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Optical Cavity Inference Techniques for Low Noise Interferometry

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Abstract

Gravitational waves are being detected more and more frequently by the Advanced LIGO interferometers due to the improvements made to their precision. To improve the rate at which we detect gravitational waves, one method would be to reduce the noise that is intrinsic to these signals, so that more signals can be extracted with confidence. To achieve this, a deeper understanding of the noise couplings that mask these signals is necessary. This project seeks to develop statistically rigorous methods of inferring interferometric parameters which govern these noise couplings using interferometer modeling software and Bayesian inference techniques.

1 Introduction

Gravitational waves exist as a consequence of general relativity imposing a universal speed limit on the diffusion of information in the universe. Much like how the information about the electric and magnetic fields induced by a moving charge is dispersed via electromagnetic waves, the information about the gravitational field of an accelerating mass is dispersed in the form of gravitational waves [1]. The analogy between electromagnetic waves and gravitational waves carry over in several ways. Electromagnetic waves carry energy away from their source and thus do work on systems they interact with via the electromagnetic force. Similarly, gravitational waves cause regions of space to shrink and expand in specific directions, and any masses in this region of space will shrink and expand accordingly. This shrinking and expanding of masses is capable of doing work as demonstrated by the "sticky bead argument" [2], which describes a stationary rod with beads that has an incident gravitational wave induce an oscillatory strain that in turn also induces friction between the beads. Thus, it can be said that gravitational waves also carry energy and do work on systems of masses via the gravitational force. The analogy between electromagnetic waves and gravitational waves breaks down when considering the relative strength of the forces and the charge properties of the force carriers, but it serves as a foundation for understanding gravitational waves.

The physical effect that gravitational waves have on masses is quantified by the strain that occurs in any mass located in that system. The strain is expressed, by convention, as the relative change in length as a fraction of the original length. For example, a 1.0 meter by 2.0 meter rectangle with an incident gravitational wave of strain = 0.1 will oscillate between having one side of 0.9 meters and 1.1 meters and the other side 1.8 meters and 2.2 meters. This oscillation will have sinusoidal components that correspond to the frequency of the incident gravitational wave.

In addition to the strain, gravitational waves also have a period and frequency associated with their wave-like nature. These properties are inherent to the system from which they are radiated from; a binary system of supermassive black holes radiate gravitational waves with a frequency of 10^{-8} to 10^{-2} Hz, objects captured by those same black holes can radiate gravitational waves of 10^{-5} to 10^{-1} Hz, binary systems of regular black holes can radiate gravitational waves from 10^{-3} to 10^{3} Hz, and rotating non-spherical neutron stars and non-symmetric supernovae [3].

The usefulness of being able to detect gravitational waves is apparent when reviewing the sources of gravitational waves and their ability to propagate. Current observational astronomy is traditionally done through the measurement of electromagnetic waves of varying wavelengths. Gravitational waves are an entirely different class of waves that can be observed to learn more about the universe. The two main drawback of observing electromagnetic waves is that their energies fall off as an inverse square law. Gravitational waves are unique in that although their energy falls off as an inverse square law, the strain they induce in detectors falls off only as an inverse law. This property has profound implications for the field of observational astronomy- an upgrade that increases the sensitivity of an electromagnetic detector by 100 times will increase its range by 10, whereas the same upgrade made to a gravitational wave detector will have its range increase 100 times. [4, 5]

Gravitational wave detectors have been prototyped since the 1970's by famous scientists such as Joseph Weber and Rainer Weiss [6]. The most successful detectors today belong to the LIGO Scientific Collaboration and employ laser interferometers to detect gravitational waves that pass through the earth by clever measurements of the strain induced on two perpendicular long laser beams. With these detectors, and upgrades being made to their sensitivity and noise reduction, gravitational waves are being detected more consistently than ever before.

The current goal for all of these gravitational wave detectors is to improve the quality of signals that we detect. This can be accomplished, broadly speaking, in two major ways: we can either improve the quality and precision of our mirrors and detector optics inside the interferometer, or we can develop better control systems and noise reduction systems that can actively deal with the background noise.

The background noise associated with gravitational waves detected by these laser interferometer detectors is a thorough blend of many different unrelated noise sources. Some of these noise sources are: *seismic noise*, environmental noise like cars or earthquakes, *thermal noise*, microscopic fluctuations of the individual atoms of the detector, *shot noise*, the quantum effect of the discrete nature of photons at the detector's photodetectors limiting their accuracy, and *laser noise*, which is noise originating from the variations in the laser's intensity and frequency.

2 Objectives

This project seeks to understand the noise coupling mechanisms that allow various noise sources to "leak" into the gravitational wave signals that are being detected, in order to develop statistically rigorous methods of determining which interferometric parameters influence these coupling mechanisms, and how they do so. The benefit of doing this is clear. If we can understand how the noises couple to our detected gravitational wave signals, then work can be done to improve our control systems to counteract these coupling mechanisms.

In preparation for this project, a simulation environment was developed in Python to analytically determine the reflectivity of a Fabry-Perot cavity based on the reflectivity and transmissivity of the cavity mirrors. The simulation was run by holding the reflectivity of one mirror in the cavity constant, and then modifying the other, and recording how the reflectivity of the cavity changes as a function of the frequency of laser used [7].

$$r_{cav}(\omega) = -r_i + \frac{t_i^2 r_e e^{-i\frac{2L\omega}{c}}}{1 - r_i r_e e^{-i\frac{2L\omega}{c}}}$$

In this equation, r_{cav} is a complex number that has the cavity amplitude and phase encoded. $|r_{cav}|$ is equal to the reflected cavity amplitude, and the angle θ of r_{cav} is the phase of the reflected electric field. r_i and t_i are the reflection and transmission coefficients of the input mirror, r_e and t_e are the reflection and transmission coefficients of the end mirror, L is the length of the cavity (3.7 cm), and c is the speed of light.

$$r_{cav}(\omega) = \frac{-r_i + r_e e^{-i\frac{L\omega}{c}}}{1 - r_i r_e e^{-i\frac{L\omega}{c}}}$$

In this simulation, cavity losses were neglected so $t_e^2 + r_e^2 = 1$ and $t_i^2 + r_i^2 = 1$ was assumed to produce the simplified equation above. The results of the simulation are shown below.

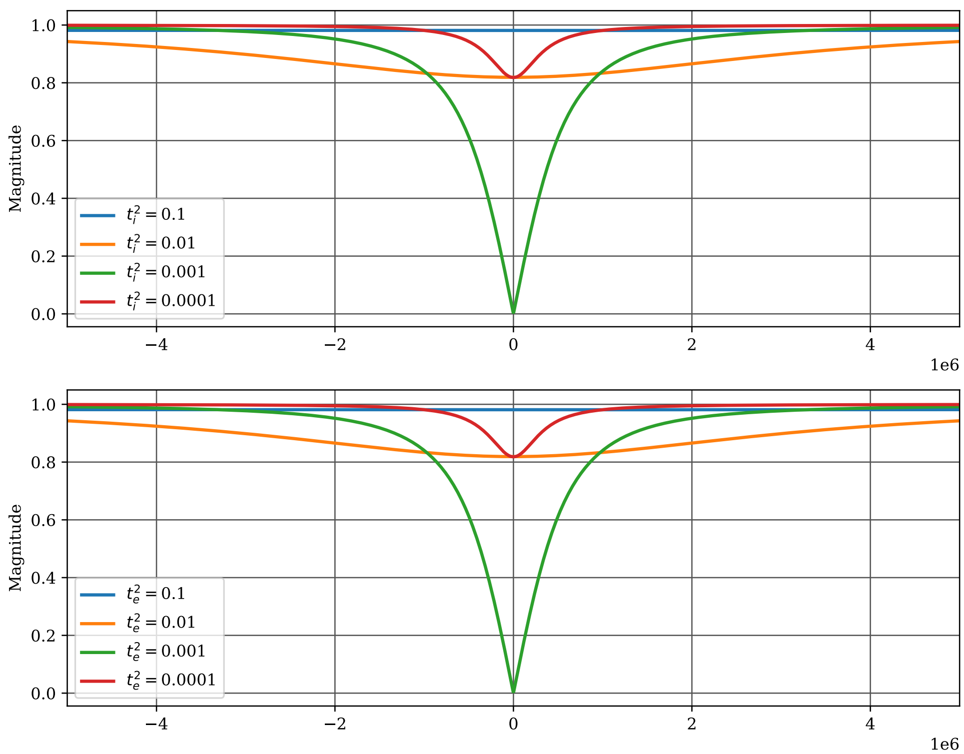

Figure 1: The first plot holds the input mirror transmissivity at 0.1% and modifies the end mirror's transmissivity and shows the results for values of 10%, 1%, 0.1%, and 0.01%. The second plot does the reverse, and holds the end mirror transmissivity at 0.1% and modifies the input mirror for the same values. The plots are identical due to cavity losses being ignored.

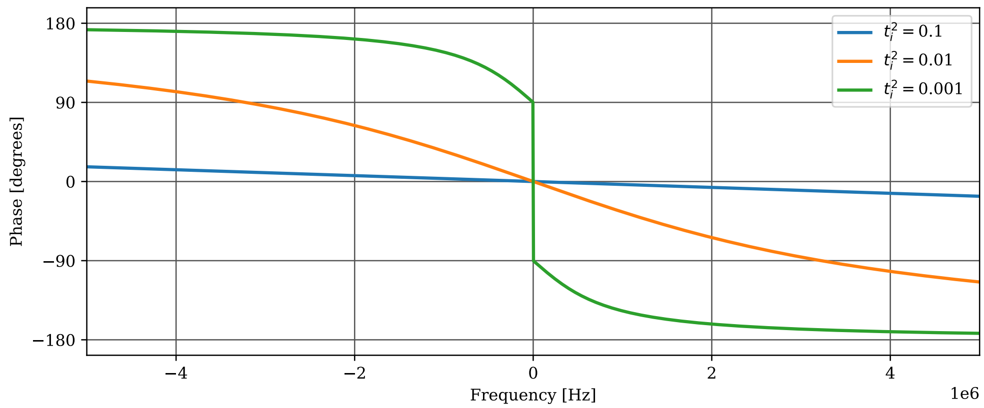

Figure 2: This is a plot showing the phase of the Fabry-Perot cavity as a function of the frequency, where $t_e = 0.001$. When $t_i = t_e$ for the green curve, there is a 180 degree phase change, indicating that the cavity is critically coupled. The other curves are undercoupled, i.e. $t_i > t_e$.

3 Approach

This project will begin by utilizing a sophisticated interferometer simulation software known as Finesse analyze more accurately the reflectivity behavior of a Fabry-Perot cavity.

Then, real data will be gathered at an interferometer located at the Coatings Thermal Noise Lab at Caltech. This data will be then analyzed using Bayesian inference techniques to correct and adjust the Finesse simulation for a more realistic simulation of a cavity.

After that, a thorough statistical analysis of the various noise couplings will be conducted on several different cavity configurations. This will lead to a better understanding of the mechanisms in which noise sources couple to gravitational wave signals.

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