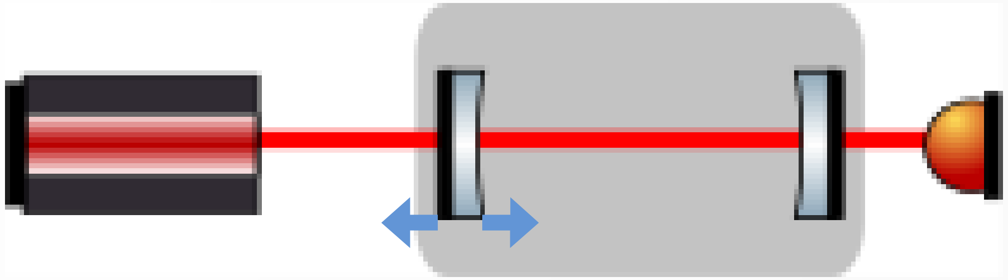
Optical Cavity Inference Techniques for Low Noise Interferometry

Jorge L. Ramirez Ortiz

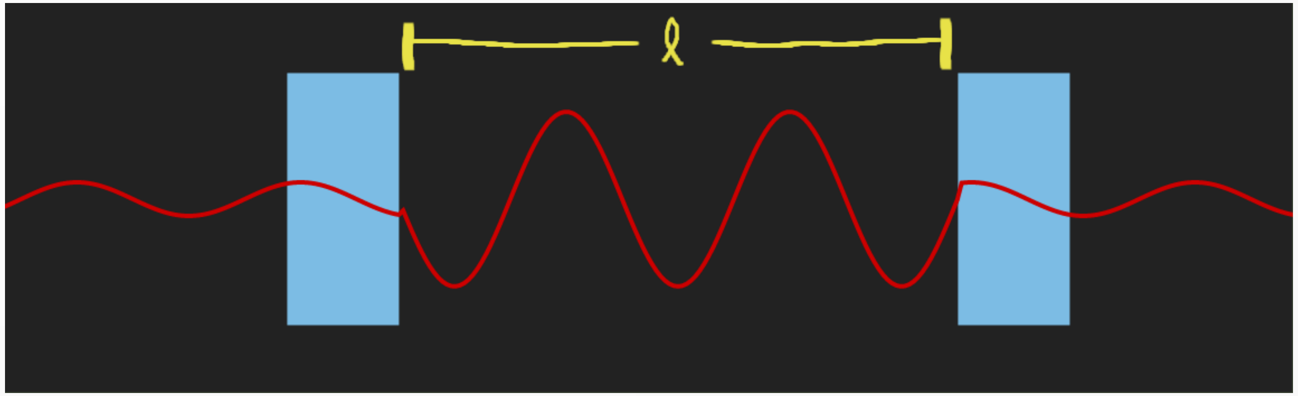
Mentored by: Craig Cahillane, Anchal Gupta, Rana Adhikari

What is a Fabry-Pérot Cavity?

A Fabry-Pérot cavity is a type of cavity that consists of two planar mirrors separated by an adjustable distance ℓ.



If the mirrors are separated by an integer number of laser wavelengths, the laser will constructively interfere inside the optical cavity and no light will be reflected.



Fabry-Perot Cavity Simulator

Fabry-Pérot Parameters

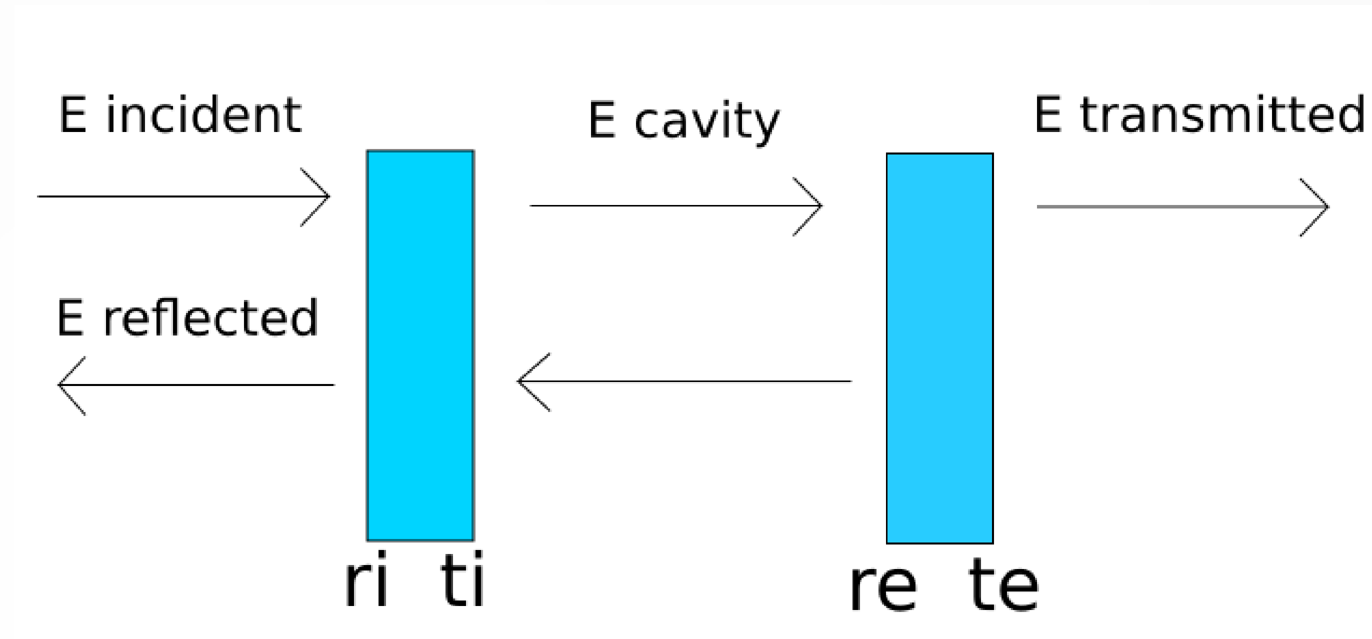
Parameter	Sym	Description	Values
Cavity Length	ℓ	Space between mirrors	~3.68 cm
Cavity Poles	f_{pole}	Frequencies at which cavity is resonant [Hz]	$f_{pole} = \frac{f_{FSR}}{2\pi} \ln(\frac{1}{r_i r_e})$
Free Spectral Range	f_{FSR}	Distance between cavity poles [Hz]	$f_{FSR} = \frac{c}{2L}$
Finesse	$\mathfrak{F}$	Gives an estimate of cavity sensitivity	$\mathcal{F} = \frac{\pi\sqrt{R}}{(1-R)^2}$
Reflectivity & Transmissivity	r t	Intrinsic properties of the mirror coatings and mirror material	Advanced LIGO $T_{\text{input}} = 0.000004$ $T_{\text{end}} = 0.14$

Fabry-Pérot Cavity Equations

$$E_{tran} = E_{inc} \frac{t_i t_e e^{2i\phi}}{1 - r_i r_e e^{2i\phi}}$$

$$E_{cav} = E_{inc} \frac{t_i}{1 - r_i r_e e^{2i\phi}}$$

$$E_{refl} = E_{inc} \left(r_i - \frac{t_i^2 r_e e^{2i\phi}}{1 - r_i r_e e^{2i\phi}}\right)$$



Why use a Fabry-Pérot Cavity at LIGO?

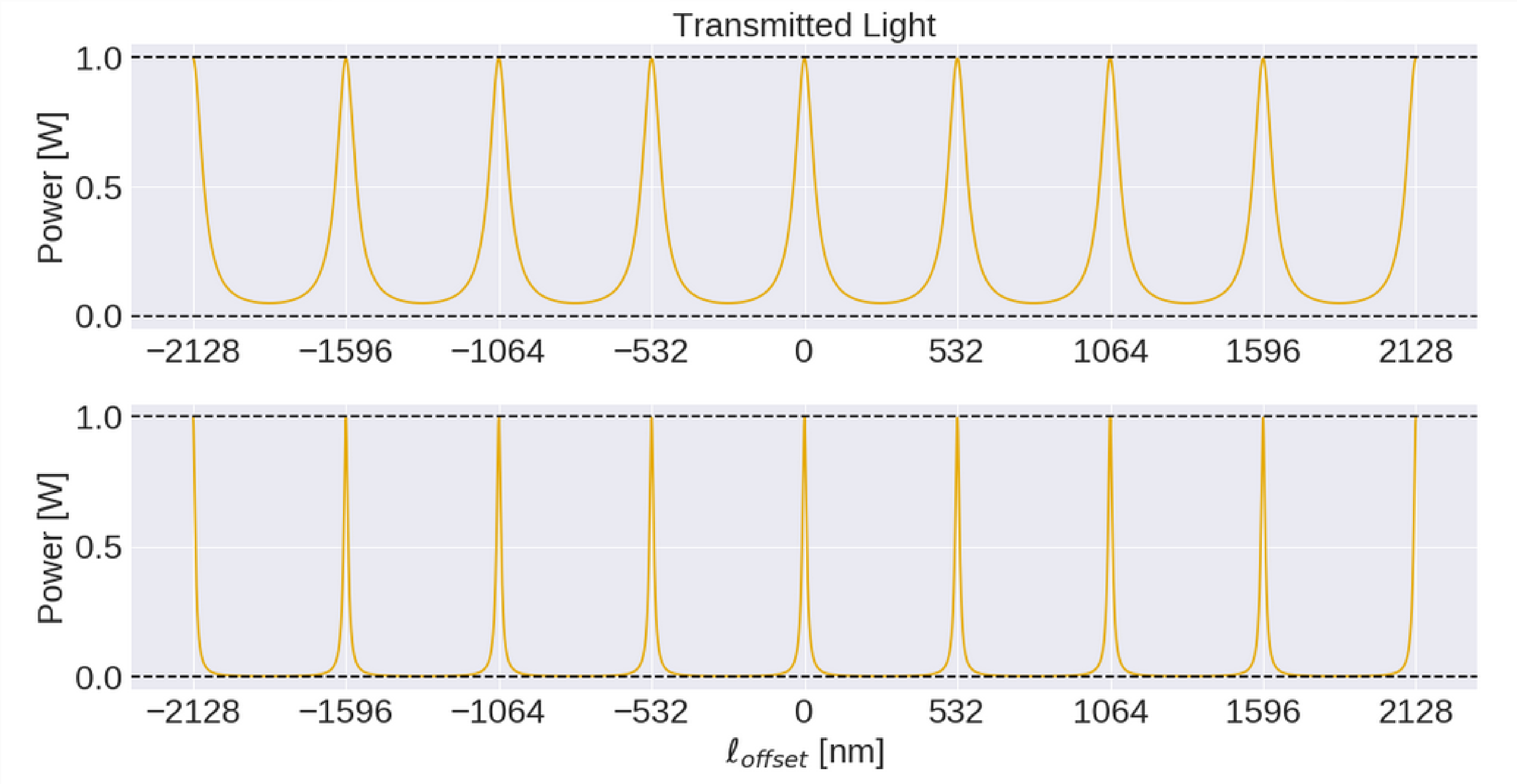
A Fabry-Pérot is extremely sensitive to microscopic shifts in length- sound familiar?

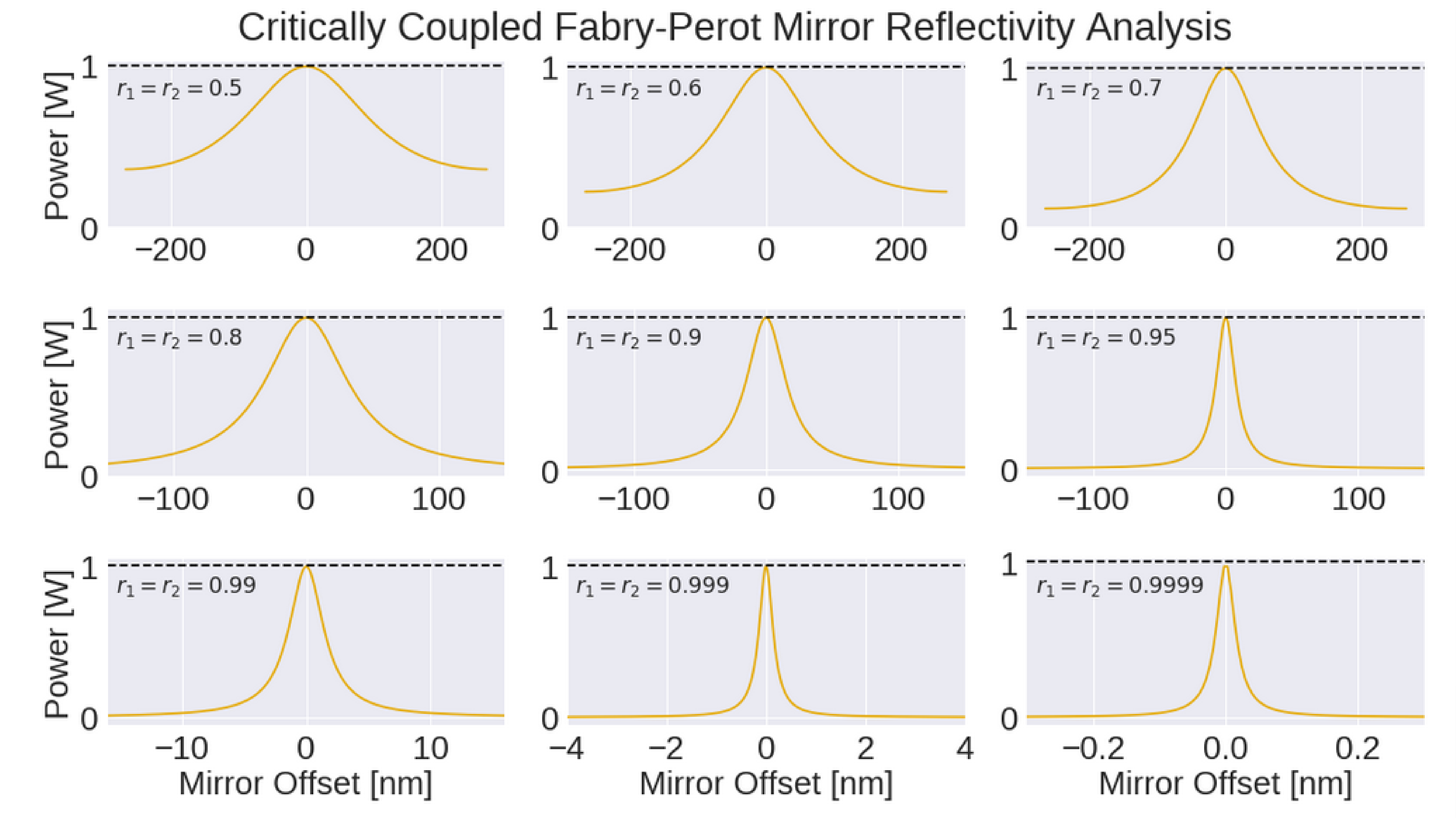
$R = 0.8$

$\mathcal{F} = 14$

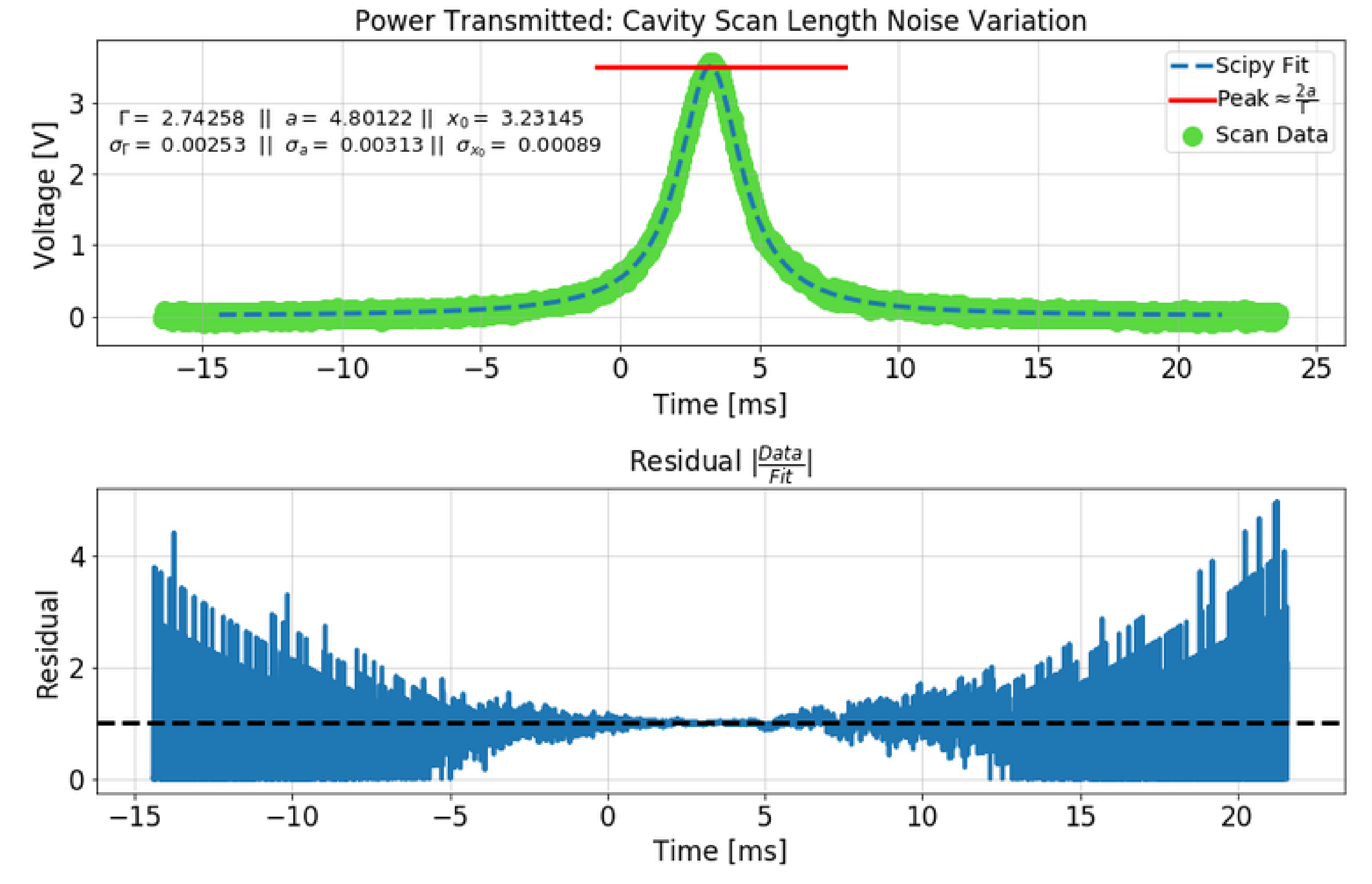
$R = 0.95$

$\mathcal{F} = 312$




Critically Coupled Fabry-Perot Mirror Reflectivity Analysis
$r_1 = r_2 = 0.5$
$r_1 = r_2 = 0.6$
$r_1 = r_2 = 0.7$
$r_1 = r_2 = 0.8$
$r_1 = r_2 = 0.9$
$r_1 = r_2 = 0.95$
$r_1 = r_2 = 0.99$
$r_1 = r_2 = 0.999$
$r_1 = r_2 = 0.9999$
Power [W]
Mirror Offset [nm]

Real Cavity Scan Fit



What are some noise sources?

Utilizing the Fabry-Pérot in a LIGO detector for its extreme sensitivity to length changes means we must isolate the power signal from the cavity from outside influences.

Seismic Noise – vibrations in the ground supporting the instrument

Thermal Noise – thermal “wiggling” of the matter composing the mirrors

Laser Noise – “dirty” laser beams whose purported frequency can wander

Shot Noise – poissonic nature of counting photons leading to discrepancies

Length Noise – “catch-all” for unwanted Gaussian noise in movement of mirrors during a sweep

Laser Noise:
Pound-Drever-Hall Technique

PDH uses the laser to regulate its own frequency by forming a negative feedback control loop

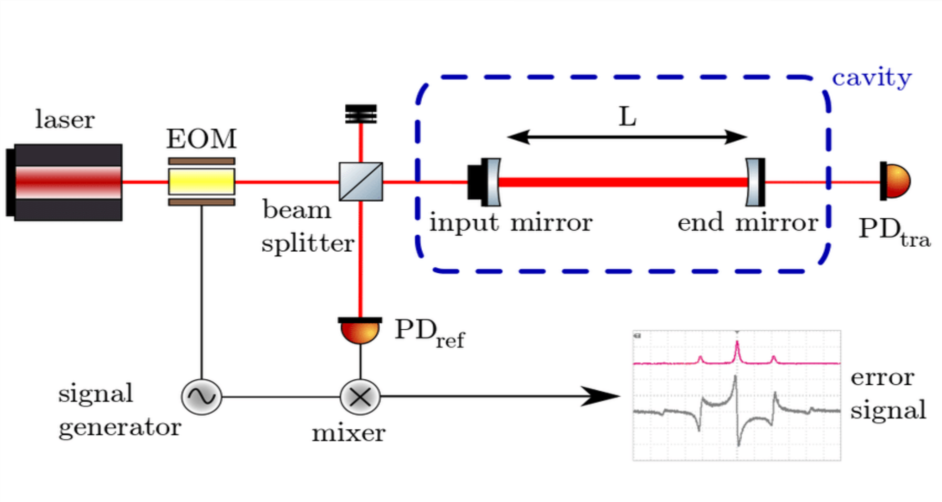
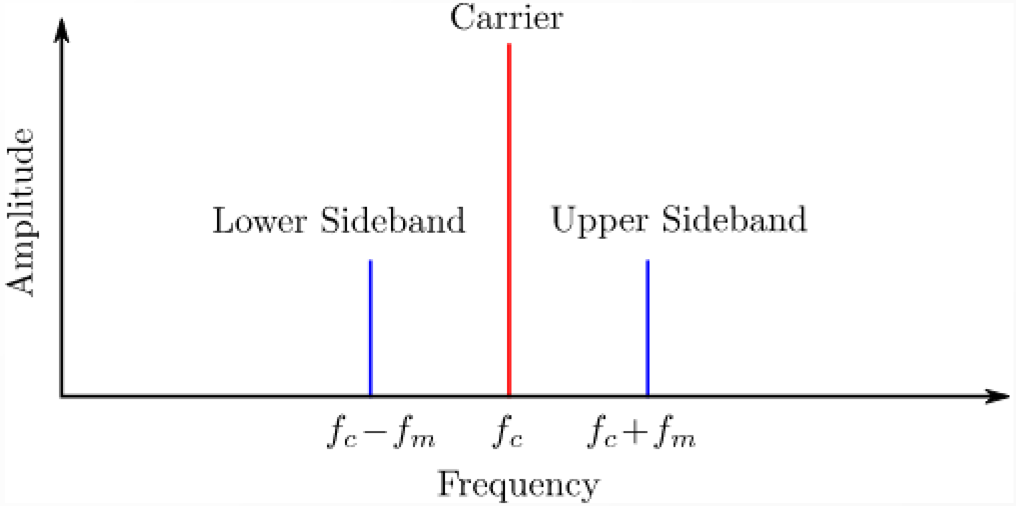
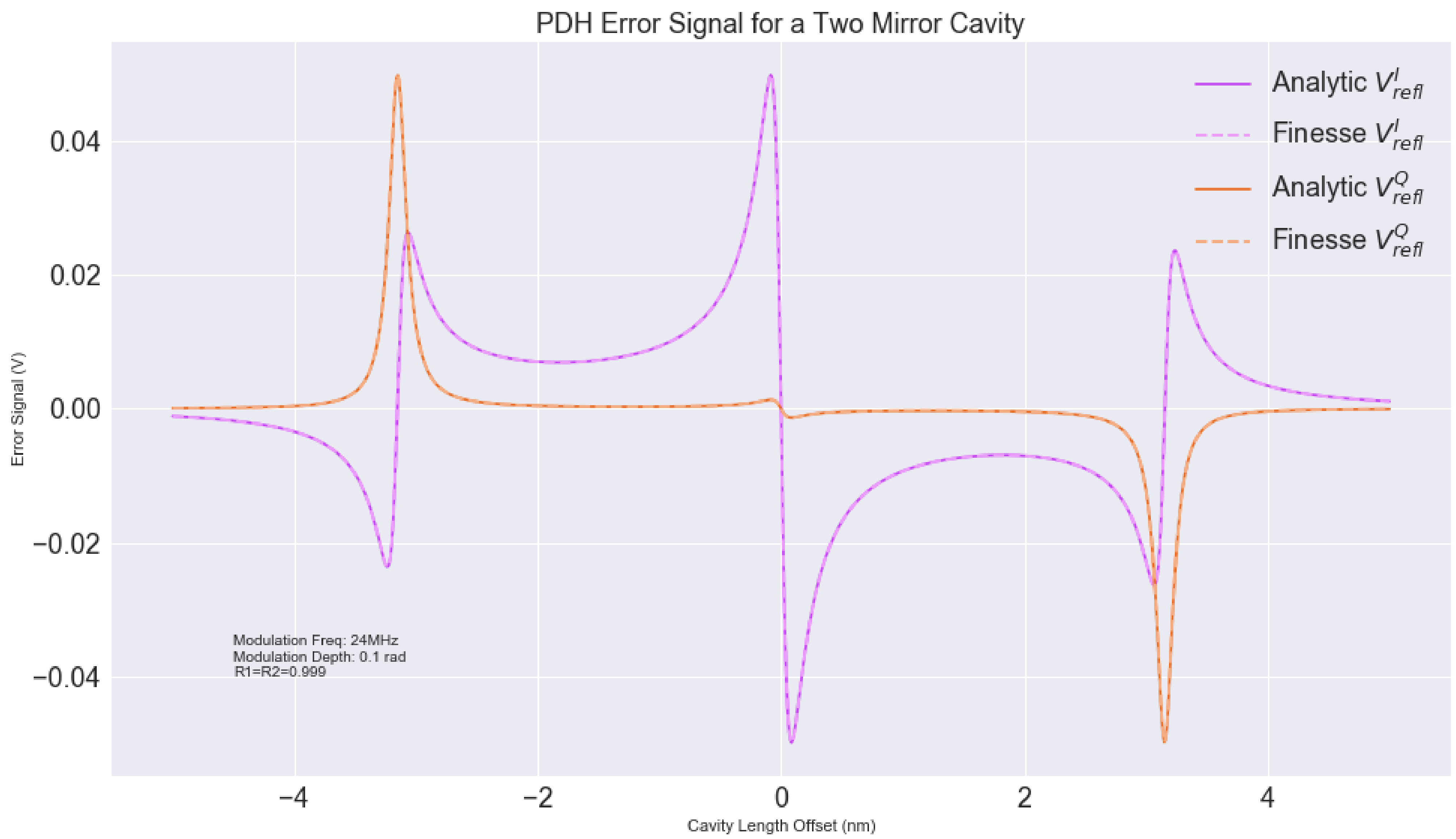


Image Source: Lock Acquisition & Commissioning of the Advanced Virgo detector

Phase modulate the main laser beam, use the newly-created sidebands as reference points for how close you are to resonance





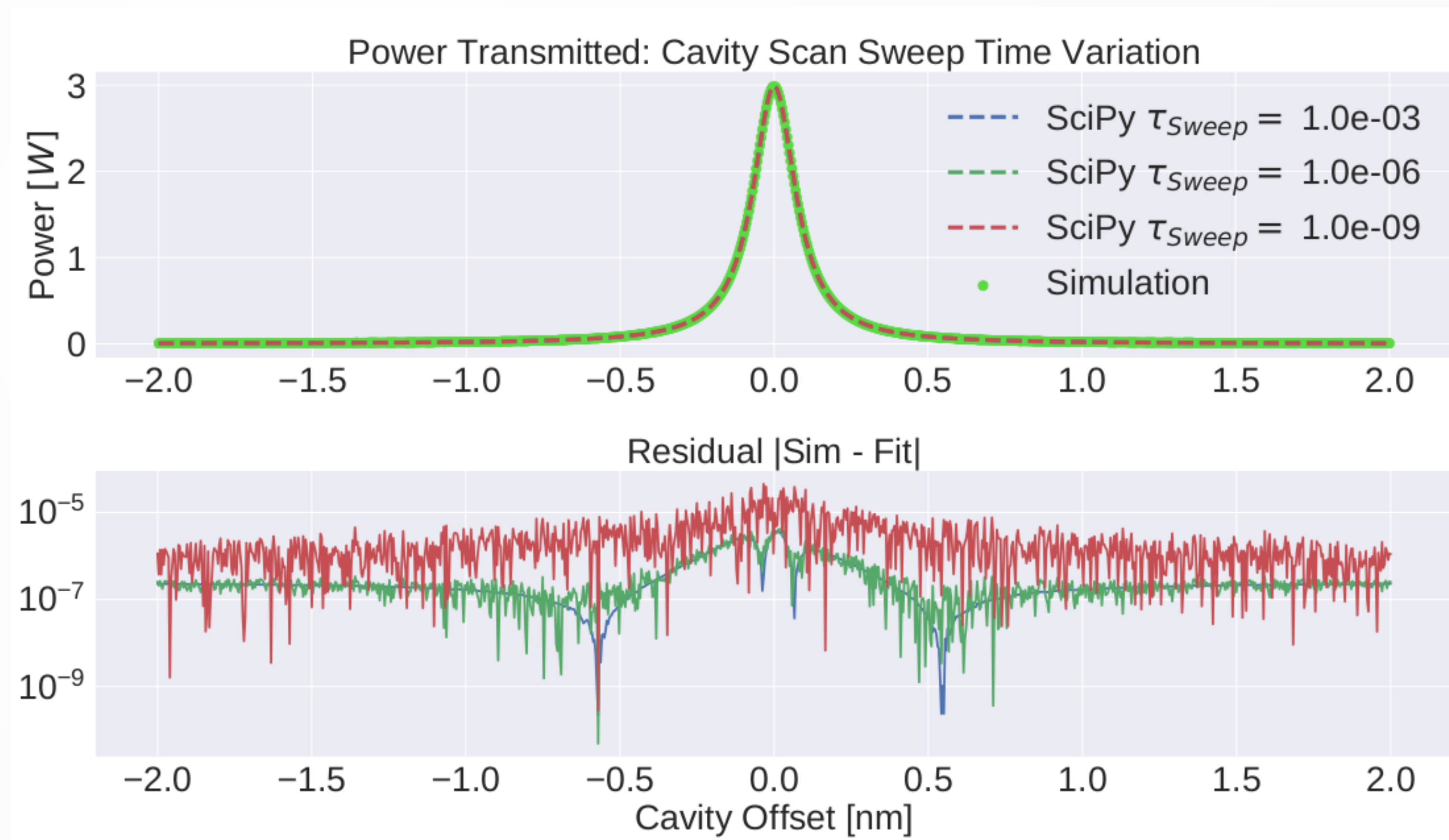
PDH Error Signal for a Two Mirror Cavity
Analytic V^{I}_{refl}
Finesse V^{I}_{refl}
Analytic V^{Q}_{refl}
Finesse V^{Q}_{refl}
0.04
0.02
0.00
−0.02
−0.04
Error Signal (V)
Modulation Freq: 24MHz
Modulation Depth: 0.1 rad
R1=R2=0.999
−4
−2
0
2
4
Cavity Length Offset (nm)

Shot Noise

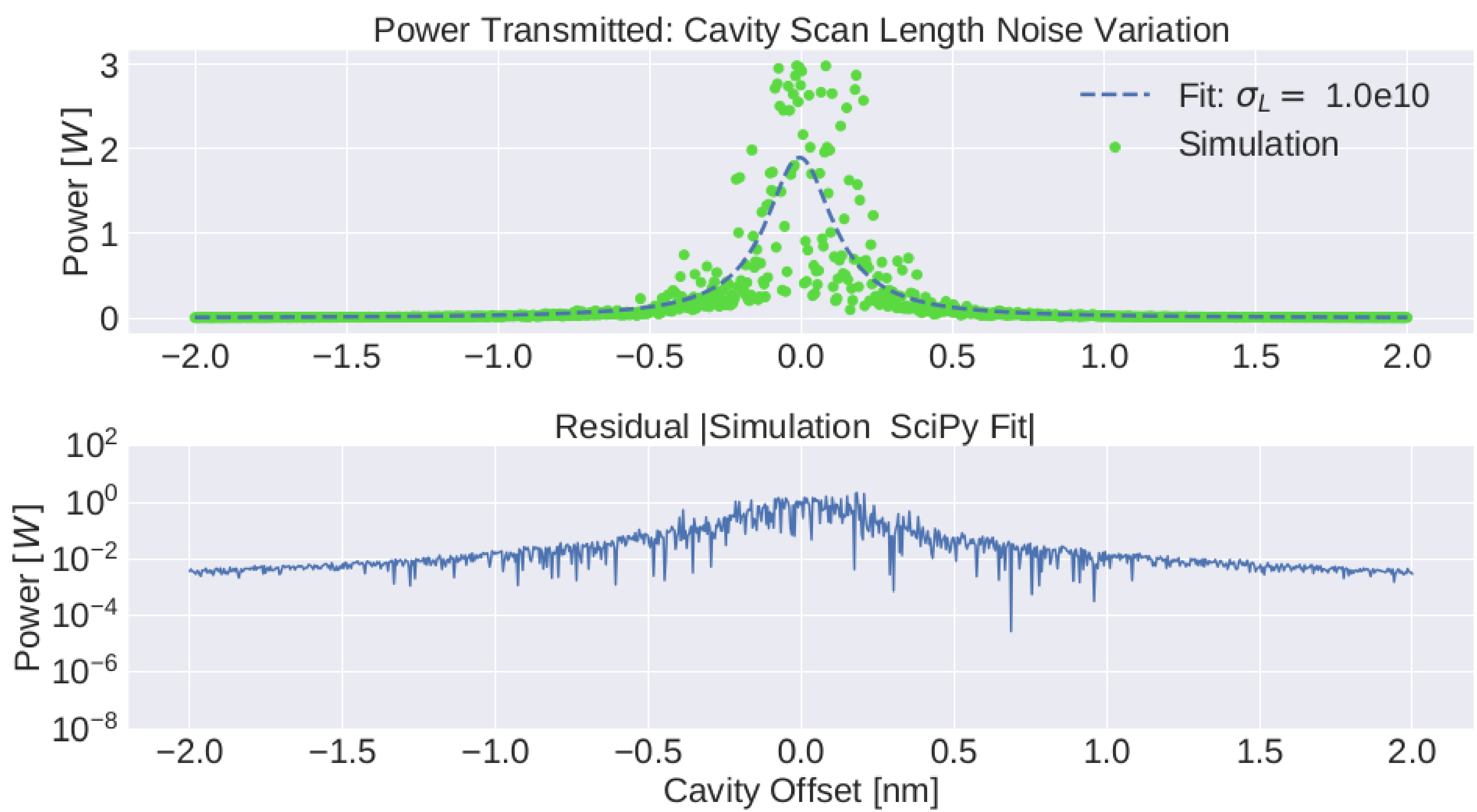
Simulating a cavity scan with a 100MHz sampling rate, three sweeps at 1ms, 1µs, and 1ns intervals are shown.

Slower sweep leads to more counts, but introduce their own technical problems

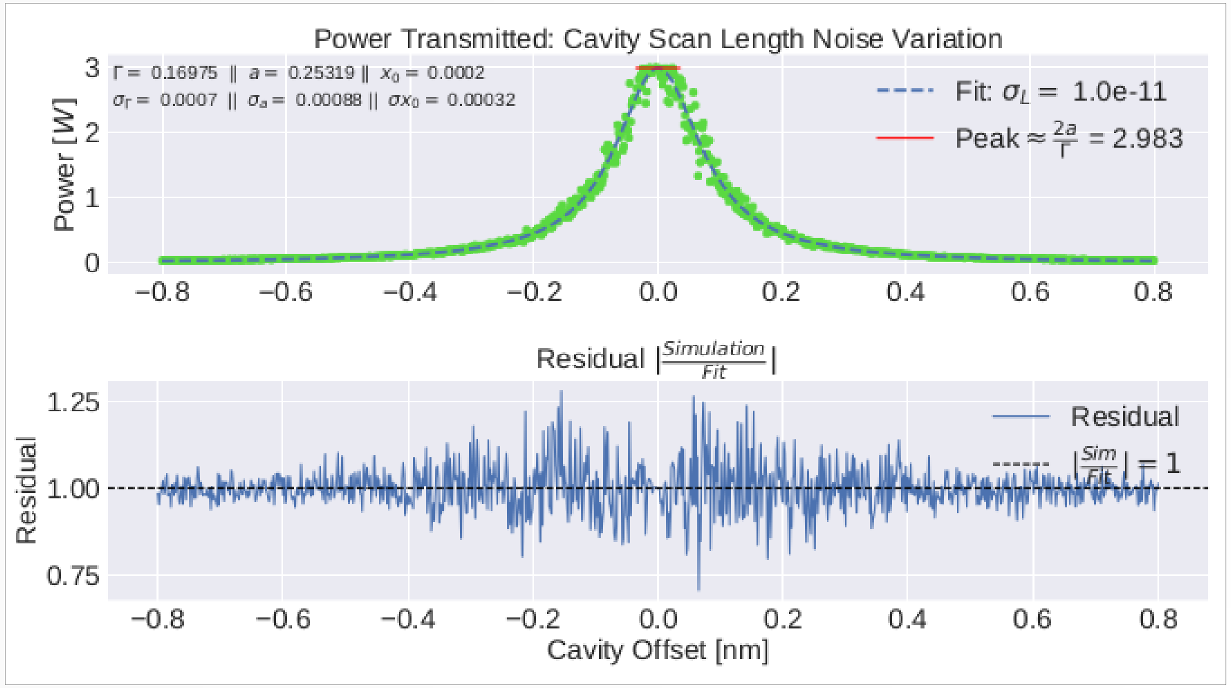
$$\sigma_{shot} = \sqrt{\hbar \omega P \tau_s}$$

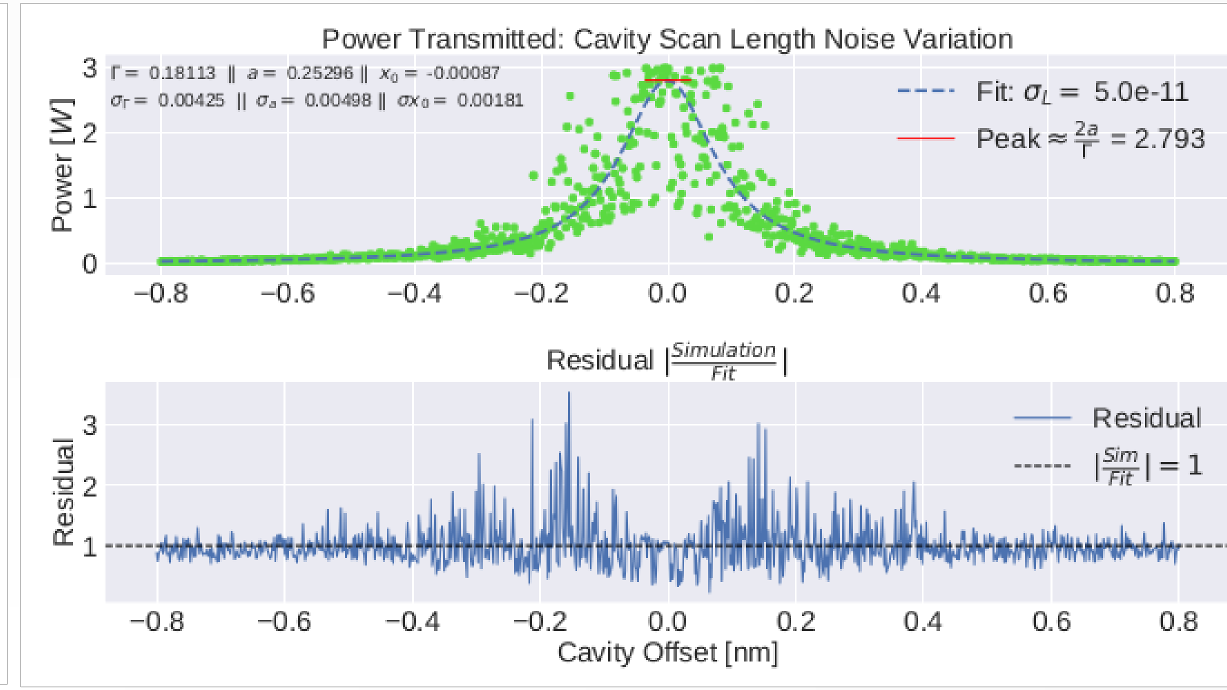


Length Noise



Servo controlling ℓ isn't perfect, this creates an uncertainty in the x-value of each data point. Above is fit with no weights





Propagation of Errors for Length Noise

$$E_{tran} = \frac{[TR - T\cos(2\phi)] - [T\sin(2\phi)]i}{1 - 2R\cos(2\phi) + R^2}$$ where $T = t_i t_e$ and $R = r_i r_e$

$$P_{tran} = E_{tran}(E^*_{tran}) = \frac{T^2}{(1-R)^2 + 4R\phi^2}$$ using $\cos(2\phi) = 1 - 2\phi^2$

$$\sigma_P = \sigma_L \frac{\partial \phi}{\partial L} \frac{\partial P}{\partial \phi}$$

$$\sigma_P = \sigma_L \frac{2\omega}{c} \frac{-8RT^2\phi}{((1-R)^2 + 4R\phi^2)^2}$$

Lesson learned:

when fitting, assigning and weighing errors is critical!

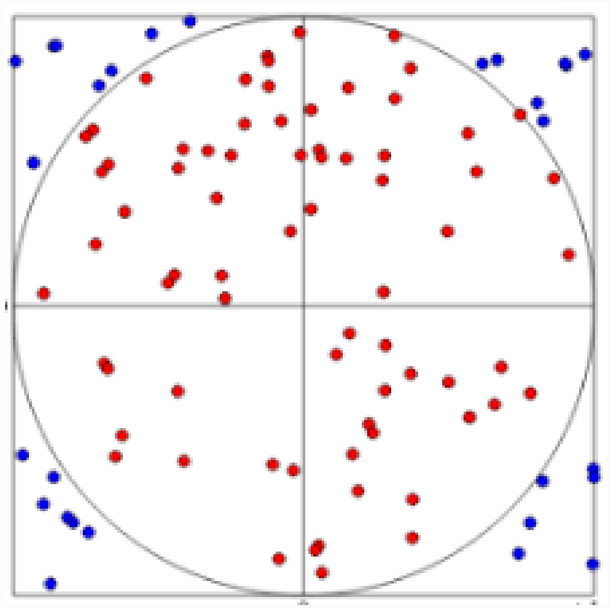
```
--------------------------------------------------------------------
Fit Parameters (gamma, a, x_0) for noise 8e-11 :
 [ 0.192  0.26  -0.005]
St. Dev:
 [0.008, 0.009, 0.003]

gamma: 2.921 deviations || a: 0.798 deviations  || x0: -1.43 deviations
--------------------------------------------------------------------
--------------------------------------------------------------------
Fit Parameters (gamma, a, x_0) for noise 1e-10 :
 [ 0.202  0.266 -0.006]
St. Dev:
 [0.011, 0.012, 0.004]

gamma: 2.976 deviations || a: 1.095 deviations  || x0: -1.452 deviations
--------------------------------------------------------------------
```

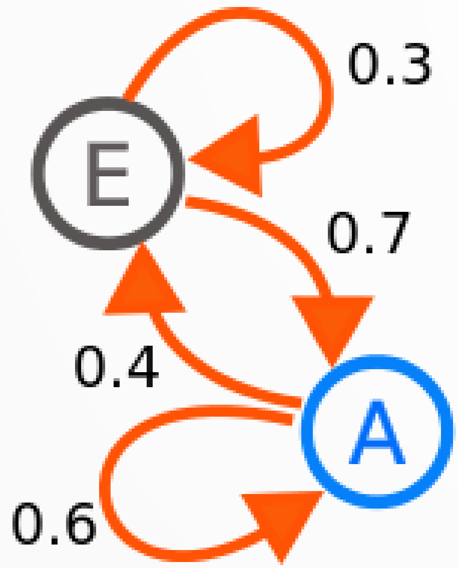
$$\mathcal{L} = a * \frac{\frac{1}{2}\Gamma}{(x - x_0)^2 + \left(\frac{1}{2}\Gamma\right)^2}$$

Still, even after carefully propagating error and including it in the least squares fitting algorithm, estimates for cavity parameters weren’t good enough!

Markov Chain Monte Carlo (MCMC)

<ins>Monte Carlo Method</ins> – utilizing random sampling to generate sets of non-random data by relying on the *Law of Large Numbers* (for a large amount of trials, results will tend towards the mean)





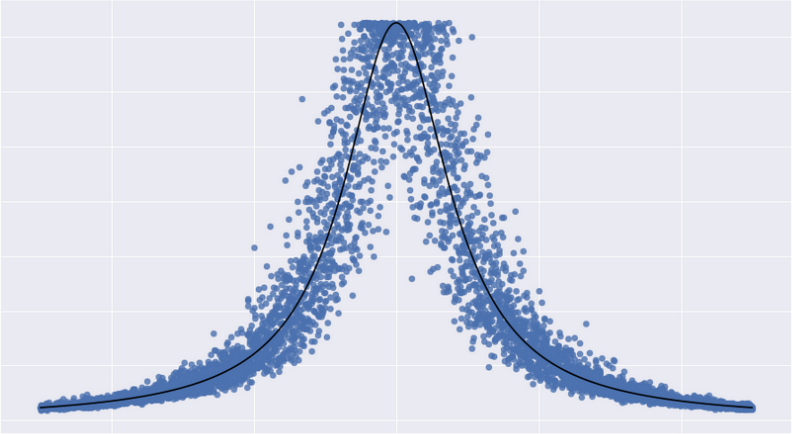
<ins>Markov Chain</ins> – a chain of events where the probability of future events is uncorrelated with past events

(1) Define a model and generate noisy data

$$E_{tran} = \frac{-t_i t_e e^{2i\phi}}{1 - r_i r_e e^{2i\phi}}$$

$$P_{tran} = E_{tran}(E^*_{tran})$$

$$T = t_i t_e$$
$$R = r_i r_e$$



(2) Define a parameter space with boundaries which will form the basis for the Markov chain

$$\theta_0 = \begin{cases} p_1 \\ p_2 \\ \dots \\ p_n \end{cases} = \begin{cases} T \\ R \\ P \\ \lambda \\ \gamma \end{cases} \in \begin{cases} 0 < T < 1 \\ 0 < R < 1 \\ 2.5 < P < 3.5 \\ 1060 < \lambda < 1070 \\ 0.18 < \gamma < 0.22 \end{cases}$$

$$T = t_i t_e$$
$$R = r_i r_e$$

(3) Define a likelihood that nudges each "walker" in the markov-chain in the correct direction

$$\text{Likelihood} = -\frac{1}{2}\,\Sigma(\text{data} - \text{model}(\theta_0))^2$$

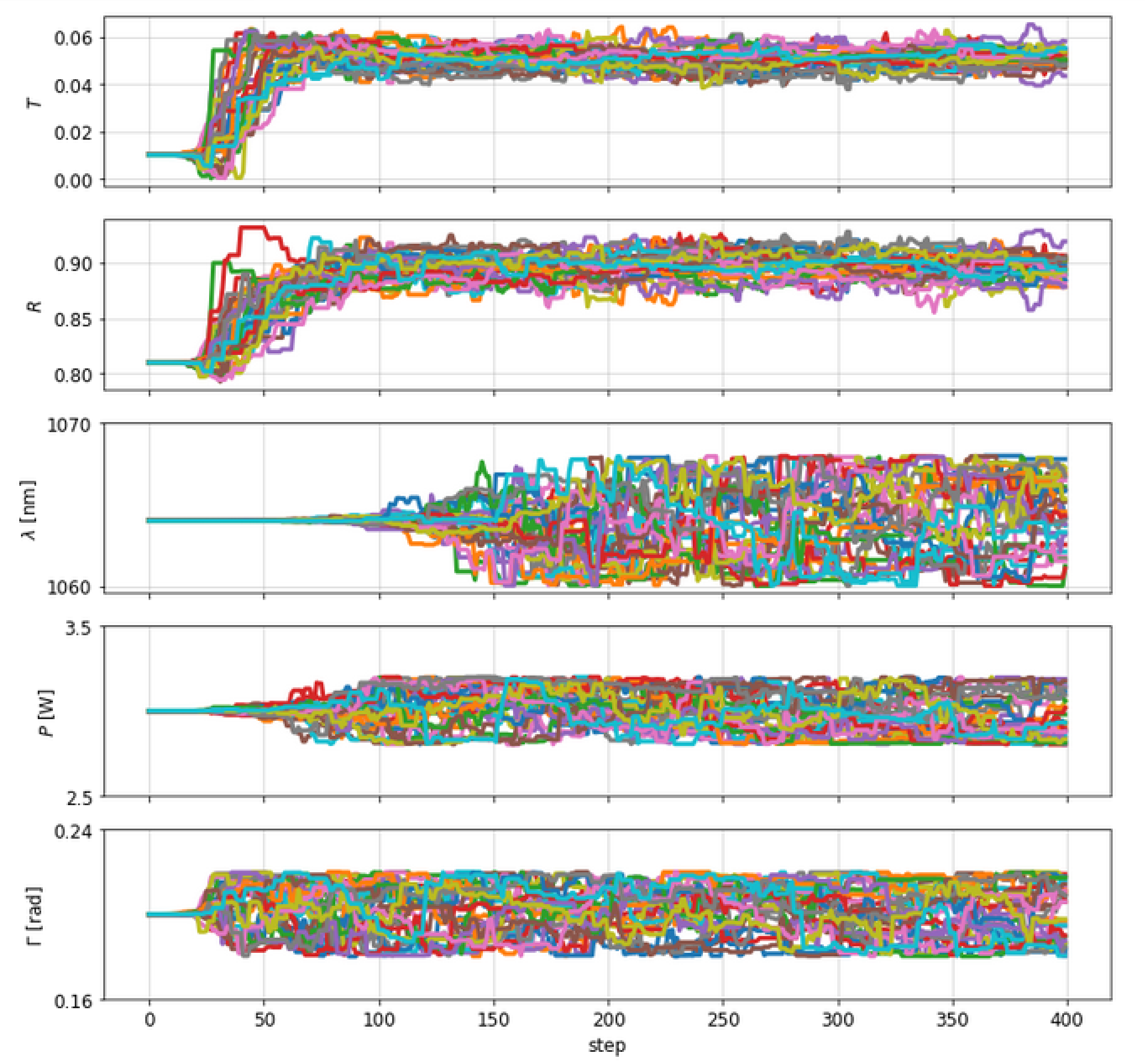
(4) Set MCMC parameters, set initial guesses, and define the size of each step:

```
#~~~ MCMC settin
ndim      = 5 # n
nwalkers = 1000 
nsteps    = 600 #
nburn     = 100 #
nthreads = 10 # 
```

```
#~~~ tell each walker where to start
# theta =[T, R, laser wavelength, laser_power, mod_depth
Tguess, Rguess, LWguess, LPguess, MDguess = (0.1**2), (0.9**2), 1064, 3, 0.2
initial = np.array([Tguess, Rguess, LWguess, LPguess, MDguess])

p0 = [np.array(initial) + 1e-6 * np.random.randn(ndim) for i in range(nwalkers)]
```

(5) Let them loose! Walkers will start at their initial guess, change it by a little bit every step, and then move towards the parameters with highest likelihood (and turning around if they ever step out of bounds)

Results

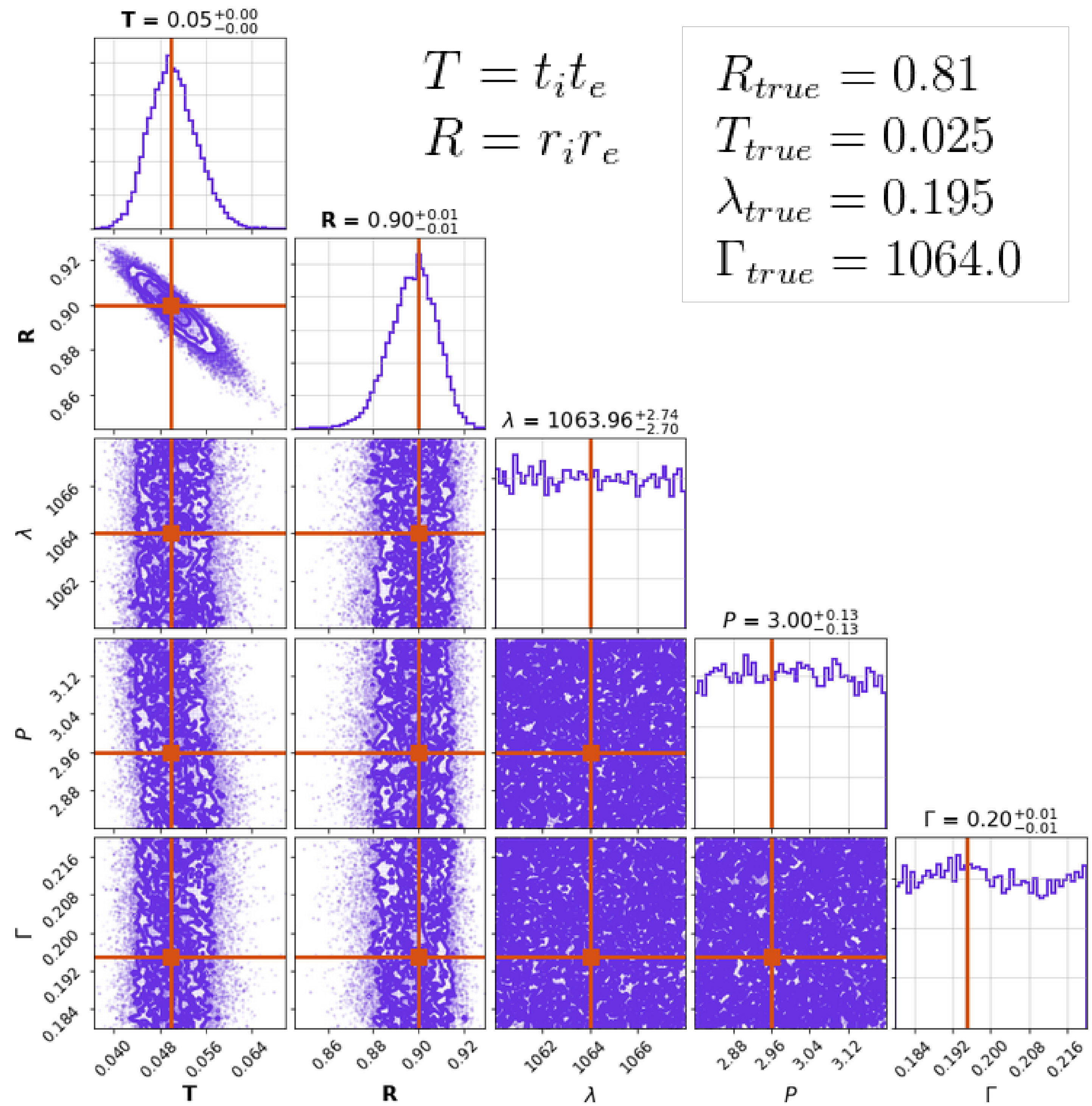


- T and R converge relatively quickly and nicely
- Laser Wavelength has a lot of variations, but most cluster around 1064
- Laser Power and Modulation Depth do not converge

- T vs R scatter plot shows a negative covariance

- Not obvious from previous plot, but wavelength converged around 1063.9

- Laser Power and Modulation Depth do not converge



Future Work

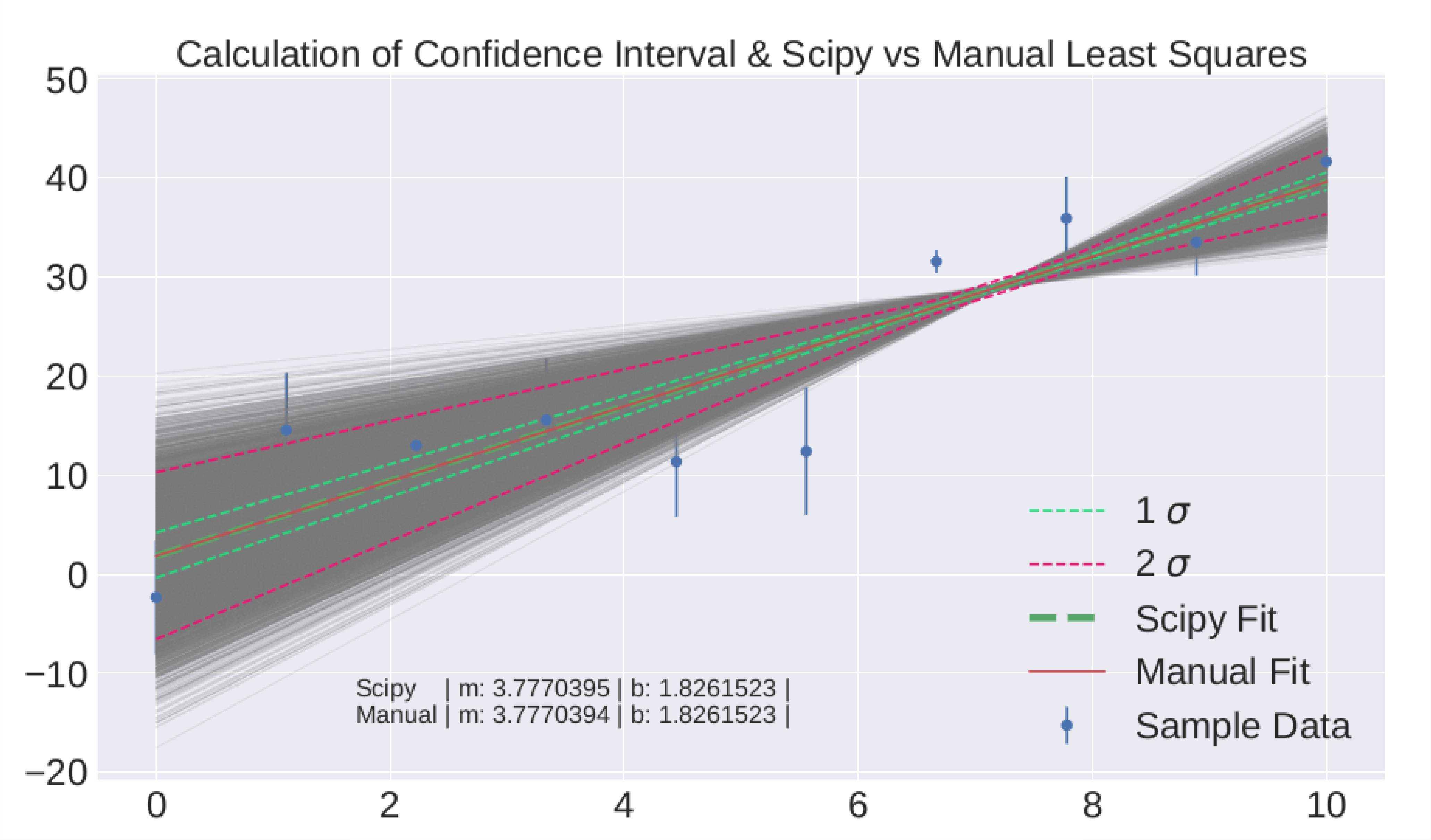
- Improving the MCMC algorithm by introducing a prior distribution that assigns probabilities to parameters
- Support for adjusting which parameters of the cavity we're trying to estimate
- Adding estimated uncertainties to likelihood function to improve the efficiency each step

- I’d like to thank Rana for his mentorship and guidance, as well as helping me realize that science isn’t only about getting the right answer, but asking the right questions

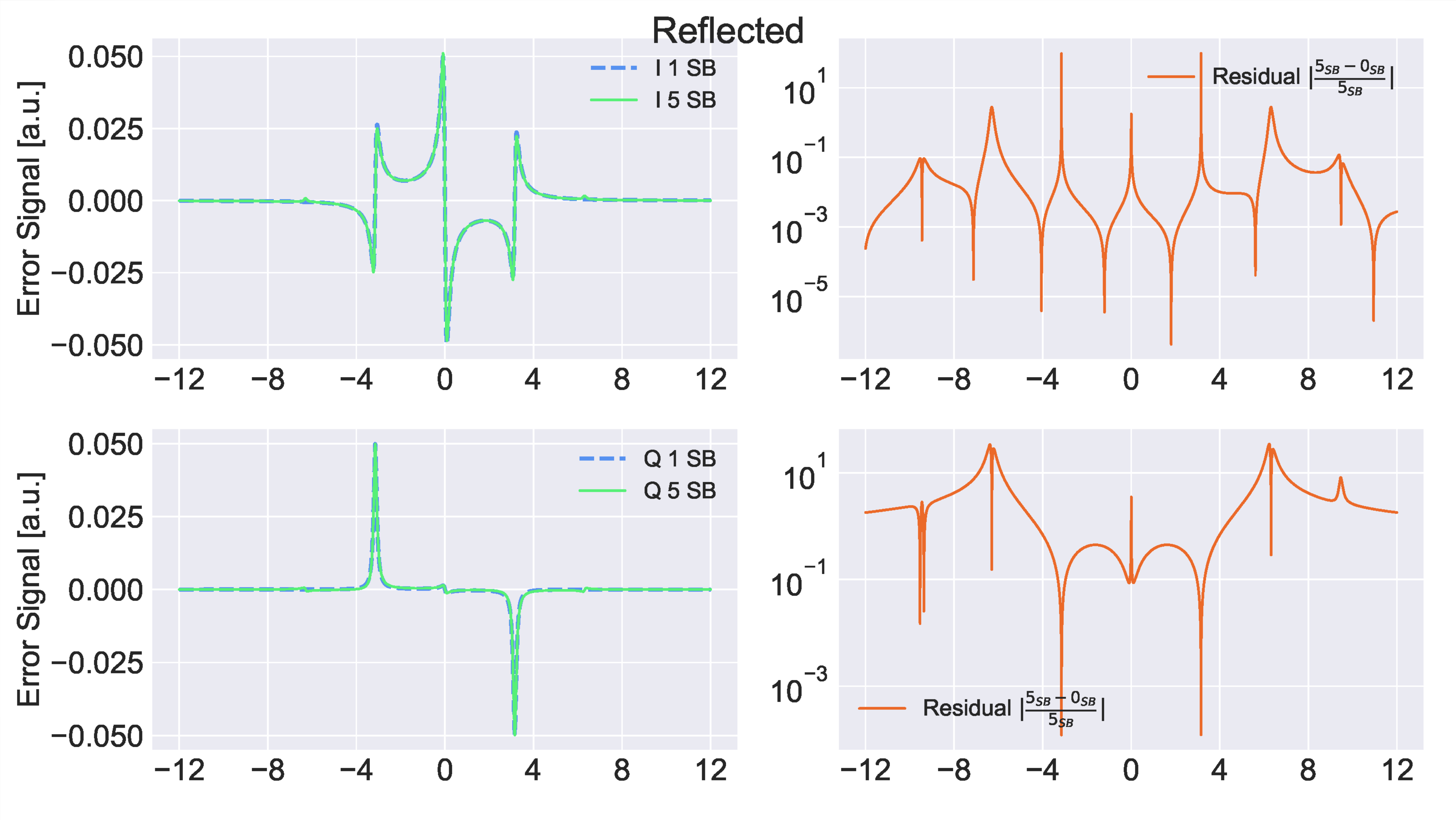
- Craig for all his advice, lessons, and for supporting me whenever I felt like I was in way over my head with this project.

Thank you!

Extra Slides



Calculation of Confidence Interval & Scipy vs Manual Least Squares
50
40
30
20
10
0
−10
−20
0
2
4
6
8
10
Scipy | m: 3.7770395 | b: 1.8261523 |
Manual | m: 3.7770394 | b: 1.8261523 |
1 σ
2 σ
Scipy Fit
Manual Fit
Sample Data


Reflected
Error Signal [a.u.]
0.050
0.025
0.000
−0.025
−0.050
I 1 SB
I 5 SB
Q 1 SB
Q 5 SB
−12
−8
−4
0
4
8
12
Residual $|\frac{5_{SB} - 0_{SB}}{5_{SB}}|$
10^{1}
10^{-1}
10^{-3}
10^{-5}

```
residual min:  0.9975015620659504
residual max:  0.9986615326479165
```