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Exploration of Metamaterial Designs for LIGO Mechanical Systems

Author: Mahiro Abe[1]
Mentors: Aaron Markowitz[2], Christopher Wipf[2]
[1] *Cornell University, Physics*
[2] *California Institute of Technology, LIGO Project*

California Institute of Technology
LIGO Project, MS 18-34
Pasadena, CA 91125
Phone (626) 395-2129
Fax (626) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project, Room NW22-295
Cambridge, MA 02139
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

LIGO Hanford Observatory
Route 10, Mile Marker 2
Richland, WA 99352
Phone (509) 372-8106
Fax (509) 372-8137
E-mail: info@ligo.caltech.edu

LIGO Livingston Observatory
19100 LIGO Lane
Livingston, LA 70754
Phone (225) 686-3100
Fax (225) 686-7189
E-mail: info@ligo.caltech.edu

http://www.ligo.caltech.edu/

Abstract

This project studies metamaterials for filtering mechanical vibration between 10Hz and 1kHz. We are interested in designing a metamaterial suspension that can mechanically isolate a silicon disk while maintaining thermal contact. Such a system will enable accurate high-Q measurements of silicon at cryogenic temperatures, which are necessary for characterizing the test mass thermal noise in next-generation gravitational detectors such as LIGO Voyager. Silicon metamaterials comprised of locally resonant structures are considered as a method of producing bandgaps near silicon disk modes, with analytical and numerical studies conducted on resonant structure candidates. Finite element analysis done on a combination of these structures shows promising filtering behavior. Future work for this project will involve fabricating and testing of candidate isolation systems. These metamaterial studies could also be useful in LIGO noise evasion, including the filtering of parametric instabilities in the test masses and seismic noise isolation.

1 Introduction

Gravitational waves, first predicted a century ago by Einstein's theory of general relativity, were detected for the first time in 2015 by the twin LIGO detectors in Livingston, Louisiana and Hanford, Washington [1]. This has spurred an era of gravitational wave physics, with multiple detections over two observation runs in the past several years [2]. The extreme sensitivity necessary to detect gravitational wave signals ($\sim 10^{-23}/\sqrt{\mathrm{Hz}}$) requires precise control of various noise sources. There is significant effort put into improving the sensitivity of current LIGO detectors, to probe fainter gravitational wave signals from sources farther away.

LIGO Voyager is a detector upgrade being explored which will replace the existing mirror system with cryogenic silicon suspensions and test masses [3]. At 123K, silicon has zero thermoelastic loss, and the total mechanical loss is limited by Akheizer damping. The minimal losses introduced by cryogenic silicon mirrors is promising for reducing noise in the LIGO optics, but we require methods of accurately characterizing these optics and optical coating losses for the Voyager noise budget. One idea for studying the material properties of silicon and the optical coatings is with an isolated silicon disk in a cryogenically controlled system. With a sufficiently low loss angle ϕ of the disk, this system will be able to measure the losses of different thin film coatings deposited on silicon.

A standard experiment setup for measuring the losses of a disk resonator is the gentle nodal suspension (GeNS) [4]. The GeNS setup has the advantage of a single-point of contact between the disk and the environment. This provides maximal mechanical isolation for modes with a node at the suspension point. However, it remains a challenge to actively control the disk temperature in a cryogenic GeNS experiment. Here we propose an alternative mechanical isolation system that addresses this issue with mechanical metamaterials. A silicon metamaterial layer placed between the disk and the external environment can behave as a bandstop filter, effectively simulating the disk isolation behavior within the bandgap. Since this system will maintain a large surface of physical contact between the disk and the environment and will consist only of silicon, conductive cooling can be used to hold the system at cryogenic temperatures. We present the results and findings for a preliminary model of this silicon metamaterial system, designed to isolate a 2-inch-diameter disk near 1kHz.

2 Spring-Mass Model

A 1-D metamaterial system can be modeled by a finite chain of springs and masses in series. This model has been explored for probing phononic bandgaps in periodic layered materials [5]. However, the layered materials for a reflection coating must be on the length-scale of a wavelength, which is on the order of a kilometer for 1kHz in silicon. For the inch-scale silicon disk we will be fabricating on, the scale of such layers would be too large. However, well-designed resonator structures can accumulate phase and utilize the same type of interference behavior used in layered materials. We therefore consider each spring-mass cell along the chain as a resonator (Fig.1), and consider many resonators arranged in series. To study the filtering behavior of such a system, a harmonic driving force is applied to one end of the spring-mass chain, and the force-to-displacement transfer function on the other end is calculated. An arbitrary damping term is introduced to the models in the form of complex spring constants.

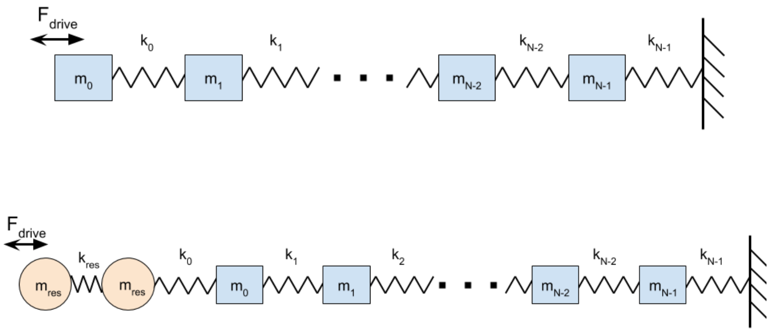

Figure 1: The top diagram is a spring-mass model in which resonators are connected in series. A driving force is applied to the mass on the left end. The corresponding transfer function is proportional to the displacement of the mass on the right end. The bottom diagram shows a high-Q resonator attached to the free end of this configuration. The resonator is comprised of two masses and a spring, with a resonance frequency reflecting the value for the silicon disk mode.

By altering the values for spring constants and masses, the filtering behavior of the spring-mass chain can be analyzed. If all of the springs and masses are identical the transfer function acts as a low-pass filter. A bandgap can be introduced in the transmissive region of the transfer function by altering the spring constant or mass values in a periodic manner (Fig.2). To position a bandgap near a frequency f, individual resonators must also have eigenfrequencies near f. For the mechanical isolation of a silicon disk, we are interested in creating a bandgap centered at the resonance frequency of the disk. To simulate this idea, we modeled the disk as a high-Q resonator comprised of two masses and a spring. This two-mass resonator is attached to the end of the transmission line, and we study its response to a harmonic driving force. The Q for this resonator is found from the peak in the resonator displacement amplitude (Fig.3).

3 Metamaterial Design

The proposed metamaterial design is inspired by the metamaterial theory for locally resonant structures [6], and.is comprised of four narrow transmission line bridges connecting the disk to the environment (Fig.4). Resonator structures are etched into each bridge such that the combined interference effects of the bridge and the resonators produce a bandgap.

To produce a bandgap near 1kHz, we require multiple resonator structures along each bridge. There are several constraints for selecting an applicable resonator geometry. First, each resonator must have individual eigenmodes also near 1kHz. Second, to fit multiple resonators on a standard substrate for nanofabrication each resonator must be no larger than 1cm. Third, the mode shapes of the resonators must be transverse to couple efficiently to the

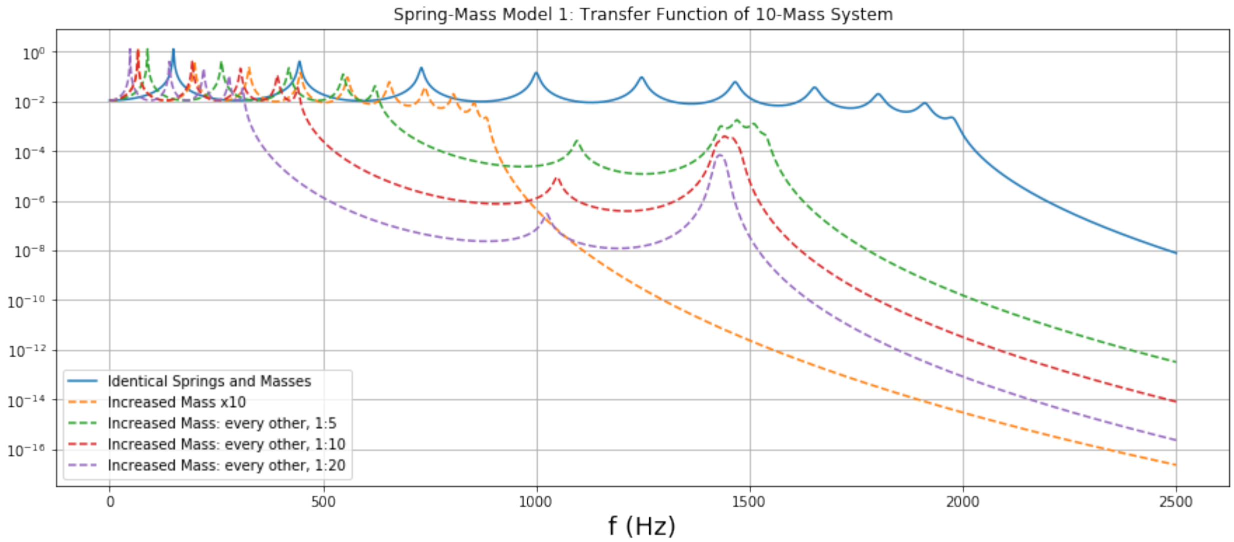

Figure 2: The transfer function for a spring-mass model where different resonators are connected in series. The blue curve is the case where all resonators are identical. For the orange curve, all masses are increased by a factor of 10. Bandgaps are present in the green, red, and purple curves, where every other mass (odd masses in Figure 1) was increased by a factor of 5, 10, and 20 respectively.

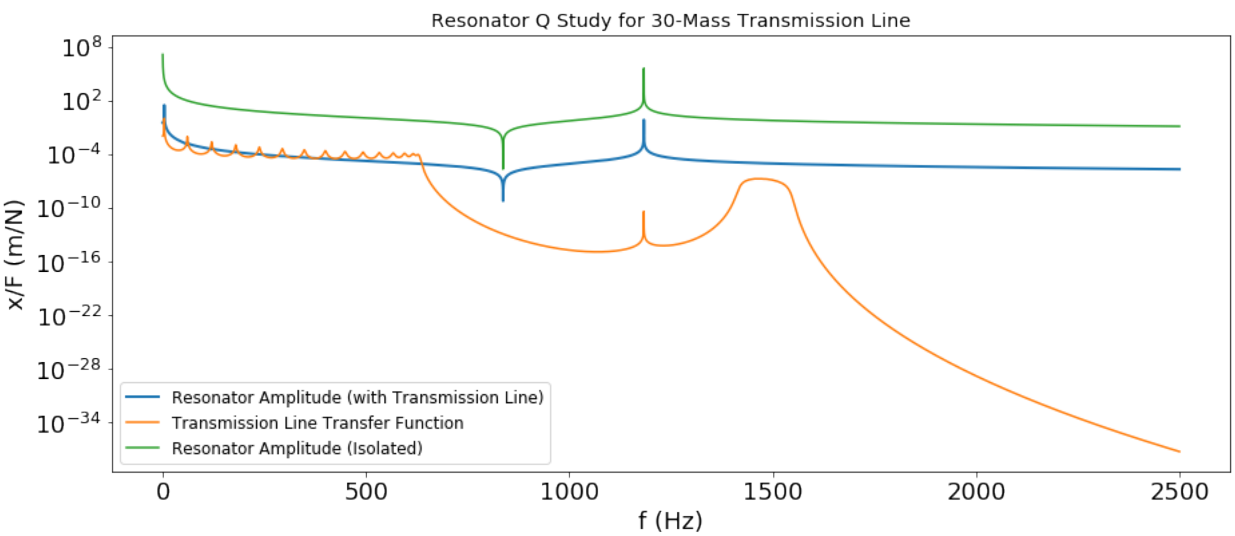

Figure 3: A comparison of the displacement amplitudes for an isolated resonator modeled by two masses and a spring, and the same resonator attached to a transmission line both subject to a harmonic driving force. The y-axis is normalized to unitary force amplitude. The resonator is modeled with a purely real spring constant, so the theoretical Q is infinite (green curve). The resonator attached to the transmission line exhibits a $Q \sim 2 \times 10^5$ (blue curve). The resonator attached to the transmission line has a smaller amplitude than the isolated resonator. This will not affect the Q of the system, and can be accounted for by applying a larger excitation.

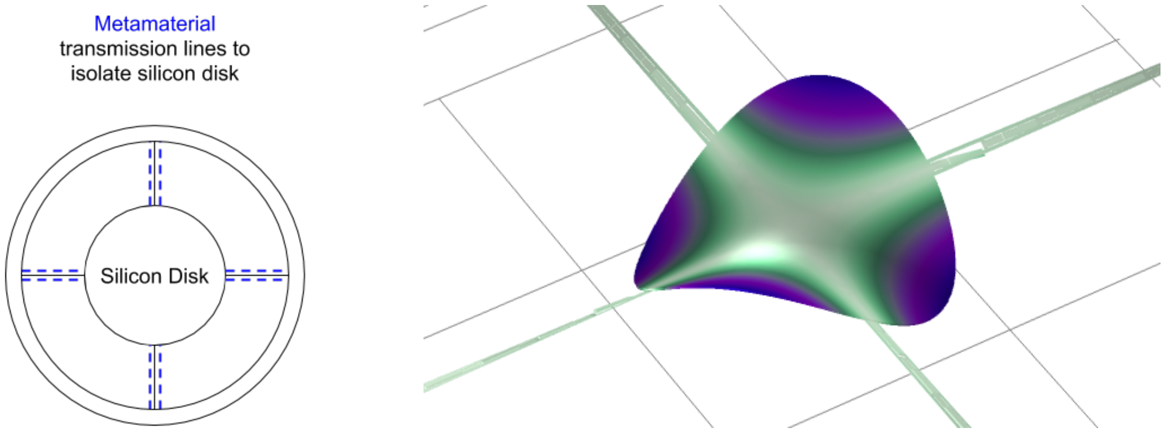

Figure 4: A schematic of the transmission line disk isolation system (left) and a visualization of the system behavior when the disk butterfly mode is excited (right). The system consists of four transmission lines patterned with resonators, connected to the disk at 90° spacings.

butterfly mode of the disk. We would also prefer a geometry with few features, that is simple to fabricate. More surface area in the geometry will increase surface loss, and a complex fabrication procedure can create additional defects in the physical system that also contribute to mechanical losses. A cantilever geometry with a thicker endpiece satisfies these requirements (Fig.5). We consider a system of five such structures in series for the transmission line design.

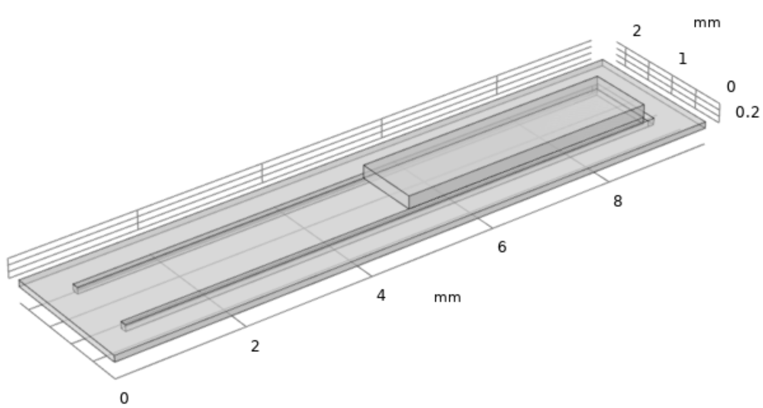

Figure 5: A model of the resonator unit cell constructed in COMSOL. The resonator consists of two components: a cantilever and a frame. The cantilever is attached to the frame on one end, and the other end has a thicker block to lower the resonance frequency. The frame is necessary to connect multiple resonators in series.

Another important consideration is the fragility of the transmission lines. Approximate calculations for the width limit of each transmission line were done by studying a rectangular silicon beam fixed at one end and subject to a transverse load on the other end. For a sufficiently narrow beam, the shear stresses are much smaller than the normal stress and the pure bending approximation can be applied to simplify calculations. The detailed derivation for the normal stress can be found in a civil engineering textbook (e.g. [7]); the formula is

$$\sigma_{x,max} = 6\frac{PL}{bh^2}$$

where P is the load force, and L, b, h are the length, width, and height of the beam respectively. For the bare transmission line system, these parameters are

$$\begin{aligned} P &= \frac{1}{4}m_{disk}g \\ L &= 0.75in \\ h &= 0.3mm \end{aligned}$$

Using silicon's tensile strength 165 MPa [8] as the maximum stress that can be handled by the beam, we find a lower bound for the bridge width to be $b_{min} = 28\mu m$. In practice, the transmission lines will be more fragile than the beam model due to the resonator structures etched into the silicon.

4 Losses

The purpose of isolating a silicon disk at 123K is to have minimal disk loss and thus be able to perform extremely low loss measurements. Therefore, additional losses introduced to the system by the metamaterial layer must be minimized. The metamaterial system has two loss sources, due to the mechanical loss of the suspension and the energy dissipated into the environment. In the GeNS system, these losses were made negligible by selecting a hard, low-loss suspension material. For the metamaterial system, it is necessary to minimize the loss of the metamaterial itself, and the energy that escapes through the metamaterial into the external environment (Fig.6). The magnitude of these loss angles are a useful scale for measuring the effectiveness of the disk isolation mechanism. The loss angles for the metamaterial, ϕ_{TL}, and the external environment which is a clamped piece of bulk silicon, $\phi_{clamped}$, are given by the following equations.

$$\begin{aligned} \phi_{TL} &= \int \phi_{TL,0}(x)\frac{dE_{strain}/dV}{E_{tot}}dV \\ \phi_{clamped} &= \phi_{clamped,Si}\frac{E_{out}}{E_{tot}} \end{aligned}$$

$\phi_{TL,0}(x)$ and $\phi_{clamped,Si}$ are geometry- and setup-dependent losses for each component of the system. dE_{strain}/dV is the strain energy density of the metamaterial transmission line, and E_{out} is the energy that escapes the transmission line. E_{tot} is the total energy of the disk-metamaterial-clamp system. Physically, the $\phi_{TL,0}(x)$ term has a complicated position dependence. More loss is introduced in regions of the resonator structure that experience more stress., since the intrinsic loss angle changes in response to a stress field. For simplicity, we do not consider this position dependence, and instead assume ϕ_{TL} as the sum of two

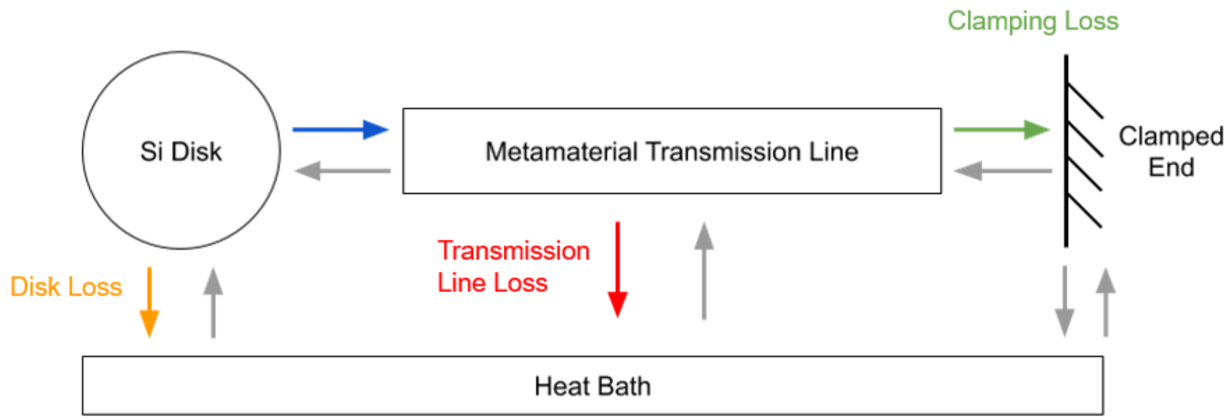

Figure 6: A schematic of the disk-metamaterial system. The energy propagating from the disk (blue) into the transmisson line will result in mechanical losses (red) due to the stored strain energy. Any kinetic energy that escapes through the transmission line to the clamped end will be dampened, resulting in an additonal clamping loss (green). We would like to design a system where these two additional loss terms are smaller than the disk loss (orange).

position independent terms corresponding to bulk and surface losses. That is,

$$\phi_{TL} = \phi_{TL,bulk} + \phi_{TL,surface}$$
$$\phi_{TL,bulk} = \int \phi_{Si,123K} \frac{E_{strain}}{E_{tot}}$$
$$\phi_{TL,surface} = \alpha_S \frac{\int_S dE_{strain}/dS}{E_{strain}}$$

where $\phi_{Si,123K} = 8 \times 10^{-9}$ and $\alpha_S = 0.5\text{pm}$ [9]. Using these constants and the strain energy densities found from finite element analysis, we can evaluate the loss angles of the system. Due to the nature of cryogenic silicon, we expect the bulk loss term to be negligible, that is, $\phi_{TL,bulk} \ll \phi_{TL,surface}$. $\phi_{TL,surface}$ becomes more significant for metamaterial designs with a greater surface-to-volume ratio.

$\phi_{clamped}$ is a measure of how well the metamaterial functions as a disk isolation mechanism. We use a high loss value for the clamped end, $\phi_{clamped,Si} = 1 \times 10^{-3}$, since we expect the stiffness of the clamped end to dampen out the energy.[1] The ideal metamaterial design will perfectly reflect and not allow strain energy to propagate through, in which case $E_{out} = 0$.

5 Finite Element Analysis

Both the resonator unit cell and the metamaterial transmission line were studied numerically using the COMSOL v5.4 Structural Mechanics package. In each case, the initial geometry

[1]This may be an underestimate for the value of $\phi_{clamped,Si}$. It should be closer to 1 if we assume most of the energy at the clamped end is dissipated

is constructed in the COMSOL software and defined by adjustable parameters. For the resonator unit cell, eigenfrequency studies were conducted to center the cantilever resonance at 1kHz (Figs. 7, 8). COMSOL has a parametric sweep feature that allows sweeps of up to two parameters with user-specified ranges. To sweep over more than two parameters, the COMSOL-MATLAB interface should be used.

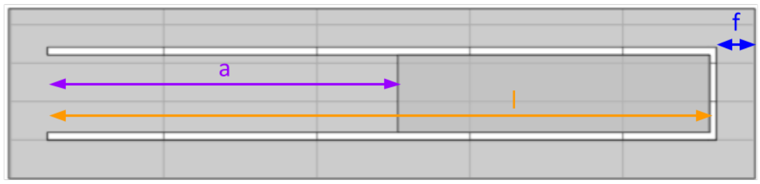

Figure 7: Top view of the cantilever geometry. a is the length of the thin part of the cantilever, l is the total length of the cantilever, and f is the frame width. These three parameters were the main focus of the optimization studies.

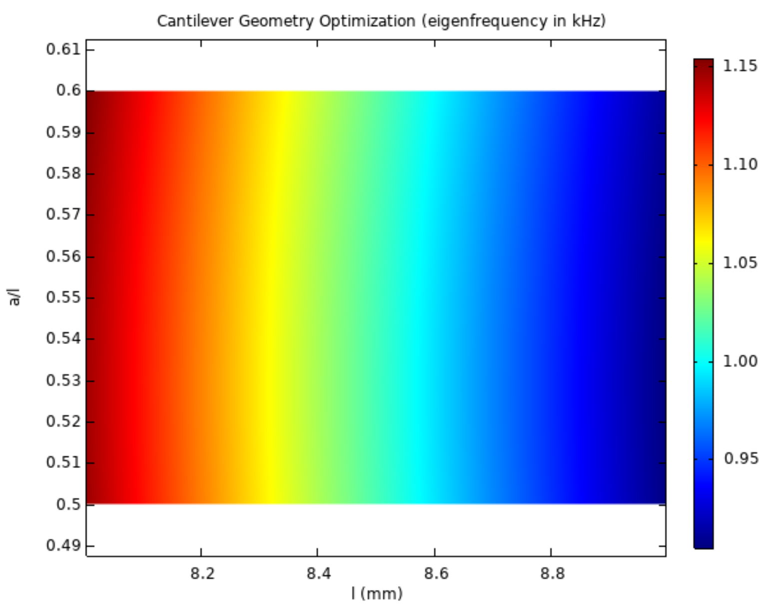

Figure 8: The results of a parameter sweep for the cantilever geometry, showing the eigenfrequency of the cantilever resonator as a function of the cantilever length and the fraction of the cantilever length that is thin. A 1kHz resonance can be achieved while maintaining a sub-cm scale resonator unit cell length.

The transmission line system is modeled by fixing the clamp-end and applying forces simulating the excited disk mode to the disk-end. We used a combination of COMSOL studies to ensure we are accurately modeling a physical system, and to determine the losses of the system. A stationary test was used to study the maximum stress on the transmission line under non-excited conditions and gravity (In all further studies, gravity is not considerered due to FEA software issues). An eigenfrequency test was used for adaptive mesh refinement of the system. Following these steps, a frequency domain analysis was conducted to study the system response to different excitations (Figs.9,10,11).

After the above studies showed promising results for producing a bandgap, we used the COMSOL-MATLAB interface to optimize the three geometry parameters a, l, and f in Figure 7. The interior-point constrained minimization algorithm was used to minimize a cost function defined by the two loss sources in the system, $\phi_{TL} + \phi_{clamped}$. Note that we can minimize this cost function without knowing the total energy in the system E_{tot}, since both terms are divided by this factor. Determining E_{tot} would require including the disk in the simulations. The optimization was constrained to maintain a resonator length of less than 1cm. We found that identical resonators can be used to produce a bandgap with the following optimized parameter values: $a = 9.268$mm, $l = 2.984$mm, and $f = 0.366$mm. This does not match the spring-mass model analysis, which suggested resonators with alternating resonance frequencies is necessary to produce a bandgap. A system where each resonator geometry is optimized independently should be studied in the future for a more comprehensive and computationally expensive optimization.

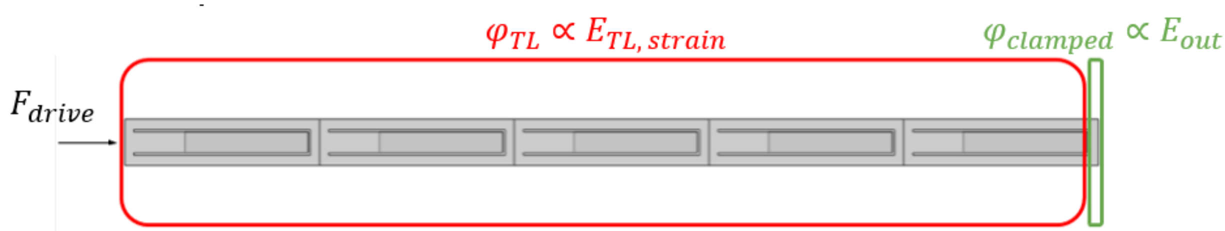

Figure 9: A top view of the transmission line model, divided into two regions where the strain energies can be measured separately. The strain energies from these two regions result are used to calculate the transmission loss and clamping loss of the system.

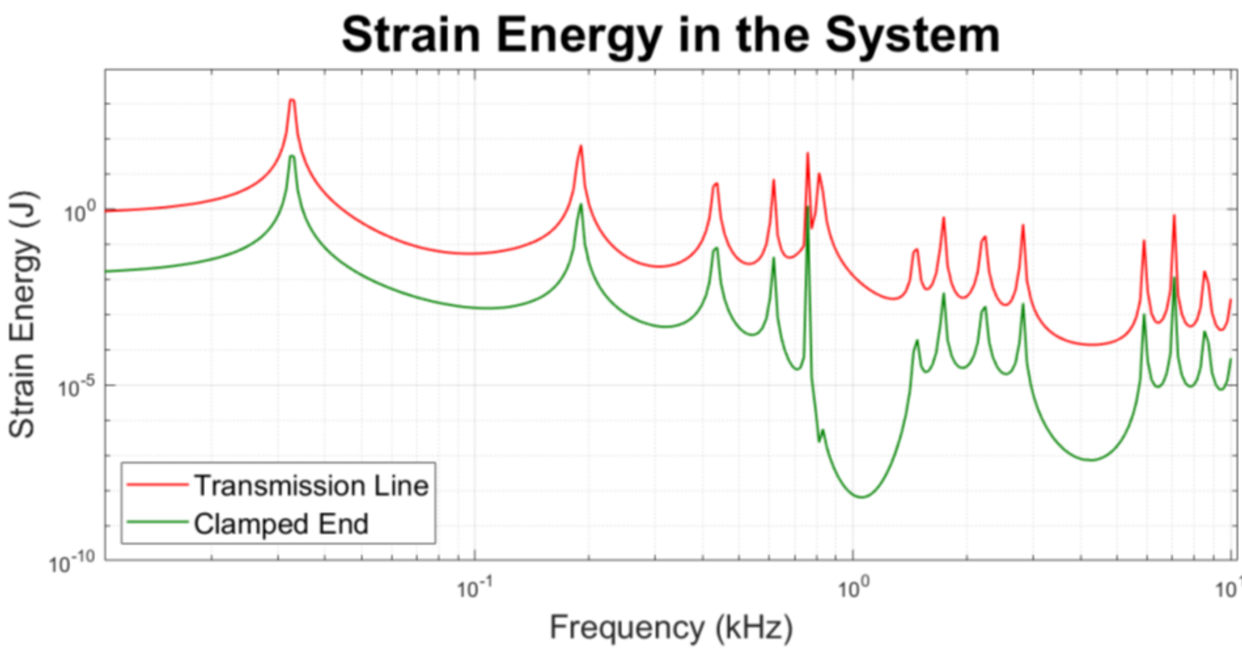

Figure 10: The strain energy response of the metamaterial system driven by a unitary harmonic load on the disk-end. The red curve is the strain energy along the transmission line and the green curve is the energy that escapes into the clamped end. The green curve has a bandgap near 1kHz, indicating an effective isolation mechanism around that frequency. There is no corresponding bandgap in the red curve because energy propagates into the transmission line before it is reflected. There is also a second bandgap at 4kHz which has more strain energy at the clamped end, but less strain energy along the transmission line.

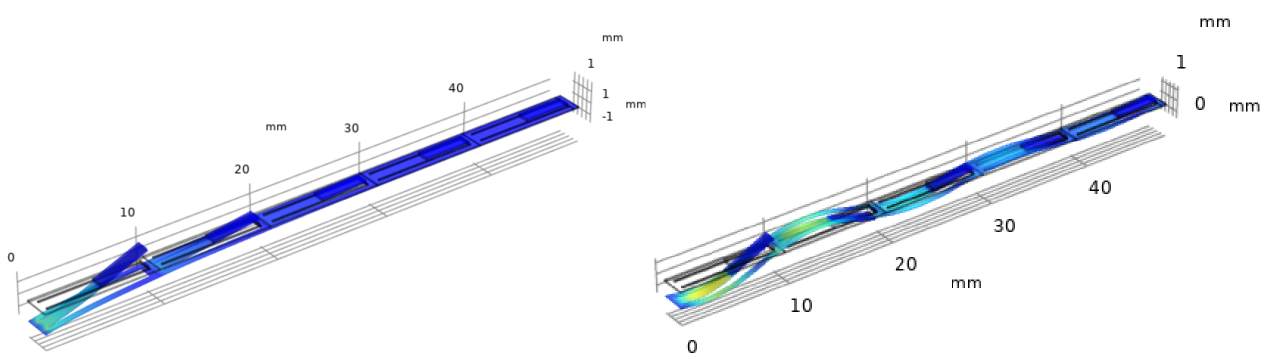

Figure 11: The transmission line response to transverse harmonic loads at 1kHz (left) and at 4kHz (right). The responses demonstrate different disk isolation mechanisms that the metamaterial can exhibit. The energy propagates further into the transmission line for the 4kHz case, which is less desirable from a mechanical loss standpoint.

6 Conclusions

We simulated a working model of a metamaterial transmission line system for mechanically isolating a silicon disk. Finite element analysis was used to numerically optimize the geometry to target the 1kHz butterfly mode of a 2-inch disk. Future simulations of the full disk-metamaterial system are still necessary to quantify the absolute magnitudes of the metamaterial loss sources and the disk loss. Other future work for this project can include prototype fabrication of the system on a silicon wafer using silicon fabrication techniques,. and designing similar metamaterial systems for the other modes of the disk.

7 Acknowledgements

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