



Constructing Echo Waveforms from Spinning Exotic Compact Objects

Shuo Xin, Tongji University

Mentors: Ling Sun, Yanbei Chen, Baoyi
Chen, Rico Lo



Overview



Introduction: What and why?

Method: How to construct echoes?

Results

Perspectives



Introduction

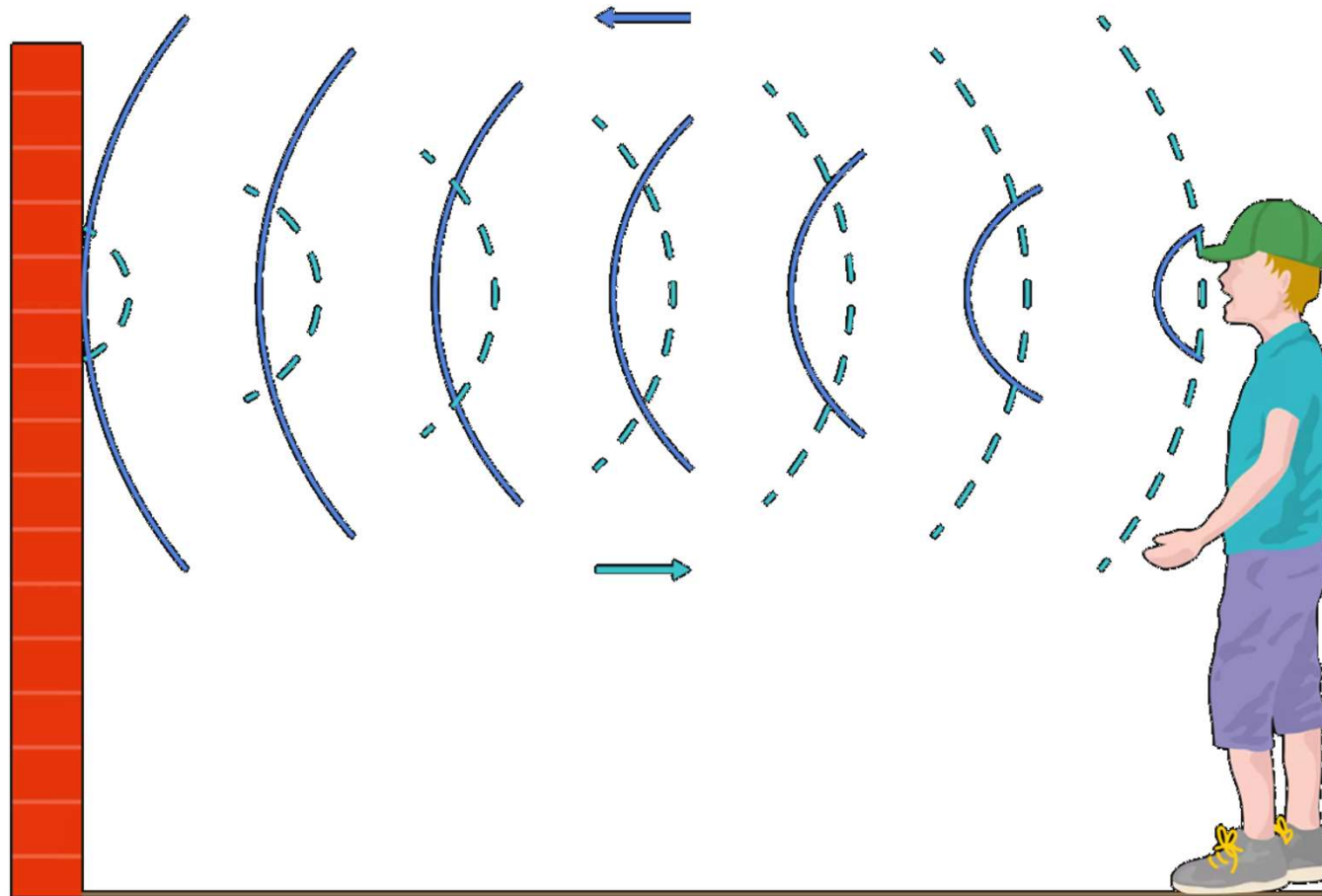


What is echo?

Why is it important?

Recent discussions

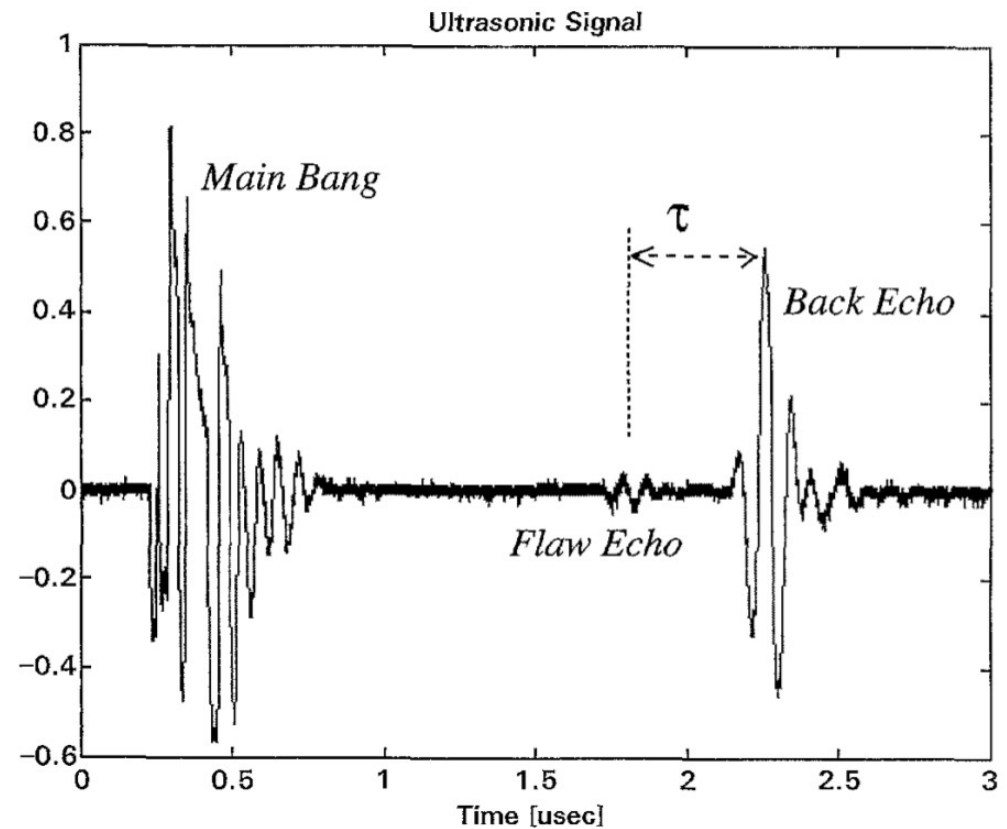
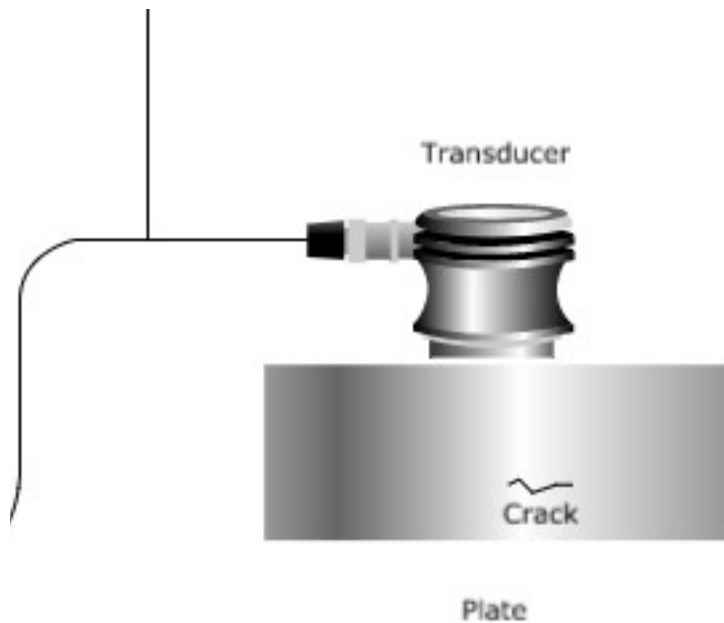
What is echo: familiar echoes



Credit: <https://www.dkfindout.com/uk/science/sound/echoes/>

SURF 2019

Find mechanical crack

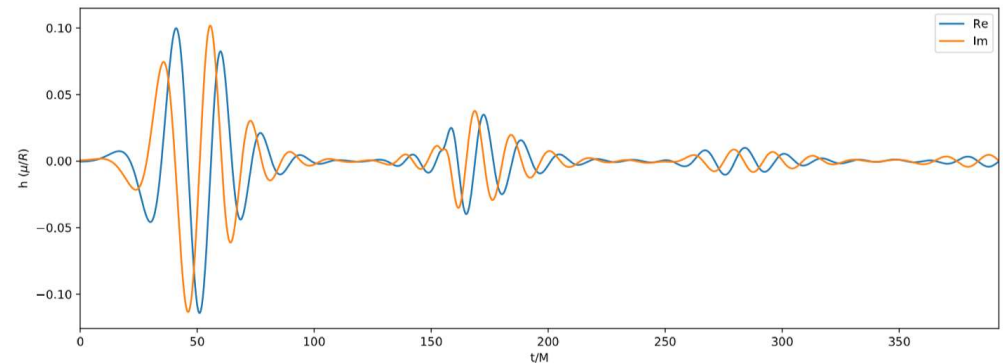
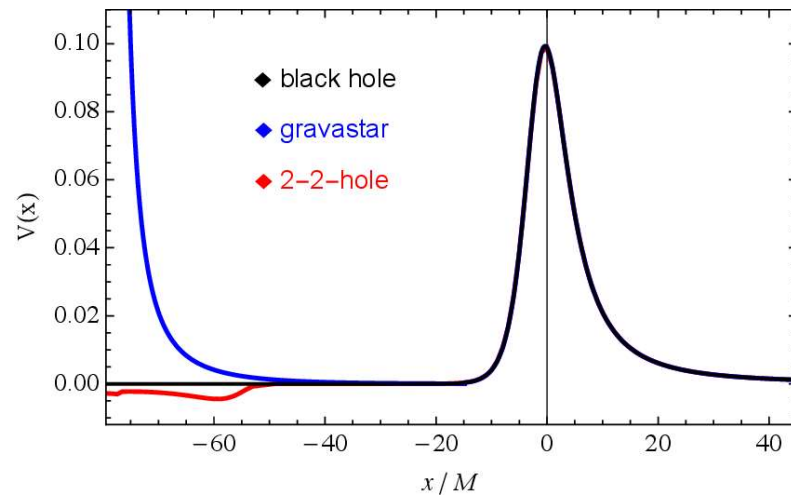


Agostino Abbate et al. *IEEE transactions on ultrasonics, ferroelectrics, and frequency control*, 44(1):14–26, 1997.



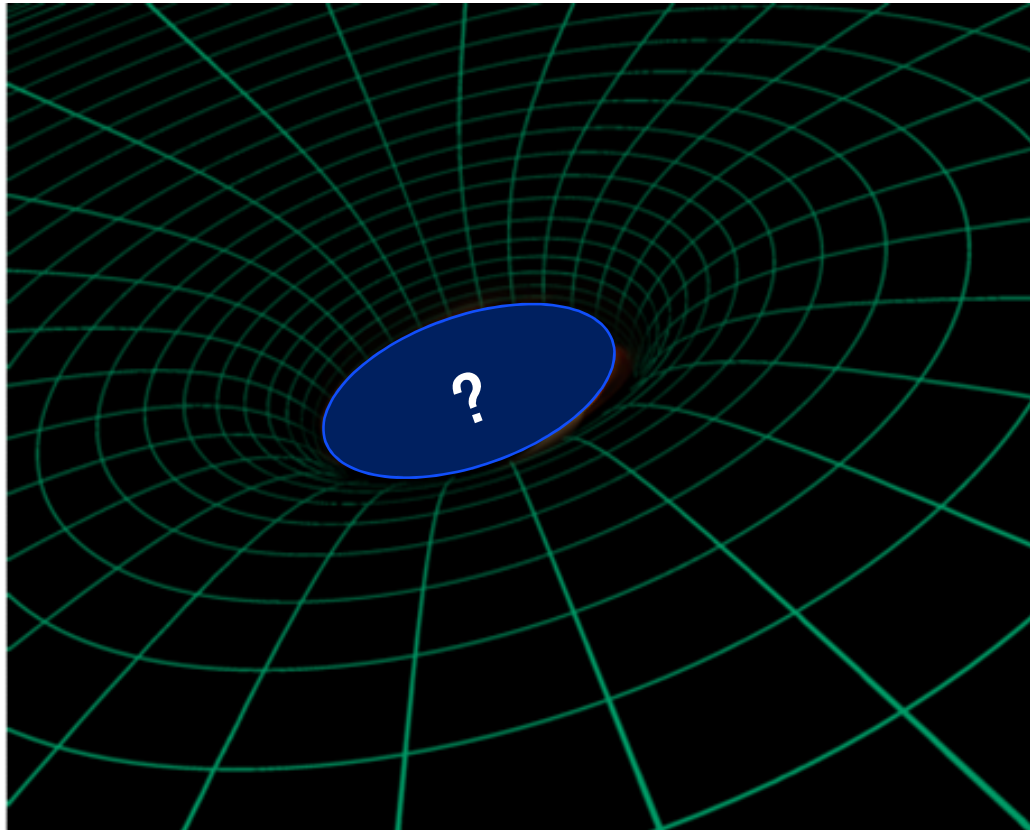
Find crack in compact object?

Horizon Crack Ringdown
← ↓ ↖
Signal



Randy S. Conklin, Bob Holdom, and Jing Ren. Gravitational wave echoes through new windows. *Phys. Rev. D*, 98:044021, Aug 2018.

Potential hint for exotic physics



What's hiding near horizon?

Black hole?

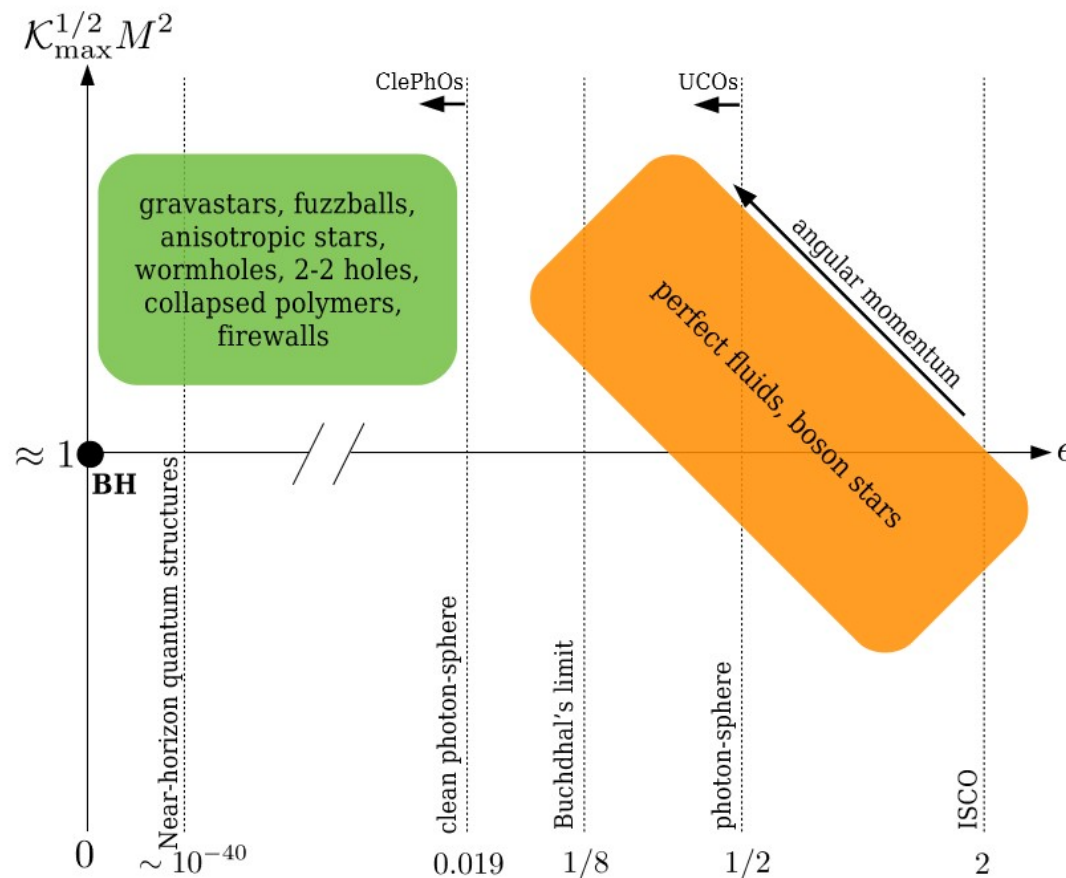
Wormhole?

Fuzzball?

...

Echo may tell.

Exotic Compact Objects (ECOs)



ϵ : Compactness, defined by

$$r_0 = r_+(1 + \epsilon)$$

r_0 : radius of ECO,

r_+ : radius of “horizon”

\mathcal{K}_{max} : Kretschmann scalar, characterizing maximum curvature in the spacetime

Vitor Cardoso, Paolo Pani. Testing the nature of dark compact objects: a status report. *arXiv:1904.05363*



Recent discussions

Data Analysis (no echo is found yet)

Jahed Abedi et al. Phys. Rev. D, 96:082004, Oct 2017.

Andrea Maselli et al. Phys. Rev. D, 96:064045, Sep 2017.

Ka Wa Tsang et al. Phys. Rev. D, 98:024023, Jul 2018.

R Ka Lok Lo et al. Phys. Rev. D, 99: 084052, April 2019.

And a lot ...

Models for echoes from non-spinning ECO

Zachary Mark et al. Phys. Rev. D, 96:084002, Oct 2017.

Song Ming Du et al. Phys. Rev. Lett., 121:051105, Aug 2018.

And a lot ...

Models for echoes from spinning ECO

Qingwen Wang et al. Phys. Rev. D, 97:124044, June 2018

Hiroyuki Nakano et al. Prog. Theo. Exp. Phys., 2017(7), 07 2017.

Elisa Maggio et al. ArXiv 1907.03091



Past work for spinning ECO

Qingwen Wang et al. Phys. Rev. D, 97:124044, June 2018

» Making echoes: ingoing wave packet from infinity

Hiroyuki Nakano et al. Prog. Theo. Exp. Phys., 2017(7), 07 2017.

» Making echoes: ingoing wave is assumed phenomenologically

$$h(t) \propto \frac{e^{-i\omega_{\text{QNM}}\tilde{t}}}{1 + \exp[-2\beta t|\Im(\omega_{\text{QNM}})|]}$$

Elisa Maggio et al. ArXiv 1907.03091

» Low frequency expansion



Method

Black Hole (BH) perturbation theory

From BH to ECO

Physical picture



BH perturbation theory

Teukolsky equation *Phys. Rev. Lett. 29: 1114, 1973:*

Describes evolution of ψ_s ($s = 0, 1, 2, 3, 4$), projection of Weyl curvature tensor on Newman-Penrose tetrads

$$\left[\frac{(r^2 + a^2)^2}{\Delta} - a^2 \sin^2 \theta \right] \frac{\partial^2 \psi}{\partial t^2} + \frac{4Mar}{\Delta} \frac{\partial^2 \psi}{\partial t \partial \varphi} + \left[\frac{a^2}{\Delta} - \frac{1}{\sin^2 \theta} \right] \frac{\partial^2 \psi}{\partial \varphi^2} - \Delta^{-s} \frac{\partial}{\partial r} \left(\Delta^{s+1} \frac{\partial \psi}{\partial r} \right) - \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) - 2s \left[\frac{a(r-M)}{\Delta} + \frac{i \cos \theta}{\sin^2 \theta} \right] \frac{\partial \psi}{\partial \varphi} - 2s \left[\frac{M(r^2 - a^2)}{\Delta} - r - ia \cos \theta \right] \frac{\partial \psi}{\partial t} + (s^2 \cot^2 \theta - s) \psi = 4\pi \Sigma T.$$

Boundary condition:

Only ingoing wave at horizon

Only outgoing wave at infinity

At infinity, solution is related to gravitational waves by

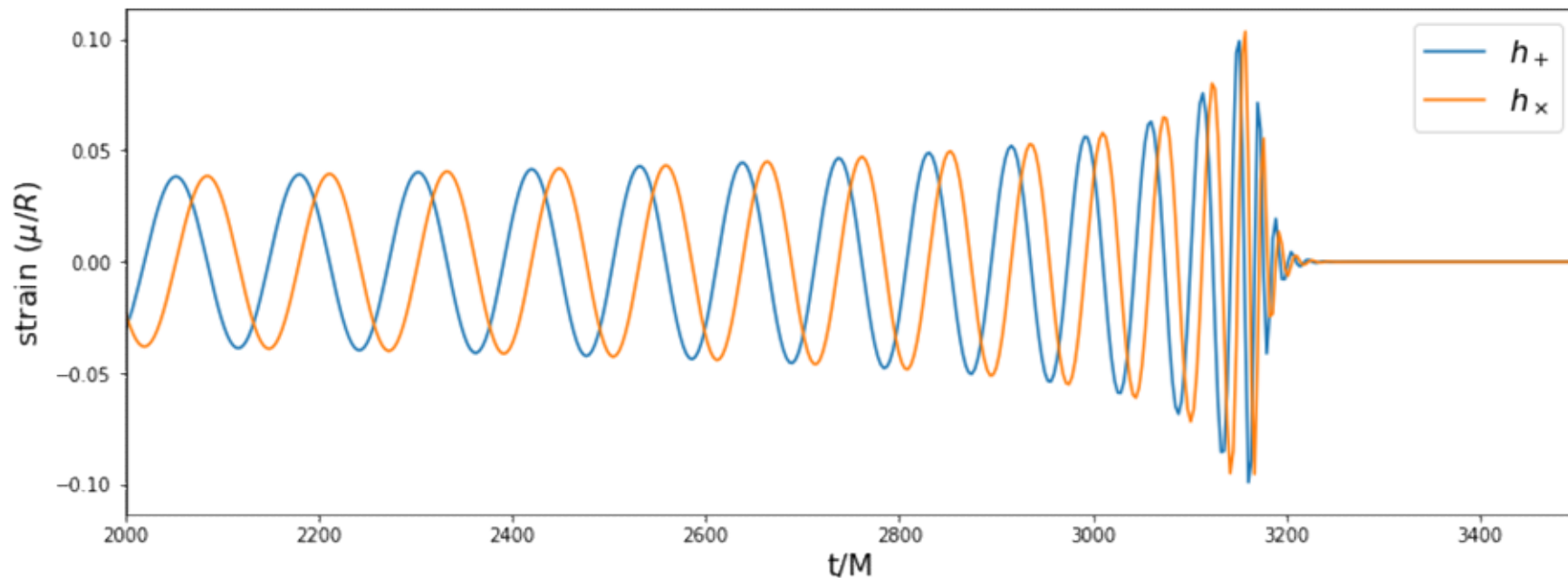
$$\psi_4(r \rightarrow \infty) \rightarrow \frac{1}{2} (\ddot{h}_+ - i \ddot{h}_\times)$$

A well tested Fortran code is already in hand

Wen-Biao Han and Zhoujian Cao. Constructing effective one-body dynamics with numerical energy flux for intermediate-mass-ratio inspirals. *Phys. Rev. D*, 84:044014, Aug 2011.

BH perturbation theory

Waveform example based on Post-Newtonian orbit
for 2 BH with total mass M and mass ratio 0.25,
Starting from an equatorial circular orbit with radius $15M$



From BH solution to ECO solution

Regard ECO as a reflecting boundary located at r_0^* with reflectivity $\tilde{R}(\omega)$

BH boundary condition:

Only ingoing at horizon

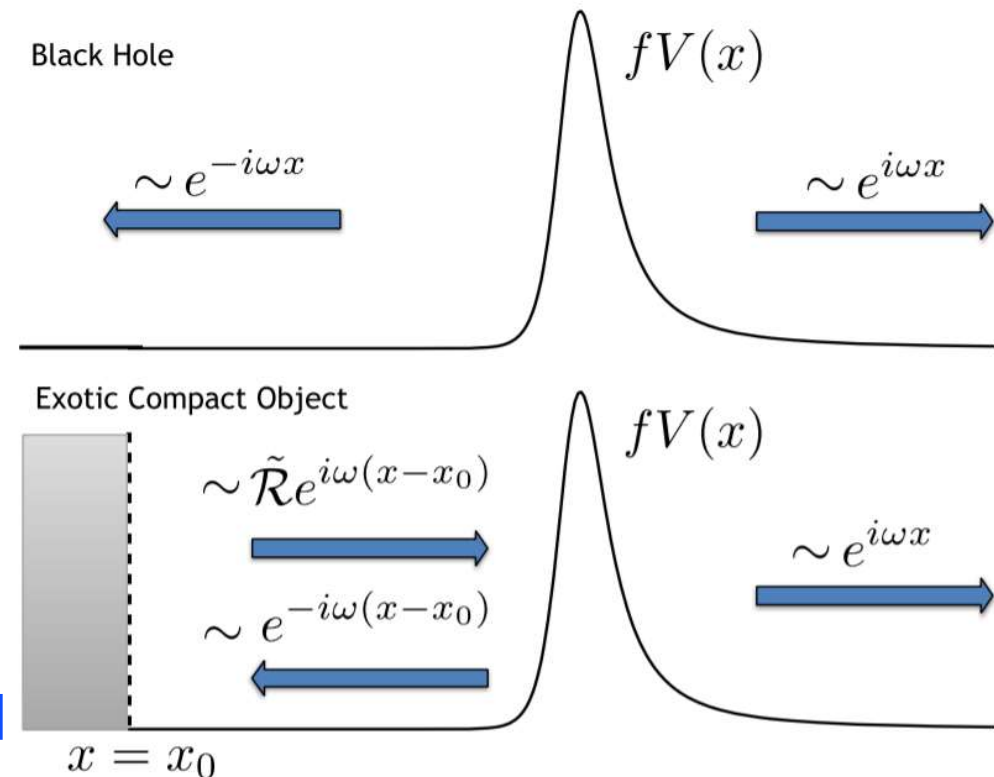
Only outgoing at infinity

ECO boundary condition:

Reflecting at r_0^*

Only outgoing at infinity

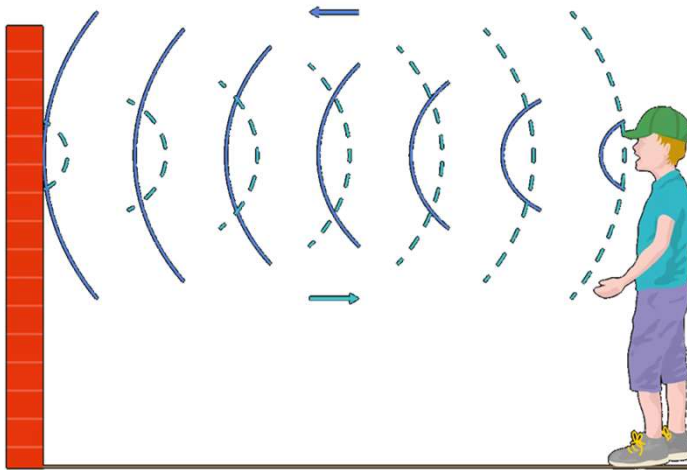
Solution in ECO background can be constructed from solution in BH background



Zachary Mark, Aaron Zimmerman, Song Ming Du, and Yanbei Chen. A recipe for echoes from exotic compact objects. *Phys. Rev. D*, 96:084002, Oct 2017.

Physical picture

Naïve echo

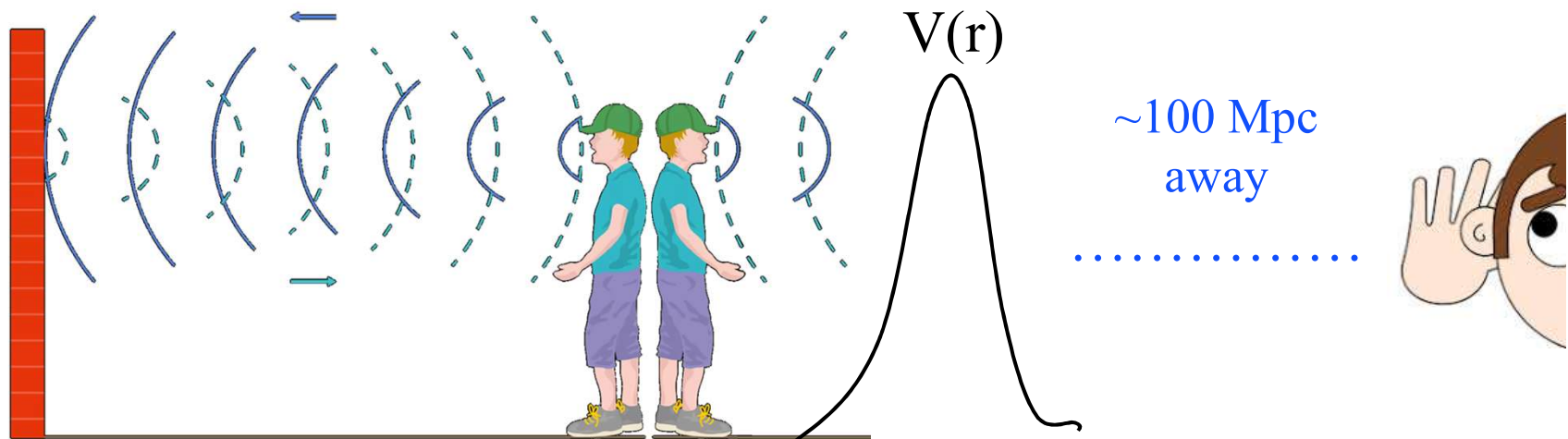


Physical picture

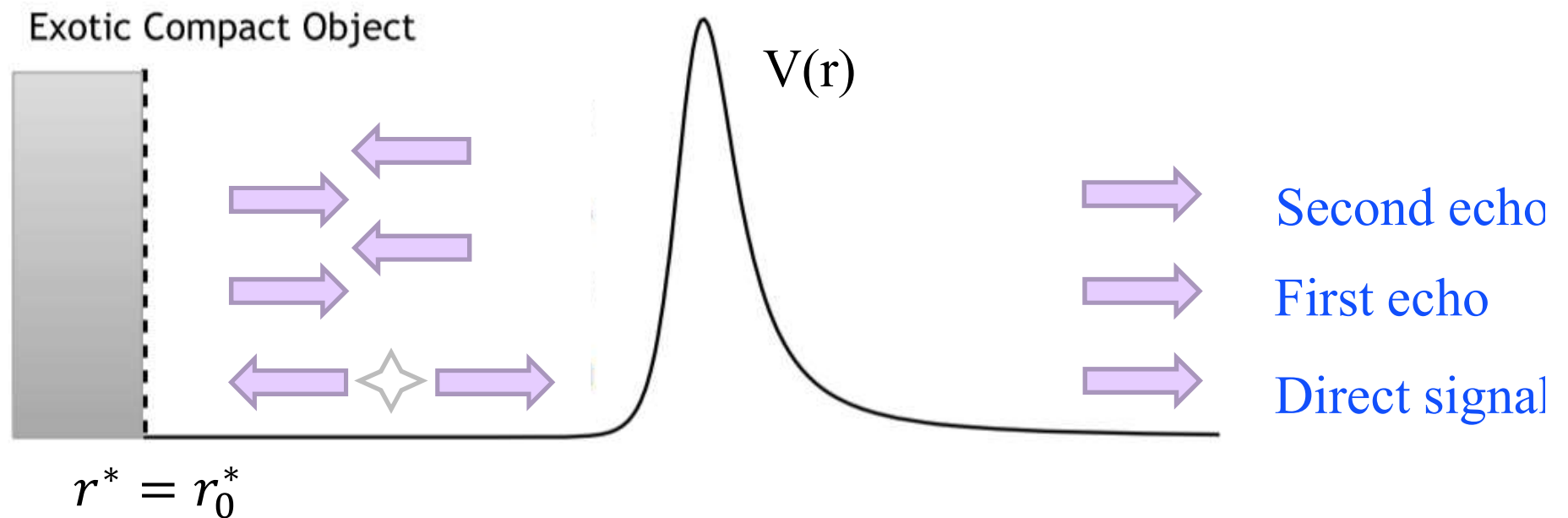
ECO
surface

Ringdown
signal

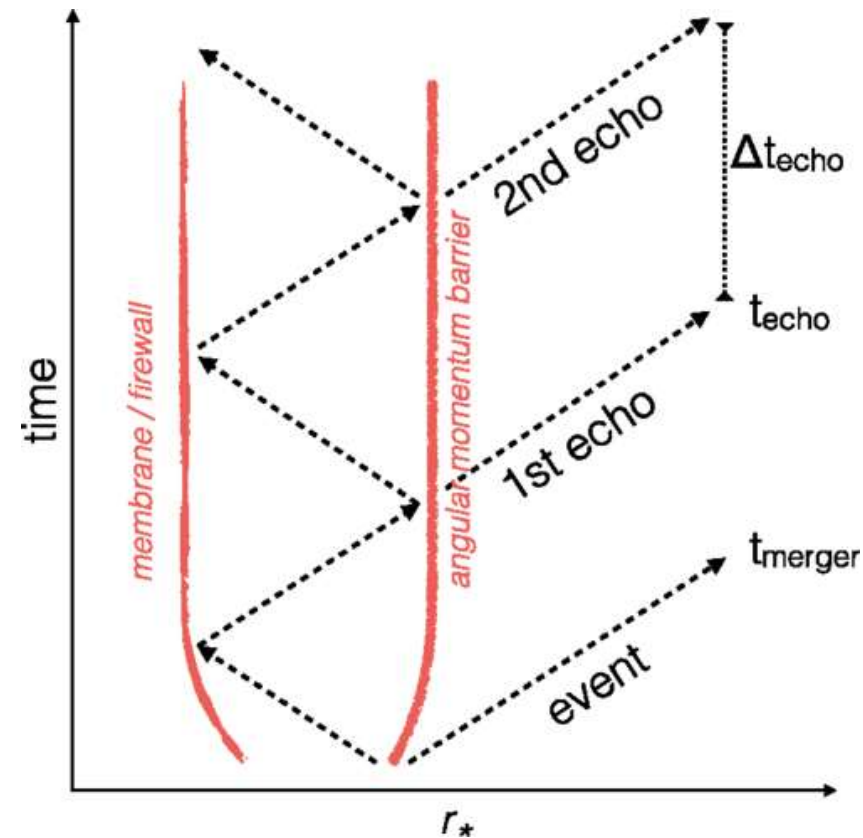
Teukolsky
potential



Physical picture



Physical picture



Jahed Abedi et al. Phys. Rev. D, 96:082004, Oct 2017.



Results



“Superradiance”

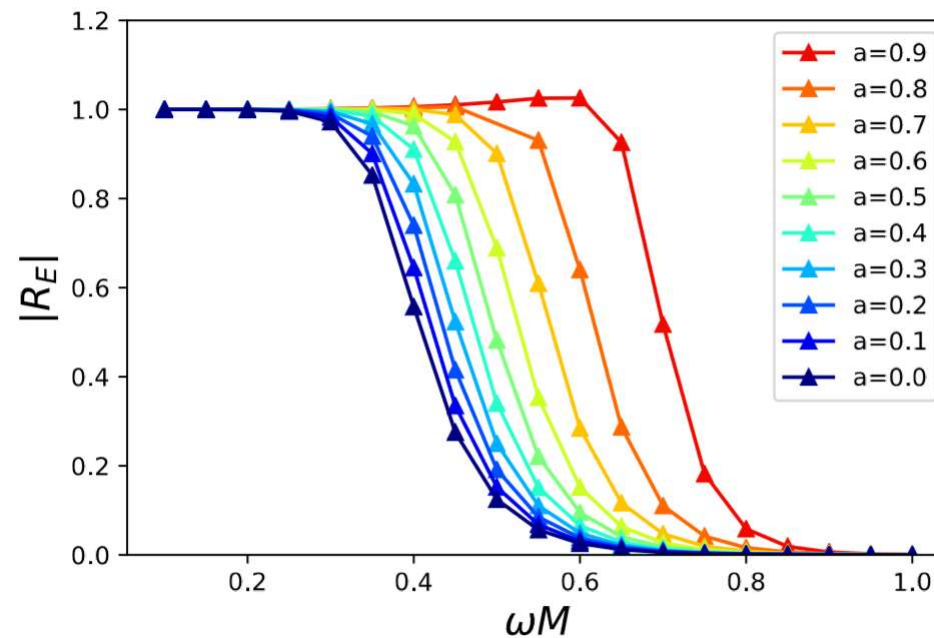
Waveform examples

Plunging particle source

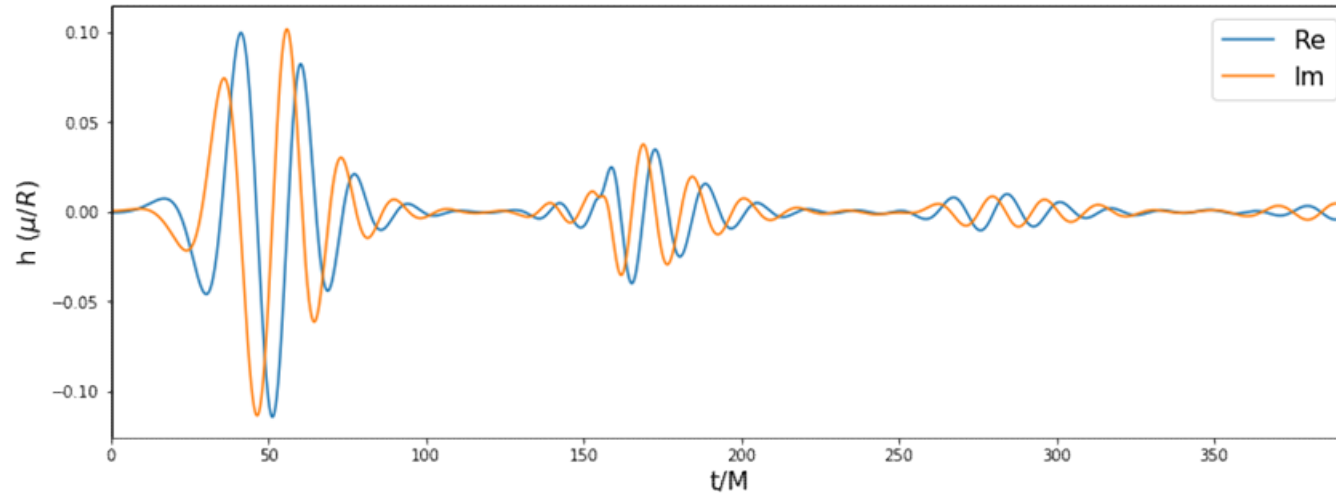
Phenomenological source

“Superradiance”

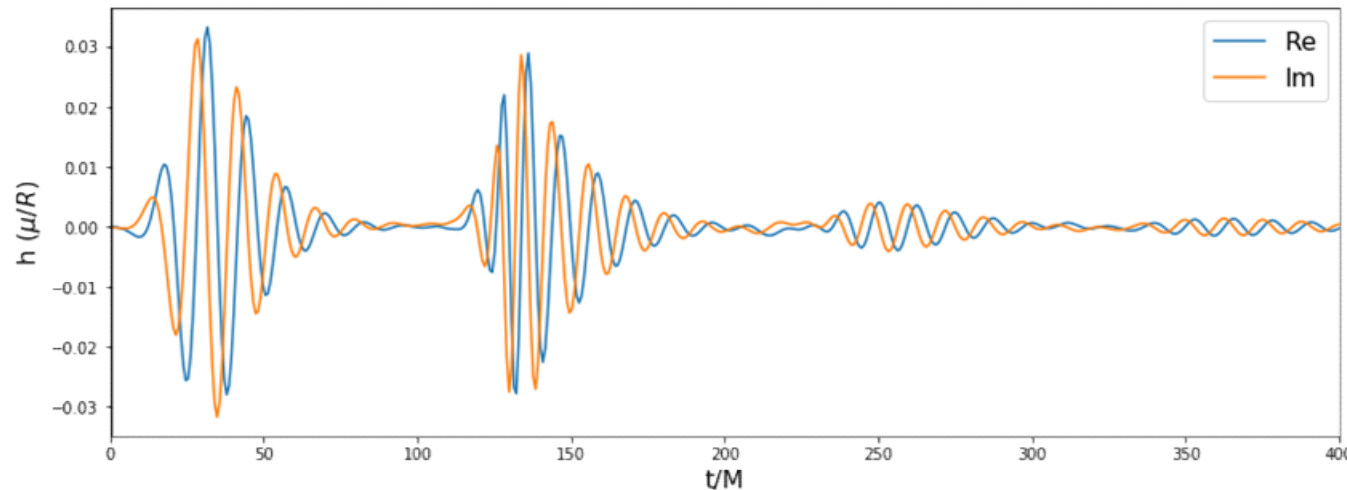
Energy reflectivity may exceed 1 when spin turns on



Waveform examples



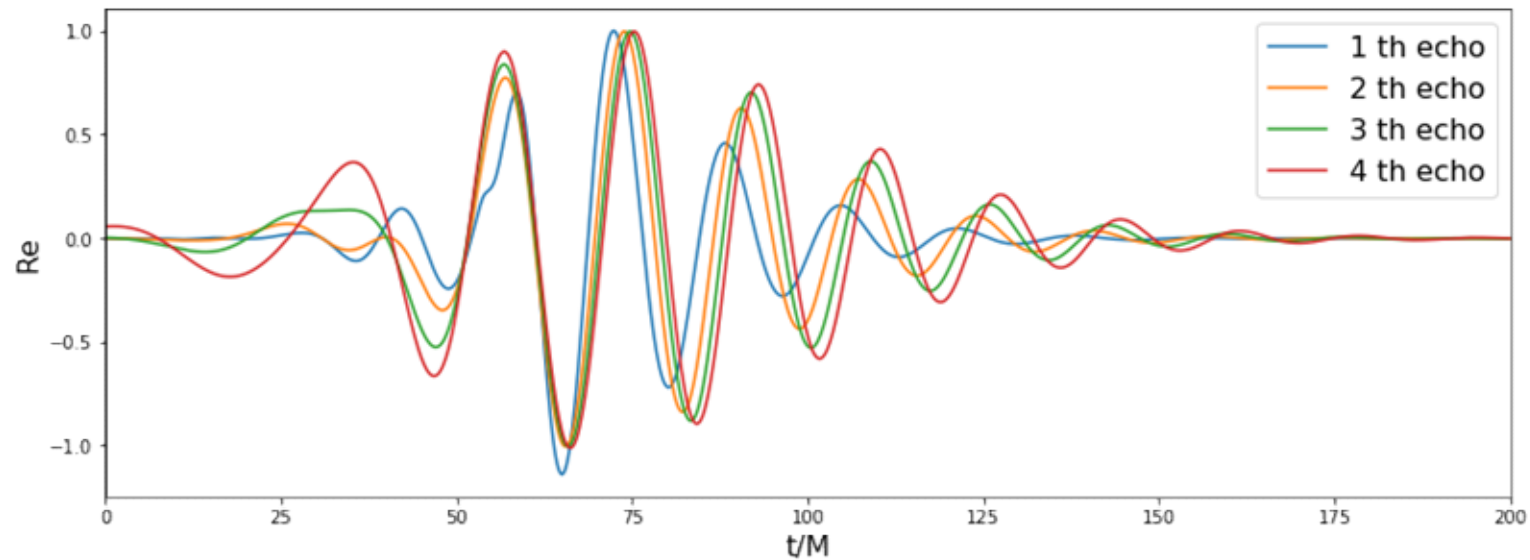
$$a/M = 0$$



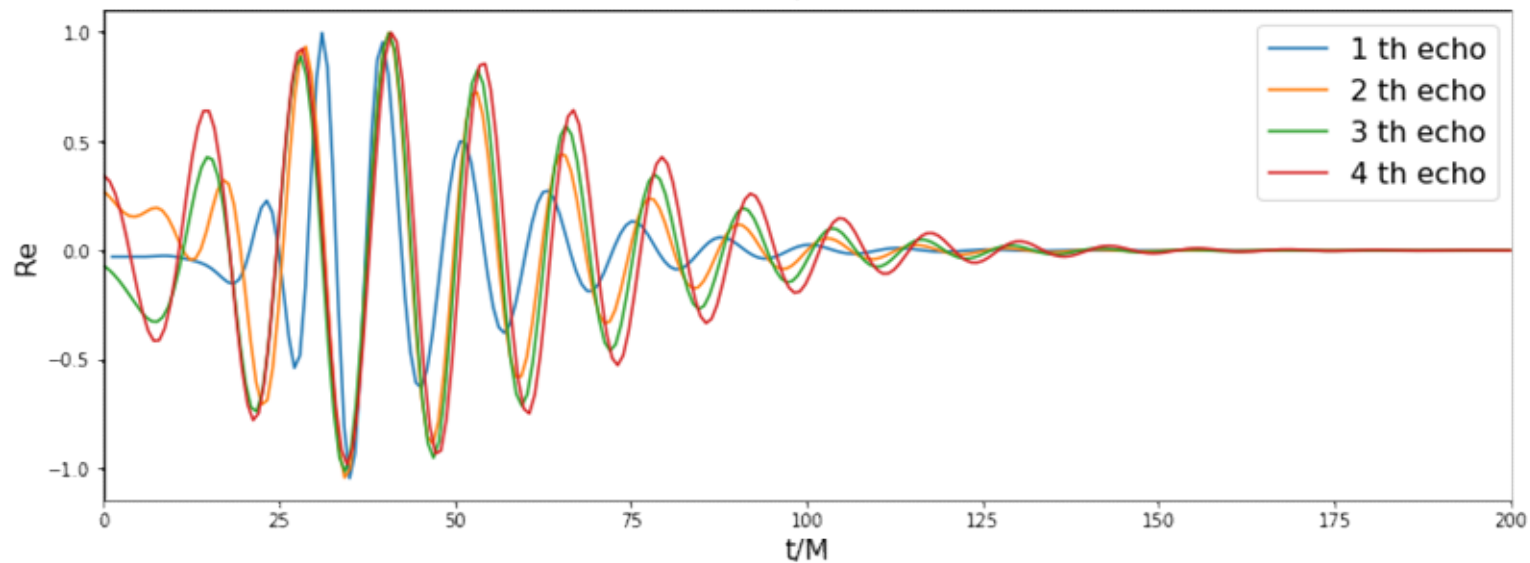
$$a/M = 0.6$$

Sourced by plunging orbit. Central ECO: $\tilde{R} = 0.5, r_0^* = -50$

Waveform examples



$$a/M = 0$$



$$a/M = 0.6$$



Phenomenological source



$$\mathcal{I}_{lm\omega}/\Delta^2 = C e^{i\omega t_s} \exp\left(-\frac{(\omega - \omega_s)^2}{2\sigma_\omega^2}\right) \exp\left(-\frac{(r^* - r_s^*)^2}{2\sigma_r^2}\right)$$

C : amplitude

t_s : arrival time

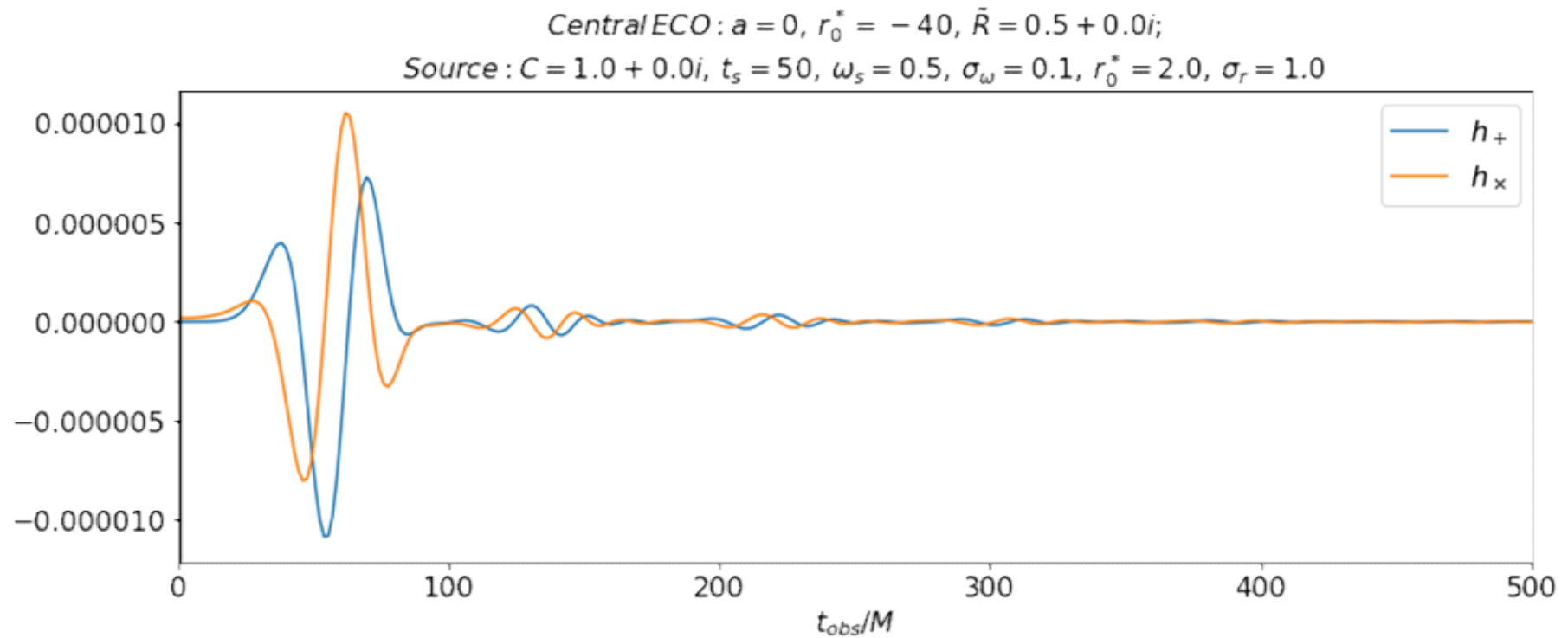
ω_s : primary frequency content of the source

σ_ω : dispersion in frequency

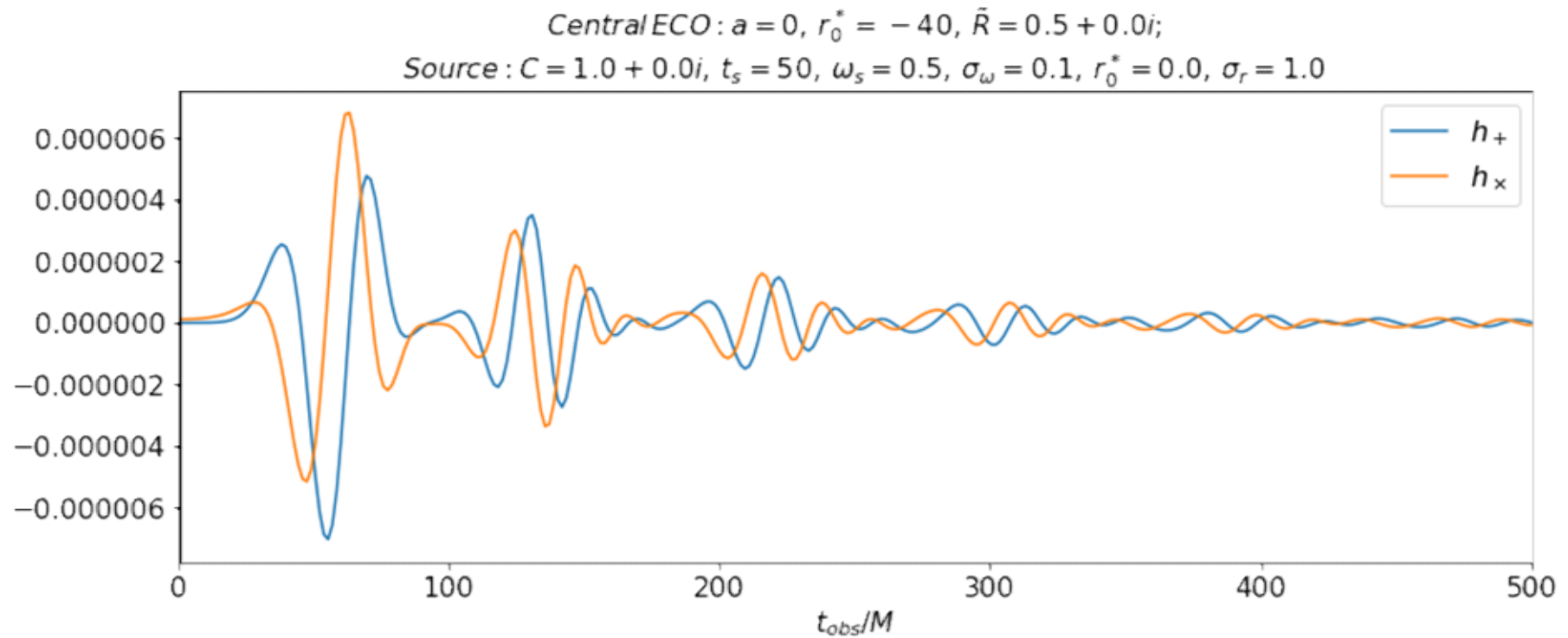
r_s^* : the source is localized near r_s^*

σ_r : dispersion in space

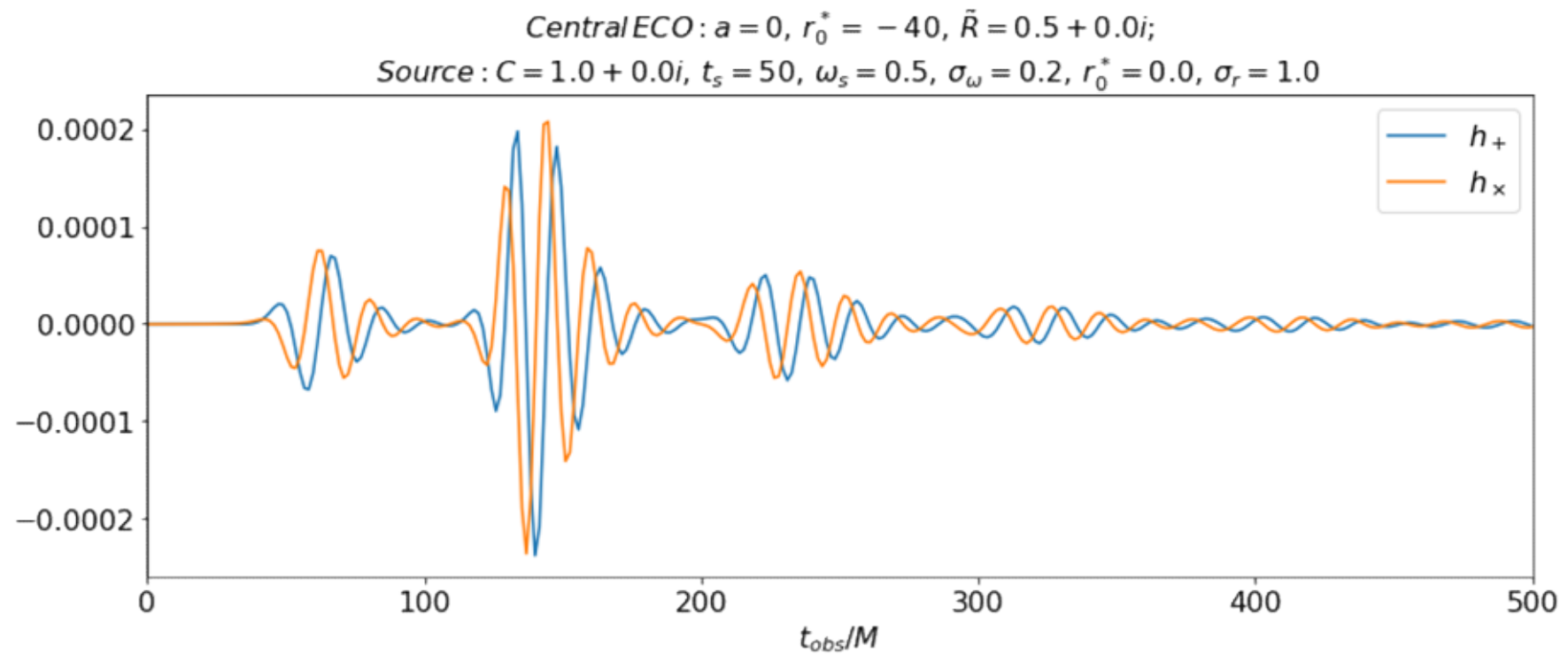
Waveform examples



Waveform examples



Waveform examples





Perspectives

Improve the formalism

The integral in Teukolsky formalism is diverging
when source is unbounded

Regularization is needed *PhysRevD.55.639*

Implement the waveform in data analysis



Thanks!