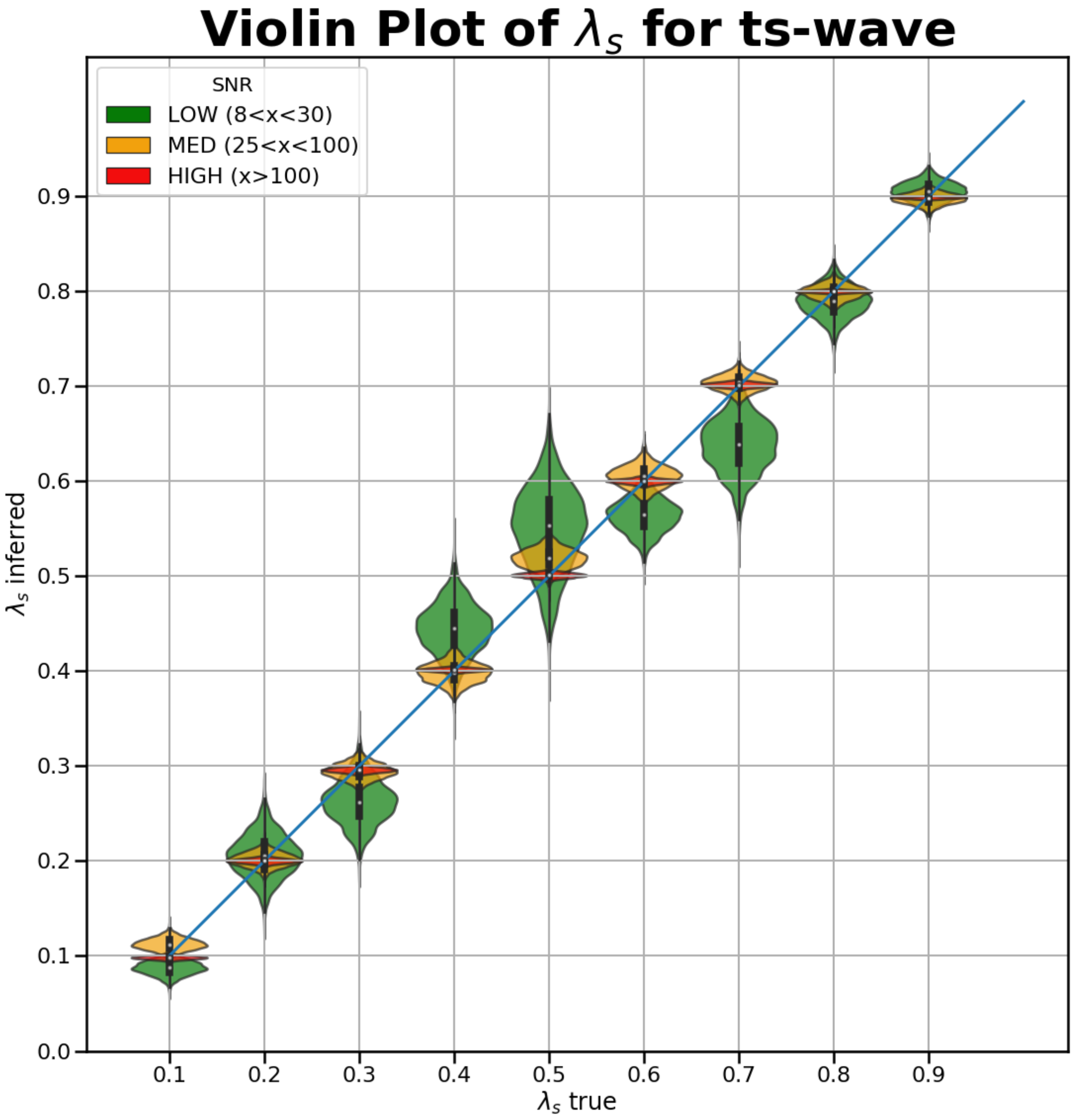


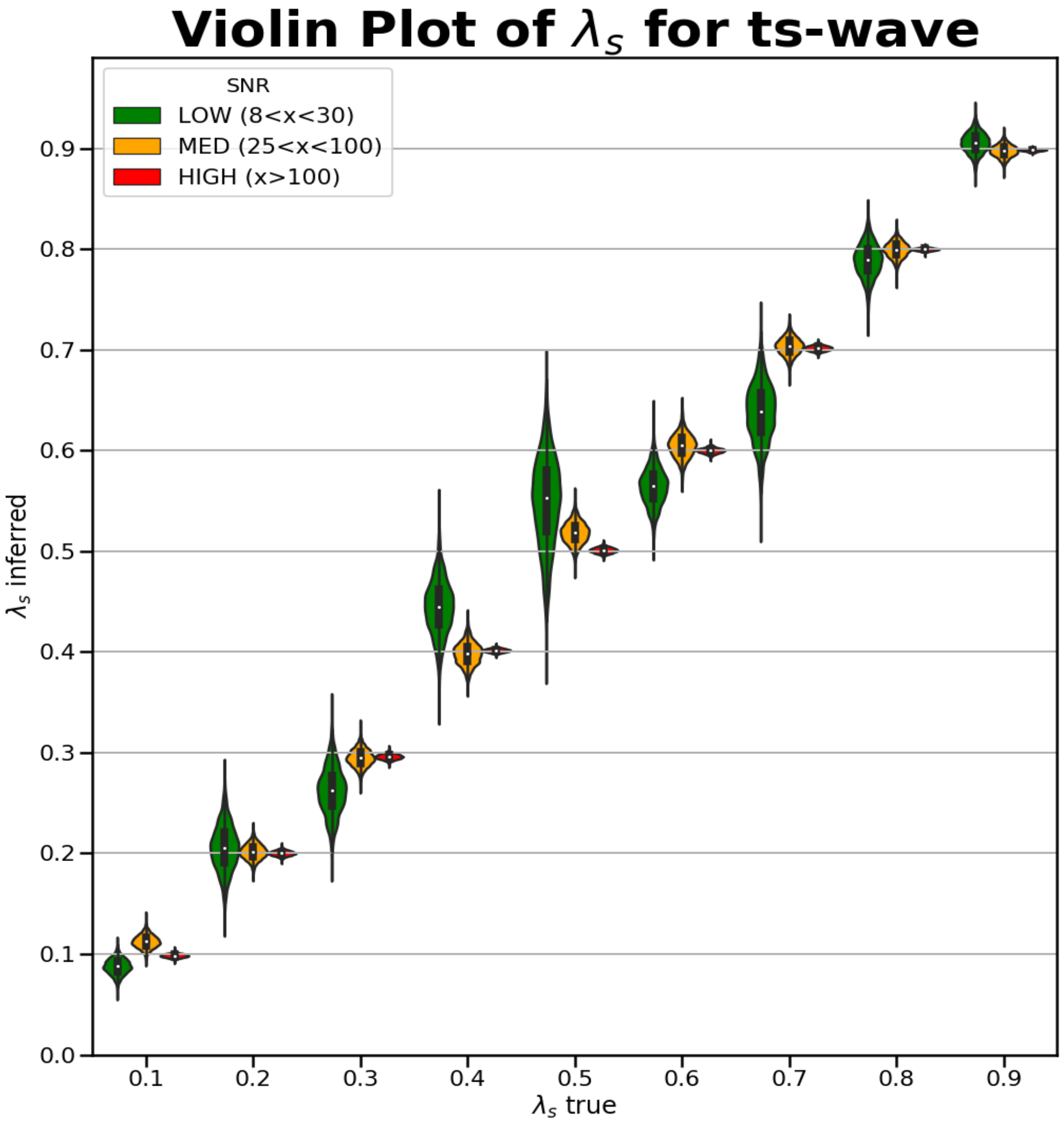


LOW SNR SOURCE

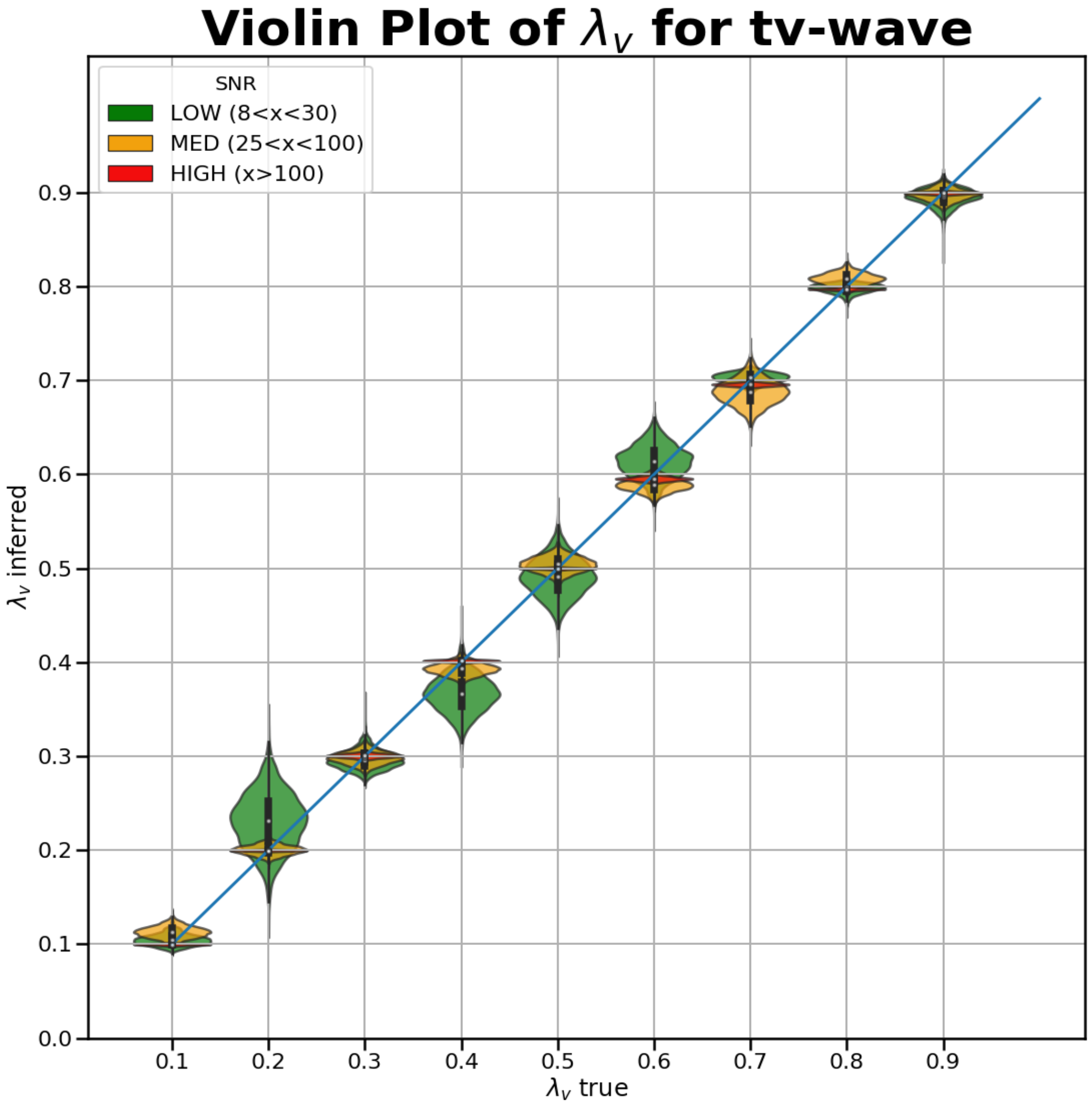
High Accuracy Lower Precision

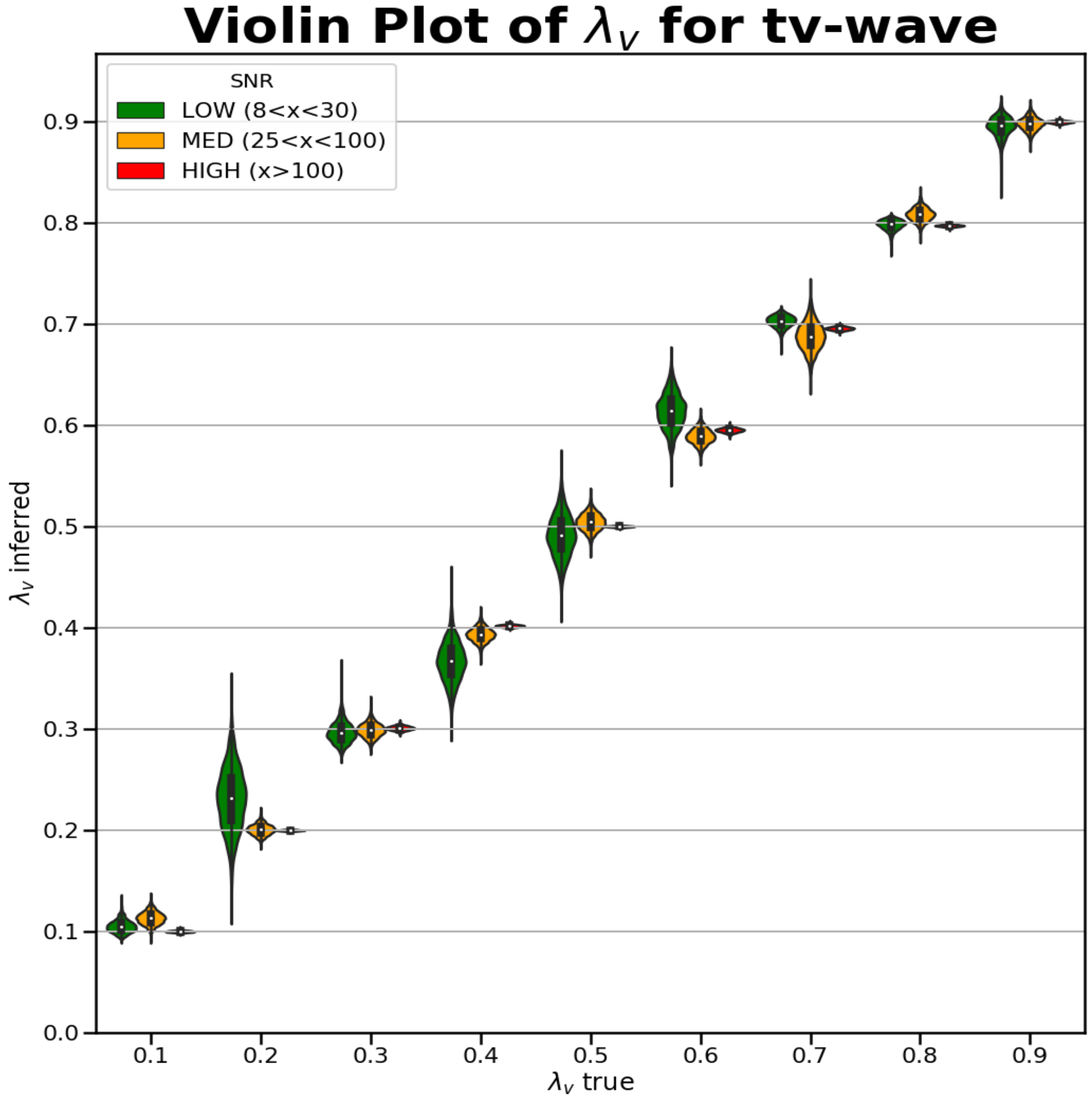
LAMBDAS ESTIMATION





LAMBDAS ESTIMATION


Violin Plot of λ_v for tv-wave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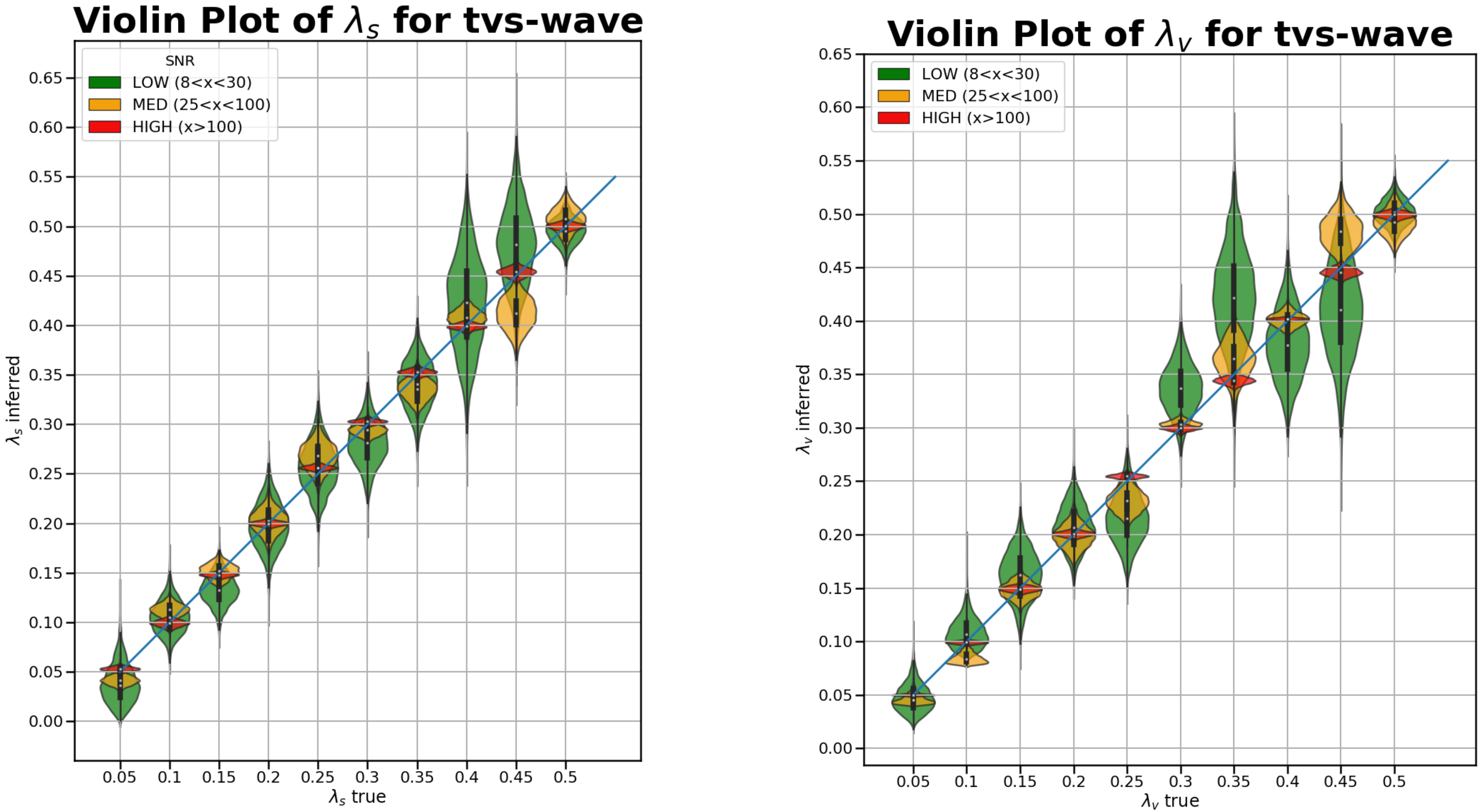
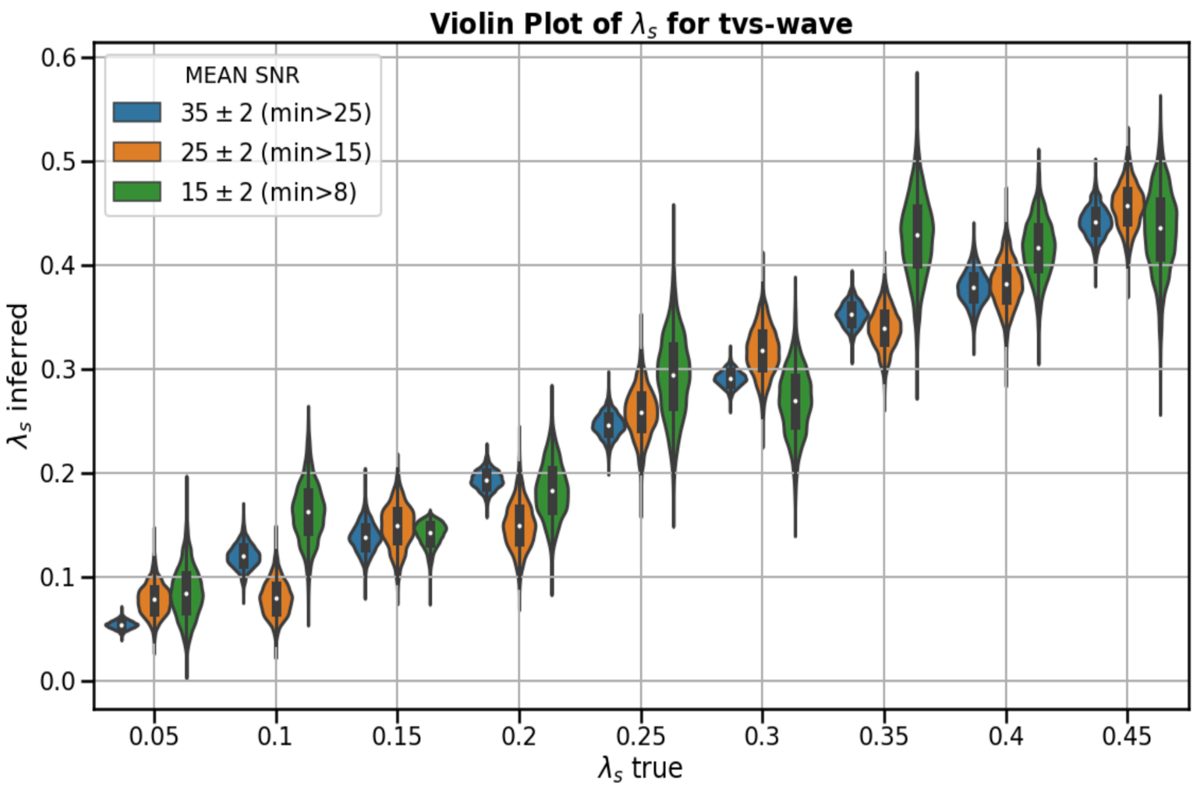
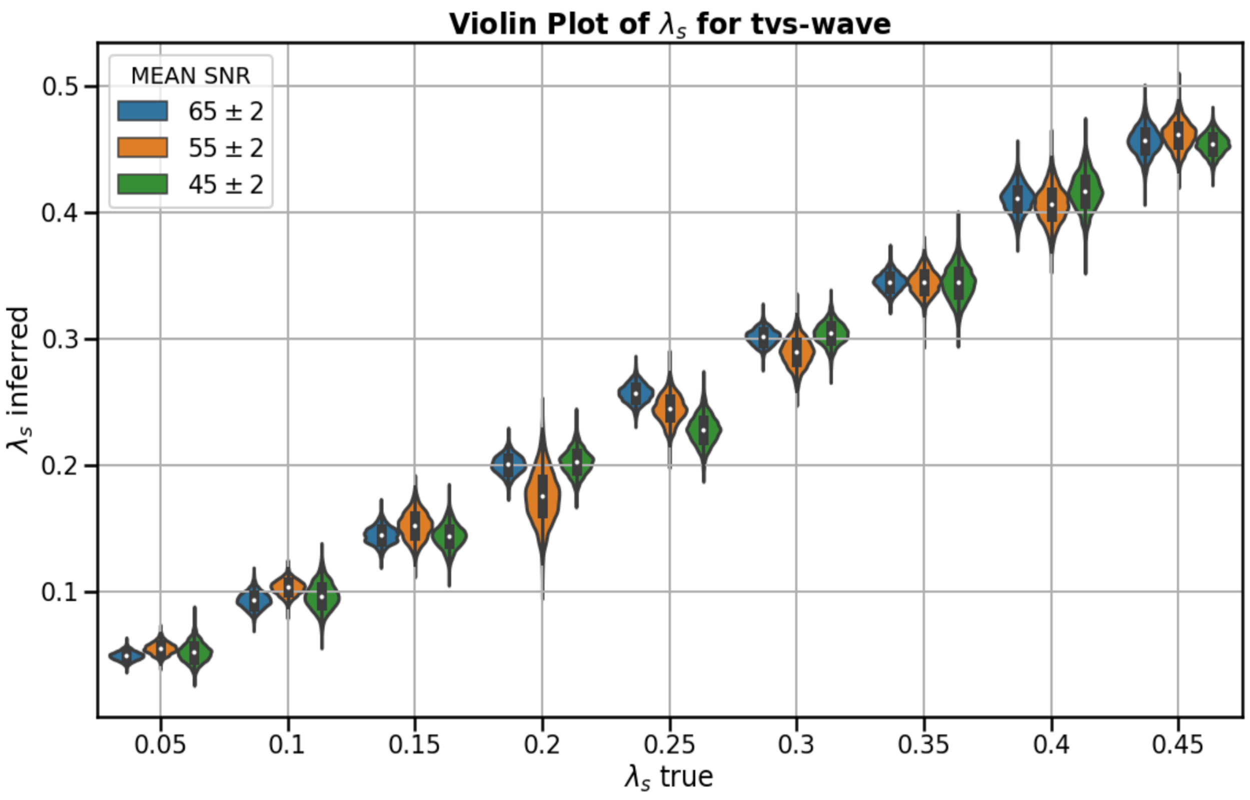


LAMBDAS ESTIMATION

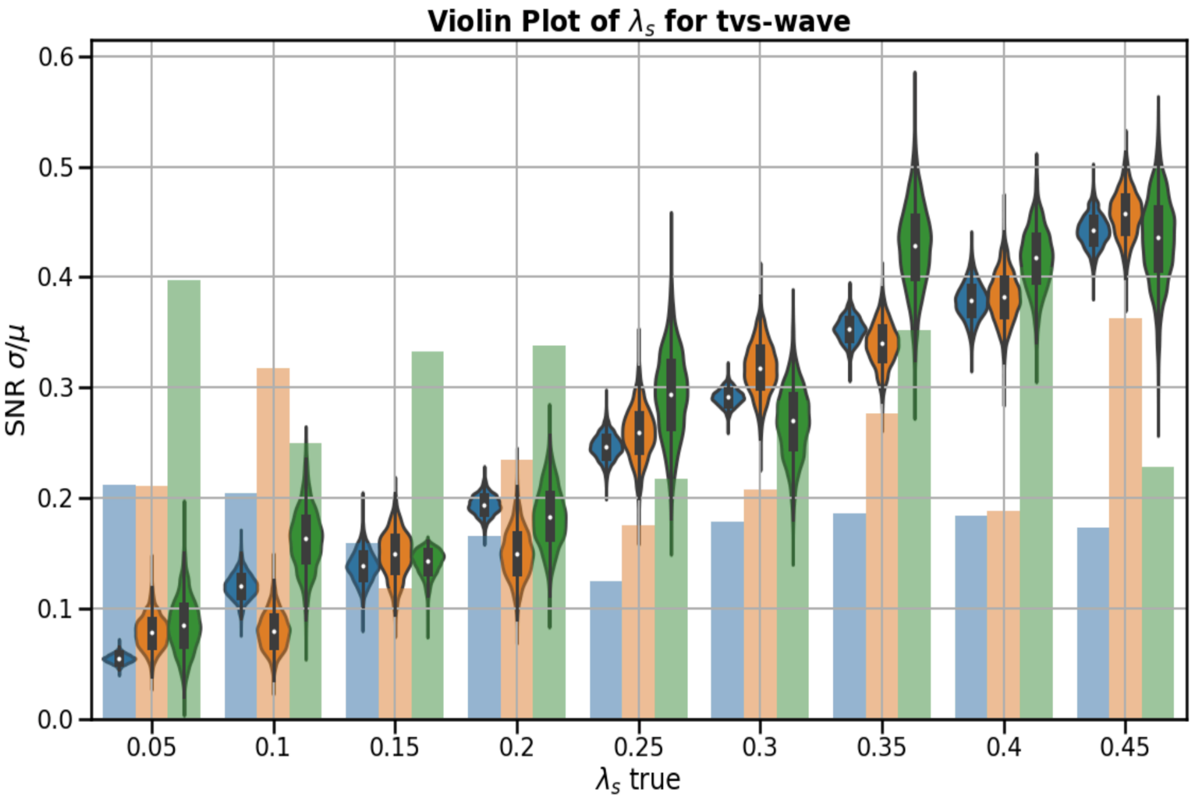


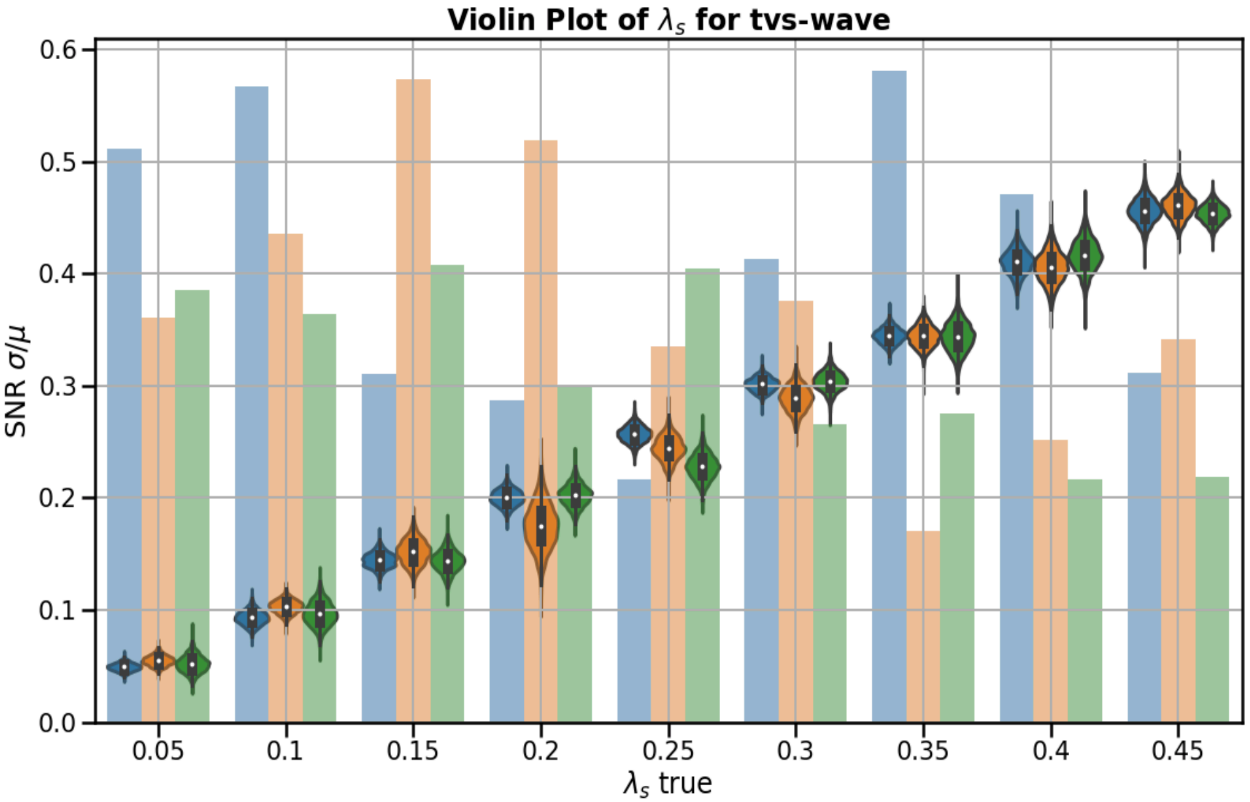
Violin Plot of λ_s for tvs-wave



Violin Plot of λ_s for tvs-wave

LAMBDAS ESTIMATION





FUTURE WORK

- More quantitative analysis on the lambdas distribution as a function of external parameters (sky position, overall amplitude, max SNR ...)

- Repeat every simulation with a pure tensor model, to compute *model selection odds*

- Using an extended post-Einsteinian Framework to compute new templates with complete polarization content (Arxiv)

Model-Independent Test of General Relativity:
An Extended post-Einsteinian Framework with Complete Polarization Content

Katerina Chatziioannou, Nicolás Yunes, and Neil Cornish
Department of Physics, Montana State University, Bozeman, MT 59718, USA.
(Dated: May 16, 2017)

THANK YOU FOR YOUR ATTENTION !

AND A SPECIAL THANKS GOES TO …

Alan J. Weinstein

Rico Lo Aaron Markovitz

Shruthi Aradhya Luca D'Onofrio

Liting Xiao Gabriele Vajente

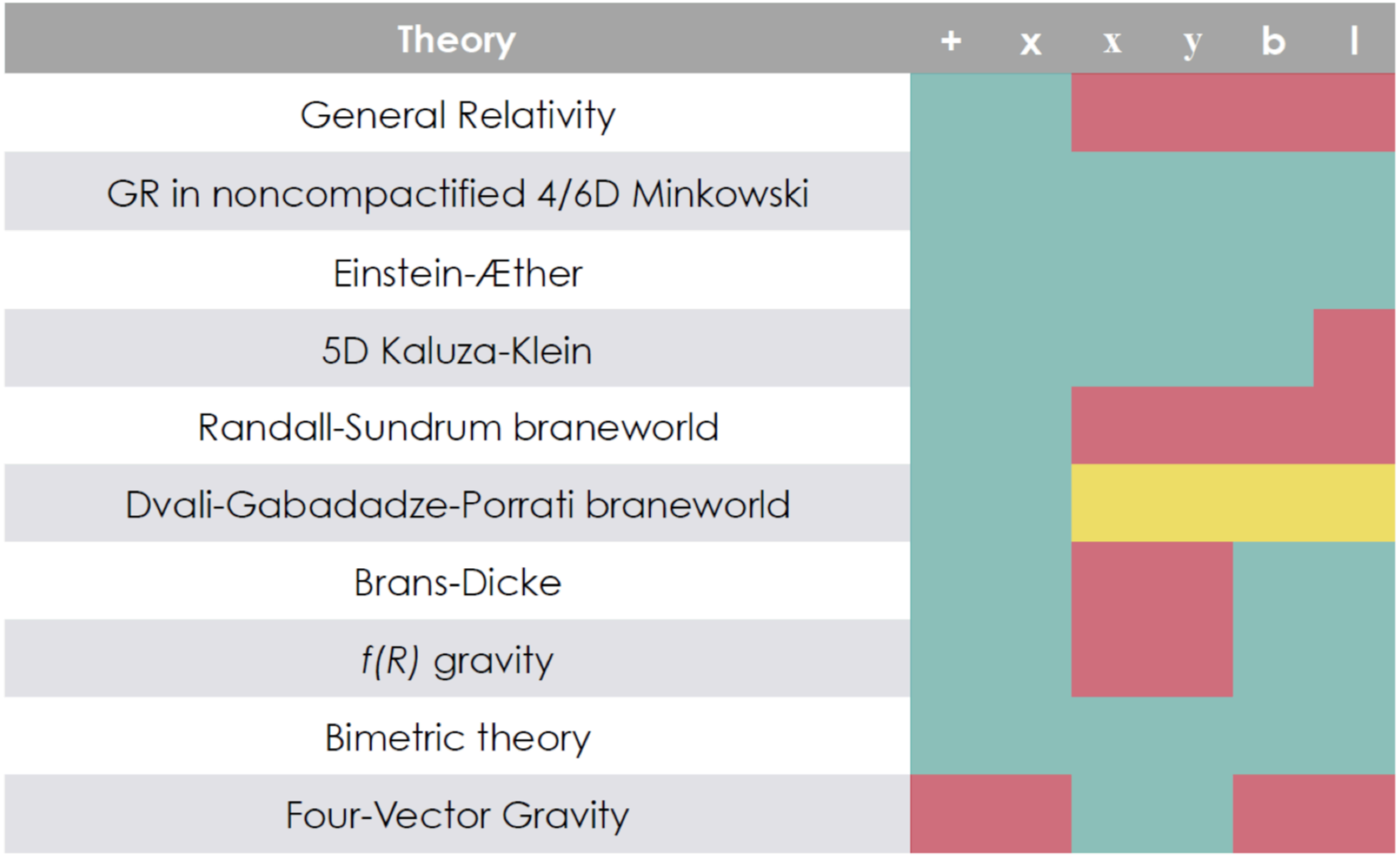
Francesco Pannarale Alvin Li

… and all my fellow SURFers !

INFN VIRGO

Caltech SAPIENZA UNIVERSITÀ DI ROMA

ALTERNATIVE METRIC THEORIES OF GRAVITY



Theory	+	x	x	y	b	l
General Relativity	🟩	🟩	🟥	🟥	🟥	🟥
GR in noncompactified 4/6D Minkowski	🟩	🟩	🟩	🟩	🟩	🟩
Einstein-Æther	🟩	🟩	🟩	🟩	🟩	🟩
5D Kaluza-Klein	🟩	🟩	🟩	🟩	🟩	🟥
Randall-Sundrum braneworld	🟩	🟩	🟥	🟥	🟥	🟥
Dvali-Gabadadze-Porrati braneworld	🟩	🟩	🟨	🟨	🟨	🟨
Brans-Dicke	🟩	🟩	🟥	🟥	🟩	🟩
f(*R*) gravity	🟩	🟩	🟥	🟥	🟩	🟩
Bimetric theory	🟩	🟩	🟩	🟩	🟩	🟩
Four-Vector Gravity	🟥	🟥	🟩	🟩	🟥	🟥

Nishizawa et al., Phys. Rev. D 79, 082002 (2009) [except G4v & Einstein-Æther].

🟩 allowed / 🟨 depends / 🟥 forbidden

POLARIZATION ANGLE

- Conceptual distinction between *triaxal GR* and *free tensor* :

$$\Lambda_{\mathrm{GR}}(t) = \frac{1}{2} h_0 e^{i\phi_0} \left[\frac{1}{2}(1+\cos^2\iota) F_+(t;\psi) - i\cos\iota F_\times(t;\psi) \right]$$

$$\Lambda_{\mathrm{t}}(t) = \frac{1}{2} \left[a_+ e^{i\phi_+} F_+(t;\psi=0) + a_\times e^{i\phi_\times} F_\times(t;\psi=0) \right]$$

- Rotation of antenna patterns:

$$F_+(t;\psi') = F_+(t;\psi)\cos 2\Delta\psi + F_\times(t;\psi)\sin 2\Delta\psi,$$

$$F_\times(t;\psi') = F_\times(t;\psi)\cos 2\Delta\psi - F_+(t;\psi)\sin 2\Delta\psi,$$

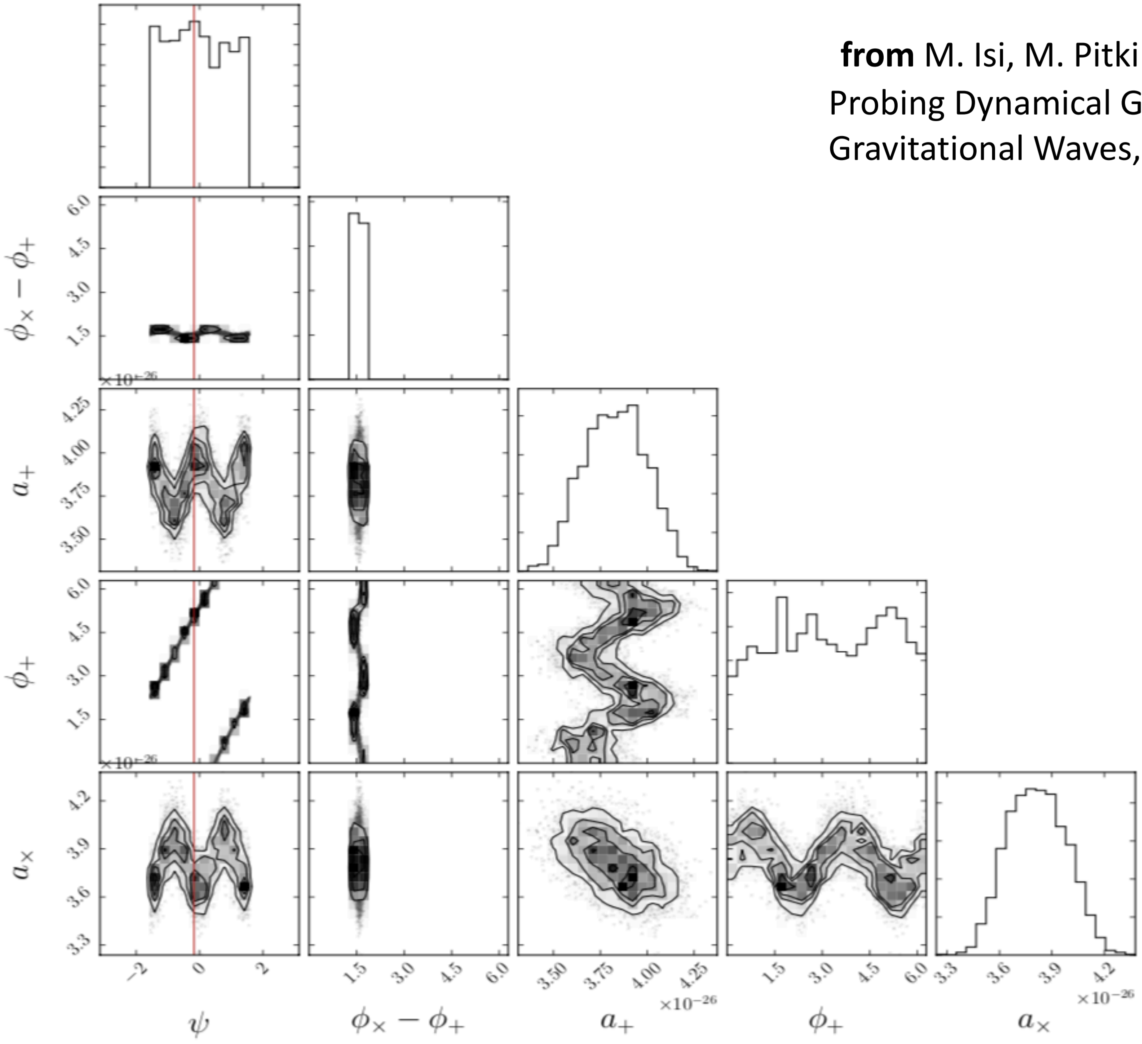
- Degeneracy between a_p and ψ:

$$a'_+ e^{i\phi'_+} = a_+ e^{i\phi_+}\cos 2\Delta\psi - a_\times e^{i\phi_\times}\sin 2\Delta\psi,$$

$$\underbrace{a'_\times e^{i\phi'_\times}}_{\psi \text{ fixed}} = \underbrace{a_\times e^{i\phi_\times}\cos 2\Delta\psi + a_+ e^{i\phi_+}\sin 2\Delta\psi.}_{\psi \text{ varying}}$$

POLARIZATION ANGLE

- Degeneracy between a_p and ψ:



from M. Isi, M. Pitkin, and A. J. Weinstein, Probing Dynamical Gravity with the Polarization of Continuous Gravitational Waves, arXiv:1703.07530

PSD & MATCHED FILTERING

- Power Spectral Density:

$$\boxed{S_y(f) \equiv \lim_{T\to\infty} \frac{2}{T} \left| \int_{-T/2}^{+T/2} [y(t) - \bar{y}] e^{i2\pi f t} dt \right|^2 .}$$

- Physical meaning:

$$\boxed{\left(\begin{array}{c} \text{rms value of } y\text{'s oscillations} \\ \text{at frequency } f \text{ in a very narrow bandwidth } \Delta f \end{array} \right) \simeq \sqrt{S_y(f)\Delta f} .}$$

- Filtering a noisy signal:

$$Y(t) = s(t) + y(t), \quad S \equiv \int_{-\infty}^{+\infty} K(t) s(t) dt , \quad N \equiv \int_{-\infty}^{+\infty} K(t) y(t) dt$$

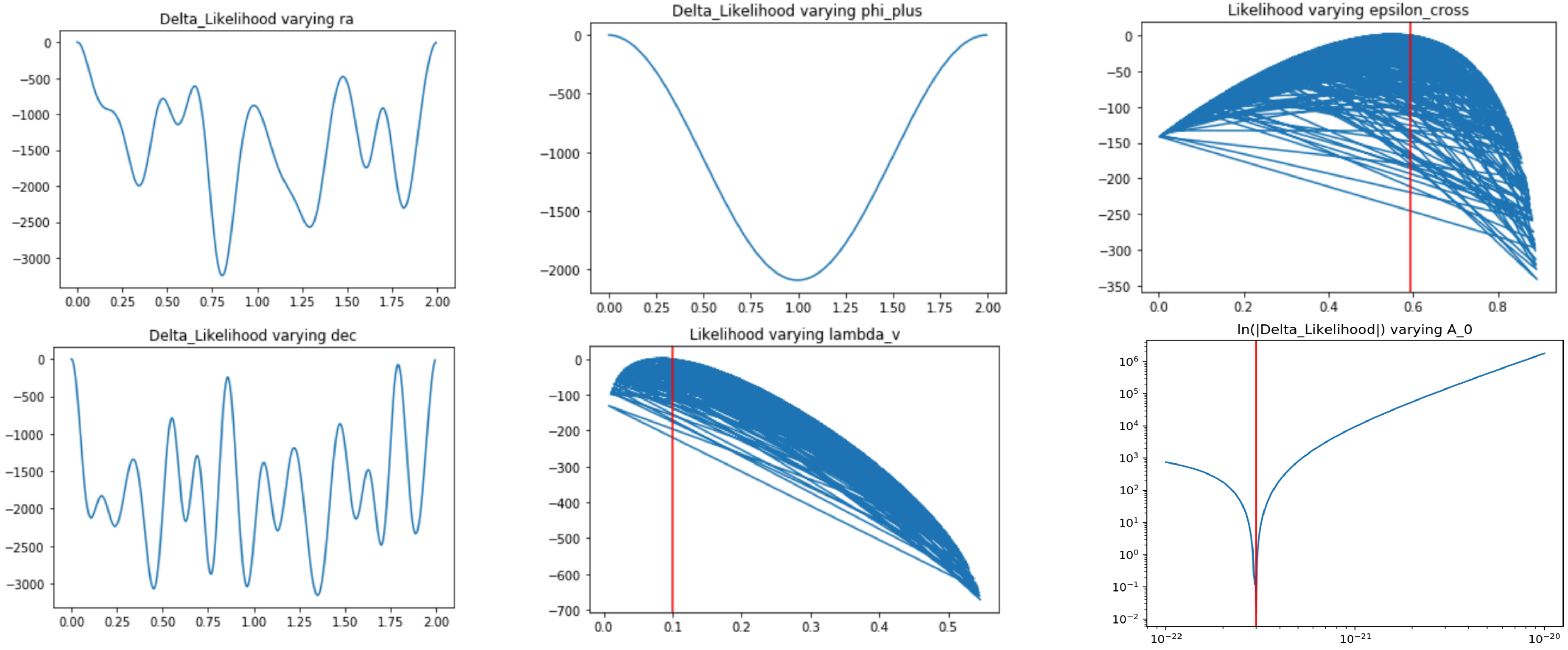
- Wiener's optimal filter:

$$\boxed{\tilde{K}(f) = \text{const} \times \frac{\tilde{s}(f)}{S_y(f)} ,}$$

maximizes

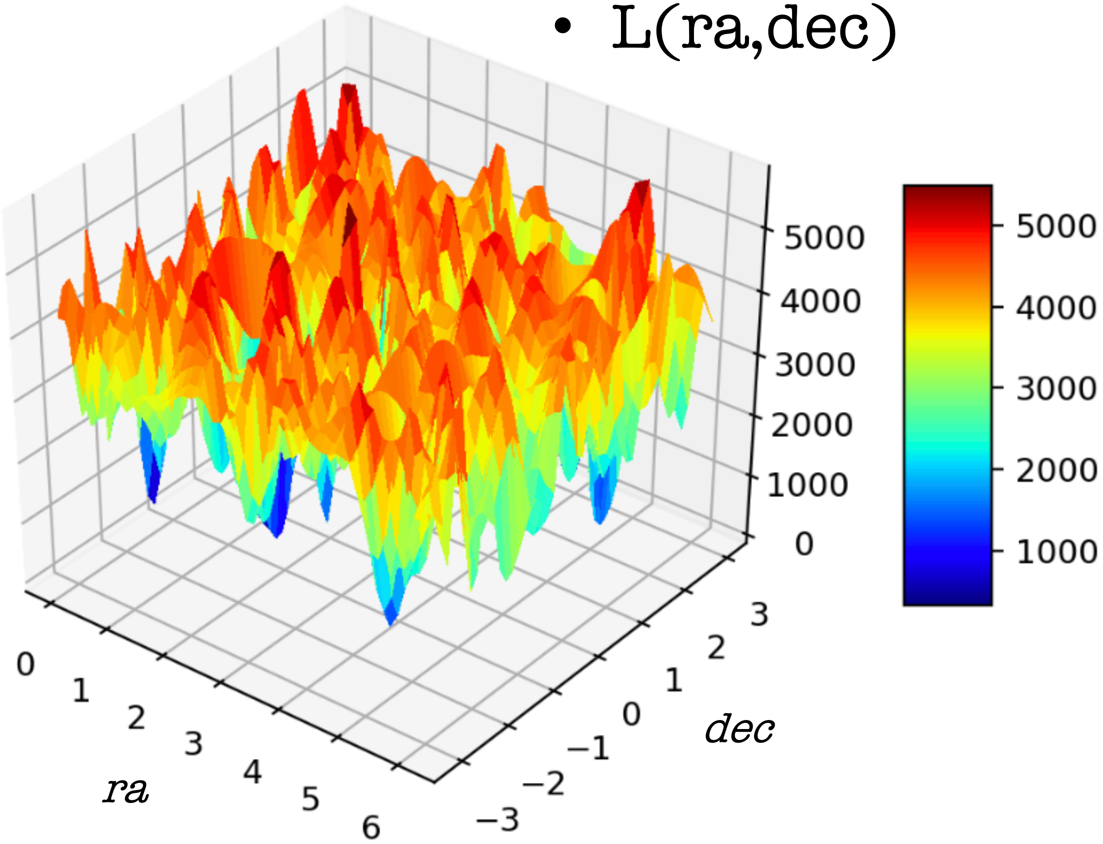
$$\frac{S}{\langle N^2 \rangle^{\frac{1}{2}}}$$

MONODIMENSIONAL STUDY OF THE LIKELIHOOD

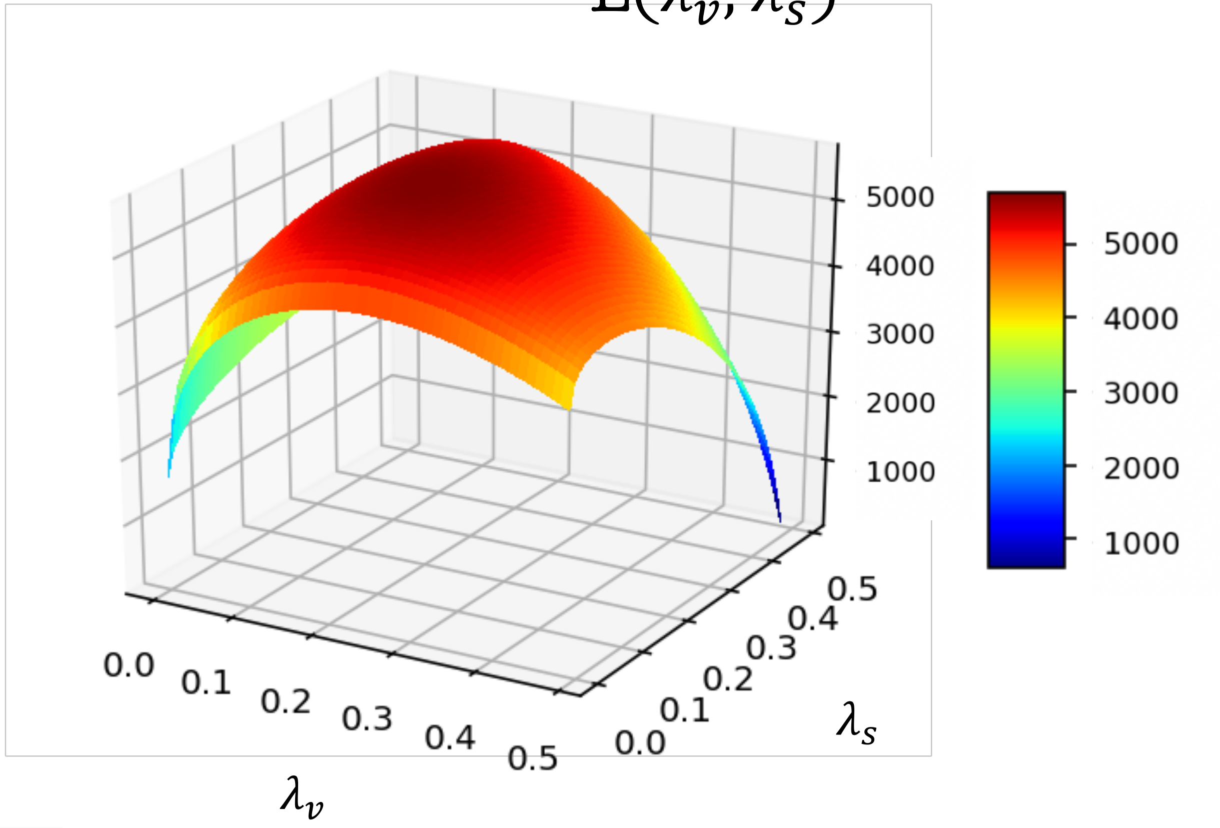


BIDIMENSIONAL STUDY OF THE LIKELIHOOD

- L(ra,dec)



- $L(\lambda_v, \lambda_s)$



NESTED SAMPLING

Evidence Numerical Estimation

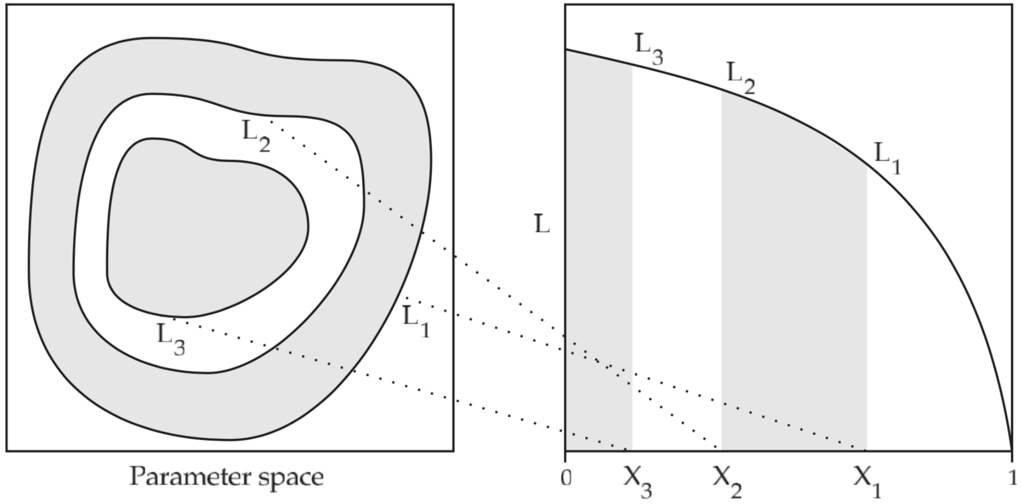
$$Z = \int_{\Theta} p(\vec{\theta}|\mathcal{H}, I)\,p(\vec{d}|\mathcal{H}, \vec{\theta}, I)\,d\vec{\theta} \approx \sum_{i=1}^{N} L_i w_i$$

Live points

Multidimensional Integral

Xi -> Normalized Volume of the prior with a likelihood greater than the lowest likelihood point of your set in each step i

1. Sample from the prior N live points
2. Find the point with the lowest Likelihood L*
3. Replace this last with another point from the prior with L > L*
4. Repeat (2)-(3)



$$X(\lambda) \equiv \int_{\Theta:\mathcal{L}(\Theta)>\lambda} \pi(\Theta)\,d\Theta$$

$$\mathcal{Z} = \int_{0}^{+\infty} X(\lambda)\,d\lambda = \int_{0}^{1} \mathcal{L}(X)\,dX$$

Monodimensional integral