LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Measuring mechanical loss in cryogenic gentle nodal suspension resonator

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1 Introduction

Thermal noise is important in areas such as gravitation wave detection and mechanical resonators since they rely upon extremely precise measurements. When measurements become extremely precise, quantum observables are being measured. These quantum observables can be measured in an isolated system, but in an environment with uncontrolled degrees of freedom, classical noise becomes a limiting factor of precision. This precision is limited by thermal fluctuations, prompting our focus on improving the quality factor of mechanical oscillators in the Gentle Nodal Suspension system. In order to minimize energy loss, we must better understand how competing frequencies depend on different mechanical losses. The resonator oscillation frequency can couple with different sources throughout the system. We will work to isolate these mechanical losses in the system, model and experimentally verify their frequency dependence, in order to lower this loss in the suspension system.

This experiment is a cryogenic version of the Gentle Nodal Suspension system for measuring Q-factors of mechanical resonators. In order to increase the Q-factor of this system, we will focus initially and substantially on the intrinsic loss in the silicon near the zero crossing point of its thermal expansion coefficient. At this zero point, some dominant losses and noise contributions drop to zero. A better understanding of the losses in the silicon could prove invaluable, as this thin film is used to coat highly reflective surfaces in the LIGO cryogenics. These silicon coatings store a small fraction of the energy in the acoustic modes, so the additional mechanical losses must be even smaller. To accurately measure the silicon coating losses, other losses must be subdominant.

The LIGO observatories in Washington and Louisiana were offline for several months while the components underwent upgrades to increase the detectors sensitivity. Research to improve the Q-factor of the silicon wafers could help to further improve the sensitivity in coming stages of LIGO development.

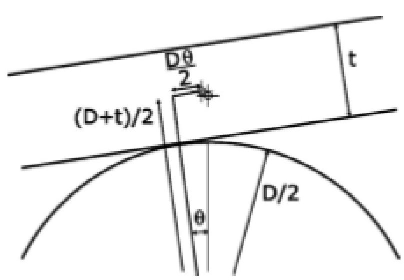

Figure 1: Diagram of Gentle Nodal Suspension system: the spherical structure touches the silicon disk at a single node. The silicon wafer is drawn as the angled piece suspended on the hemisphere.

We will calculate a loss budget for the silicon wafers, looking at surface and bulk losses and what happens at the contact point of the suspension system.

2 Background

2.1 Cryogenic silicon mirrors

The quality of gravitational wave detections in the next generation of LIGO (LIGO Voyager) facilities are limited by thermally induced vibrations in the mirrors which becomes noise in the interferometers. In LIGO Voyager, quantum noise will be brought down, which will cause thermal noise to become the limiting factor in measurement accuracy. In order to reduce thermal noise, low temperature mirrors made of high purity single-crystalline silicon will be used. One further complication comes from the potential of mechanical vibrations of the mirror and any other surfaces at these cryogenic temperatures. Such vibrations can cause a noise inducing phase shift on the light within the interferometer. Thermal noise appears in Brownian motion. Direct displacement noise in the form of Brownian noise is one form of thermal noise. Others include thermally induced shifts in the refractive index of the material and thermally induced strain causing the optics to change shapes. Brownian noise is the main focus for highly reflective coatings. To tackle this problem, thin film silicon coatings are used. Silicon is a clear choice at cryogenic temperatures its mechanical loss decreases with temperature, which is not seen in fused amorphous silica. At 124 K silicon's thermal expansion coefficient crosses zero and at this zero crossing, thermoelastic component of the thermal noise is eliminated. Also, silicon has very high thermal conductivity. This allows for higher laser powers since the thermal lensing of the mirror is reduced, and it is easier to remove any absorbed heat.

Silicon is an anisotropic crystal with cubic symmetry. Its Young's modulus varies along different directions in the material relative to the crystal orientation.

3 Experimental Overview

This experiment will measure the mechanical losses in silicon wafers, characterizing such results mathematically and experimentally. We can roughly model the silicon system as several coupled and damped harmonic oscillators representing the bulk disk, coating, and any additional domains of interest. Then, we will explore the effects of various damping sources such as pressure, thermoelastic, and others. These losses will have different frequency dependencies across the wafer. After modeling the effects of a single source of damping and experimentally finding its frequency dependence, the results can be generalized to combinations of multiple loss sources. We can represent the total loss effect as weighted sums of our various sources, with the weighting representing each difference frequency dependence.

Using an amplitude-locked loop we can calculate how much energy a mode is dissipating by measuring how much energy is put into the mode at equilibrium. This allows continuous measurement of Q-factors to be taken, which will help estimate these loss contributions with less systematic error. This amplitude-locked loop will allow us to vary the parameters of our experiment and directly see the impact on the measured loss, at many eigenfrequencies at once.

4 Calculations

As explained previously, we will model the silicon wafer as a system of masses and springs in order to calculate the frequency of oscillations in response to different damping values, representing our sources of mechanical losses.

To simplify initial calcualtions, we can model a system of two masses, both under a damping force γ connected by three unique springs. We can calculate our generic equations of motion for this system.

(Eq. 1)

$$\frac{d^2x_1}{dt^2}m_1 = -k_1 - k_2(x_1 - x_2) - \gamma_1\frac{dx_1}{dt}$$

$$\frac{d^2x_2}{dt^2}m_2 = -k_3 - k_2(x_2 - x_1) - \gamma_2\frac{dx_2}{dt}$$

(Eq. 2)

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x_1} \\ \ddot{x_2} \end{bmatrix} = \begin{bmatrix} -k_1 - k_2 & k_2 \\ k_2 & -k_3 - k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix}$$

To find oscillation frequencies, the general solution $x(t) = A_o e^{i\omega t}$ can be plugged back into Eq. 2 in order to solve for the frequencies of oscillation.

Taking a simple example; we can model equal masses, spring constants, and damping factors. Our matrix equation will become

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x_1} \\ \ddot{x_2} \end{bmatrix} + \begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix}\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} + \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Assuming the general solution of x(t), our two resulting eigenfrequencies are

$$\omega = \frac{\sqrt{(4km - \gamma^2)}}{2m}, \frac{\sqrt{(12km - \gamma^2)}}{2m}$$

Setting mass equal to 1 and the spring constant equal to 2, we can graph the frequency response as the damping factor is increased from 0 to 3. This results in the expected relationship, frequency decreasing as damping increases.

The damping factors in our silicon is extremely small and the effect on frequency is so small its practically neglible. Because of this we will instead analyze our time constant in ω, and include an exponetial term which depends on τ. The new general solution of $x(t)$ will be applied to the original, simplified equation of motion matrix.

$$x(t) = Ae^{i\omega t}e^{-t/\tau}$$

This general equation generates two slightly different frequencies.

$$\omega = \frac{-2im\tau - i\tau^2\gamma \pm \tau^2\sqrt{12km - \gamma^2}}{2m\tau^2}, \frac{-2im\tau - i\tau^2\gamma \pm \tau^2\sqrt{4km - \gamma^2}}{2m\tau^2}$$

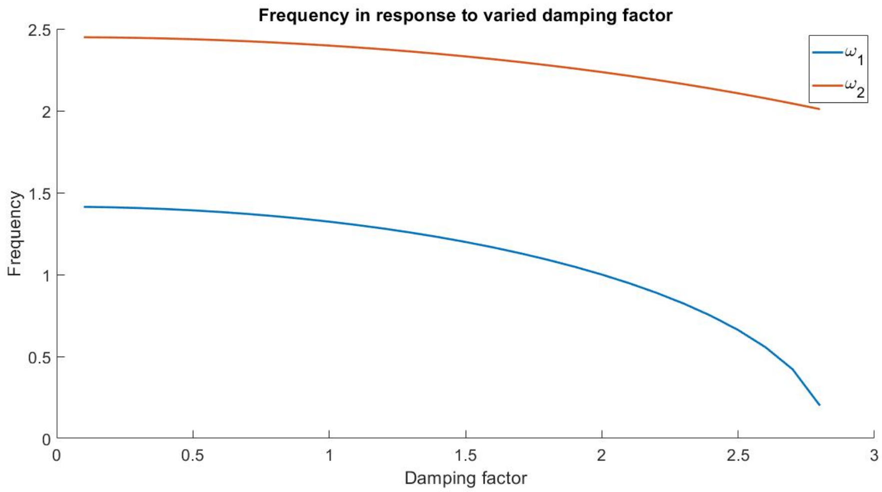

Figure 2: Decreasing frequency in response to increasing damping

Then you force the identification on frequency, that

$$\omega_{\text{full}} = \omega_{\text{effective}} + \frac{i}{\tau}$$

. This given a definition of frequency of the purley periodic, not decaying parts of the calculated ω oscillations. τ is defined as the rate of exponetial decay envelope of the oscillation. The non-decaying part of the frequency is the same as calculated previously,

$$\omega = \frac{\sqrt{(4km - \gamma^2)}}{2m}, \frac{\sqrt{(12km - \gamma^2)}}{2m}$$

And the exponetual rate of decay of the envelope in our ω is $\frac{-2im/\tau - i\gamma}{2m}$. Following the previous equation for ω_{full} we define this expoential decay as $\frac{i}{\tau}$ which results in our time constant of the envelope decay as

$$\tau = 2m/\gamma$$

Since calculting the time constant of the frequency, we can not measure the response of Q-factor to changing τ- which is a much more helpful measurement with our small damping factors. Since τ dependends on γ, were are still looking at a response to changing damping - but the change in Q-factor is more significant that measuring frequency effects directly.

$$\text{Loss Angle} = \phi = \frac{1}{Q} \text{ where } Q = 2\pi f\tau$$

As in the previous calculations, setting mass to 1 and spring constant to 2, we can plot the expected response to Q factor and loss angle as damping increases. Plotting γ from 0.1 to 2, Q factor decreases with increasing damping and phase angle has the inverse response.

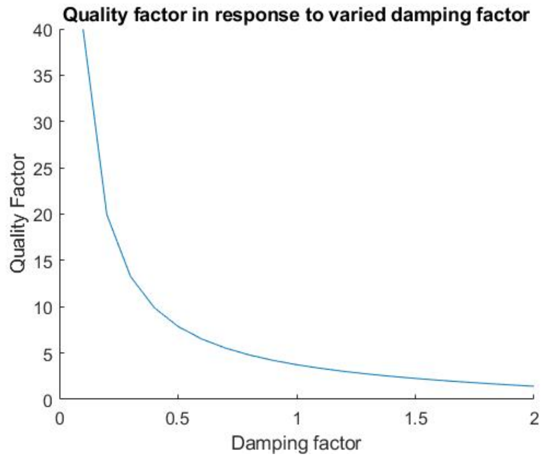

Figure 3: Q-factor decreases as damping increases, follows logic of zero damping leading to very high Q-factors

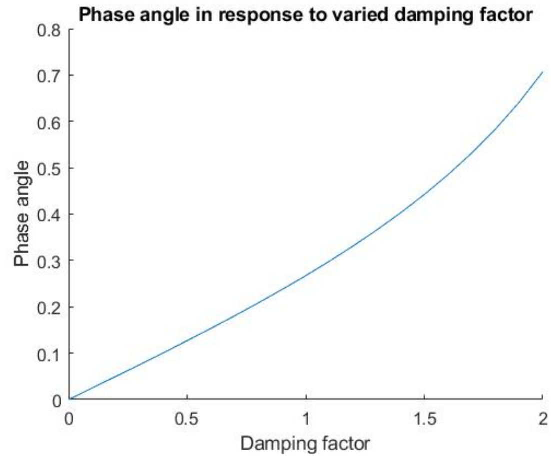

Figure 4: Phase angle increase with increasing damping, since phase angle ϕ is the inverse of Q-factor

5 Frequency dependence of mechanical losses

Friction causes multiple frequencies in the resonator. To analyze these frequencies, we can read out the resonator's oscillations and Fourier transform the background subtracted signal to see the different peaks, frequencies, in the system. Such eigenfrequencies interact and couple, causing an exponential ring down in the time domain. In the frequency domain, we can analyze the Lorentzian peak's overlap. A smaller width corresponds to a higher Q-value as the decay rate is due to coupling. Therefore it is important to understand the frequency dependences of the mechanical losses, because we can analyze their coupling and determine which source the loss is coming from. Surface losses will fall of like 1/f, while bulk losses will not change with frequency. This will allow us to create a detailed loss budget, describing what is causing the loss in the system, and to what degree.

To measure these mechanical losses, we first determine the loss's frequency dependence. To measure this dependence, we will increase the loss source, effectively isolating the desired mechanical factor and then measure the change in frequency. The change in the eigenfrequencies and modes from increasing the loss source corresponds to the how the loss changes with frequency.

Once we have looked at all the mechanical losses independently, we will look at multiple losses and their interactions. To do so, we can fit a weighted sum of the losses to match the experimental results and determine which sources contribute more loss at varying frequencies.

6 Project timeline

Week 1: I will begin the set-up of my experiment, work to understand the current resonator/cryogenic system and how to collect necessary data. Model our spring-mass model in Python.

Week 2-3: Multi source mechanical loss calculations and measurements. Fit weighted averages and calculate expected results, compare different mode shapes

Week 4: Learn COMSOL, use Monte Carlo simulations to model the frequency dependence of damping

Week 5-6: Thermoelastic dependence, computational modeling work and data analysis

Week 7-8: Surface loss dependence measurements, and any additional mechanical loss sources

Week 9: Take pressure dependence measurements and analyze results. Calculate expected results using Python mode

Week 10: Sum results and create final presentations.

References

[1] Brett Shapiroa, Rana X.Adhikari *Cryogenically cooled ultra low vibration silicon mirrors for gravitational wave observatories.* Cryogenics Volume 81, January 2017, Pages 83-92 (2017)

[2] E. Cesarini, M. Lorenzini *A “gentle” nodal suspension for measurements of the acoustic attenuation in materials.* Rev. Sci. Instrum. 80, 053904 (2009)

[3] J. P. Zendri, M. Bignotto *Loss budget of a setup for measuring mechanical dissipations of silicon wafers between 300 and 4K.* Review of Scientific Instruments 79, 033901 (2008)